

MENG366 SYSTEM DYNAMICS AND CONTROL

LABORATORY 7: System Frequency of First Order System

1. Introduction:

First order systems are, by definition, systems whose input-output relationship is a first order differential equation. A first order differential equation contains a first order derivative but no derivative higher than first order – the order of a differential equation is the order of the highest order derivative present in the equation.

First order systems contain a single energy storage element. In general, the order of the input-output differential equation will be the same as the number of independent energy storage elements in the system. Independent energy storage cannot be combined with other energy storage elements to form a single equivalent energy storage element. For example, we previously learned that two capacitors in parallel can be modeled as a single equivalent capacitor – therefore, a parallel combination of two capacitors forms a single independent energy storage element.

First order systems are an extremely important class of systems. Many practical systems are first order; for example, the mass-damper system and the mass heating system are both first order systems. Higher order systems can often be approximated as first order systems to a reasonable degree of accuracy if they have a dominant first order mode.

2. First Order System Model

The first order system has only one pole as shown

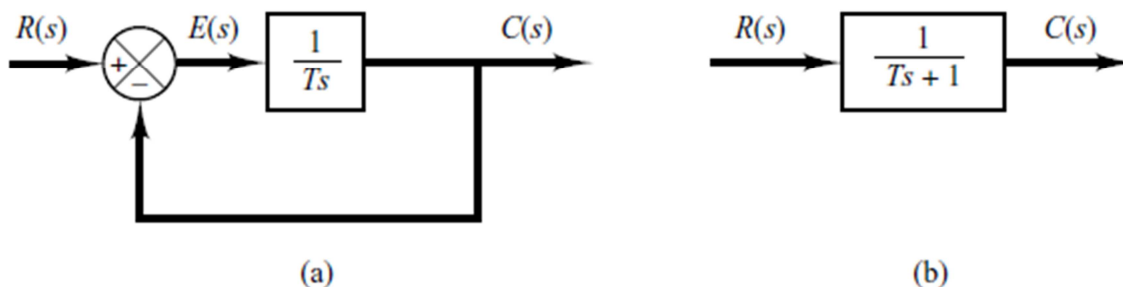


Figure 1: (a) Block Diagram of a first-order system; (b) Simplified block Diagram

$$\frac{C(s)}{R(s)} = K \frac{1}{Ts + 1} \quad (1)$$

- Where K is the DC Gain and T is the time constant of the system.
- Time Constant is a measure of how quickly a 1st order system response to a unit step input.
- DC gain of the system ration between the input signal and the steady state value of output.

Example 1:

For the first order system given below

$$G(s) = \frac{10}{3s + 1}$$

- DC gain (K) is equal 10
- Time Constant (T) is equal 3

Example 2:

For the first order system given below

$$G(s) = \frac{3}{s + 5} = \frac{\frac{3}{5}}{\frac{1}{5}s + 1}$$

- DC gain (K) is equal $\frac{3}{5}$
- Time Constant (T) is equal $\frac{1}{5}$

2.1 Impulse Response of 1st Order System

Consider the following 1st order system in figure 2

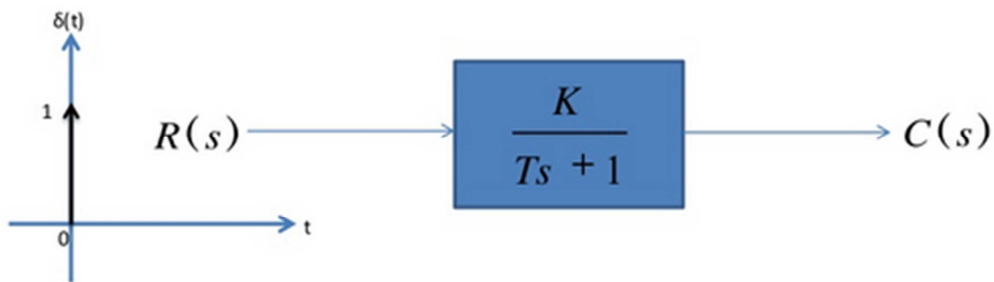


Figure 2: Impulse response of first order system

$$R(s) = \delta(s) = 1$$

$$C(s) = \frac{K}{Ts + 1}$$

In order to represent the response of the system in time domain we need to compute the inverse Laplace transform of the above equation, we have

$$c(t) = \frac{K}{T} e^{-\frac{t}{T}} \quad (2)$$

Example 3:

For the first order system given below

$$G(s) = \frac{3}{2s + 1}$$

The impulse response is shown in figure 3

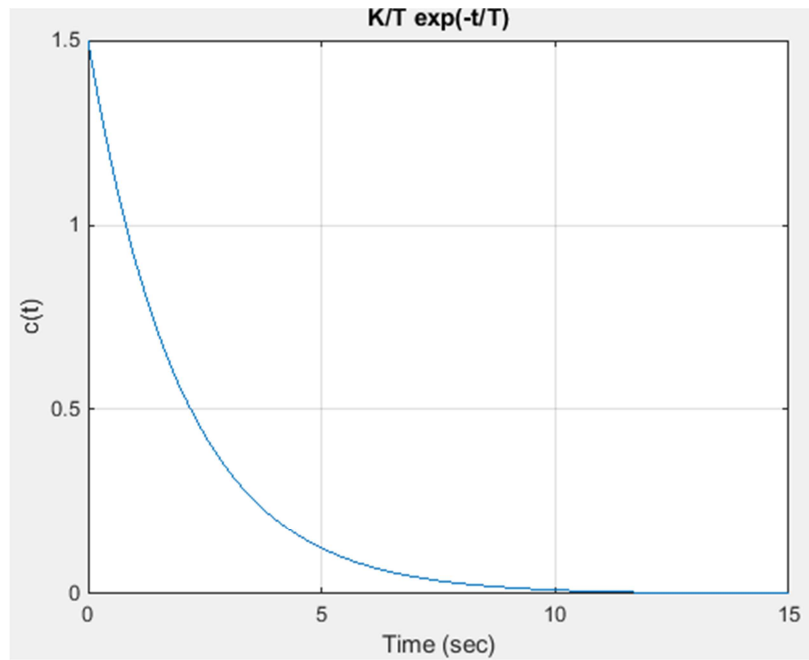


Figure 3: Impulse response of example 3

2.2 Ramp Response of 1st Order System

Consider the first order system in figure 1.

$$R(s) = \frac{1}{s^2}$$

$$C(s) = \frac{K}{Ts + 1} \frac{1}{s^2}$$

In order to represent the response of the system in time domain we need to compute the inverse Laplace transform of the above equation, we have

$$c(t) = Kt - T + Te^{-\frac{t}{T}} \quad (3)$$

Example 4:

For the first order system given below

$$G(s) = \frac{K}{Ts + 1}$$

- 1) The ramp response of system, if $K = 1$ and $T = 1$ is shown in figure 4
- 2) The ramp response of system, if $K = 1$ and $T = 3$ is shown in figure 5

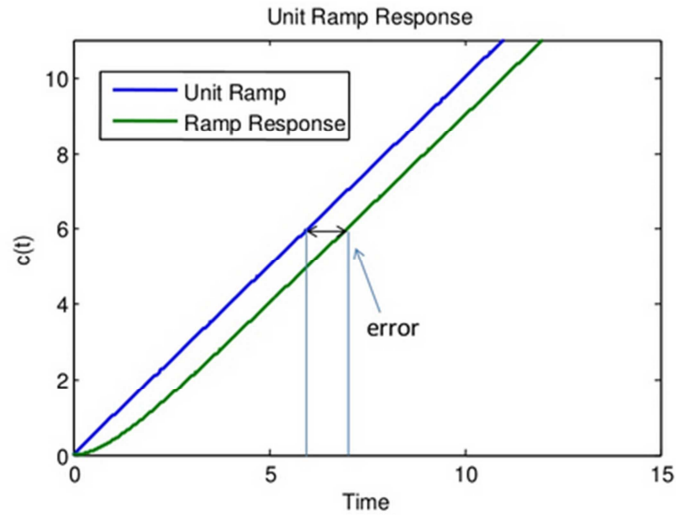


Figure 4: The ramp response for case 1, example 4

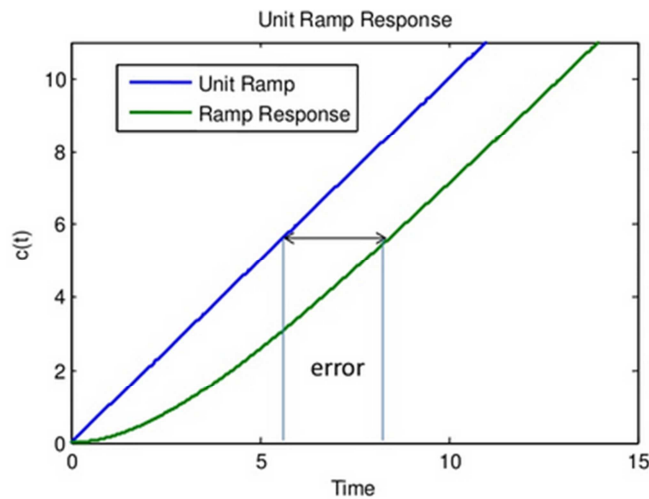


Figure 5: The ramp response for case 2, example 4

2.3 Step Response of 1st Order System

Consider the first order system in figure 1.

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{K}{Ts + 1} \frac{1}{s}$$

In order to represent the response of the system in time domain we need to compute the inverse Laplace transform of the above equation, we have

$$c(t) = Ku(t) - e^{-\frac{t}{T}} \tag{4}$$

1) Where $u(t) = 1$

$$c(t) = K - e^{-\frac{t}{T}} \quad (5)$$

2) Where $t = T$

$$c(t) = K - e^{-1} = 0.632K \quad (6)$$

- For example, assume $K = 10, T = 1.5s$

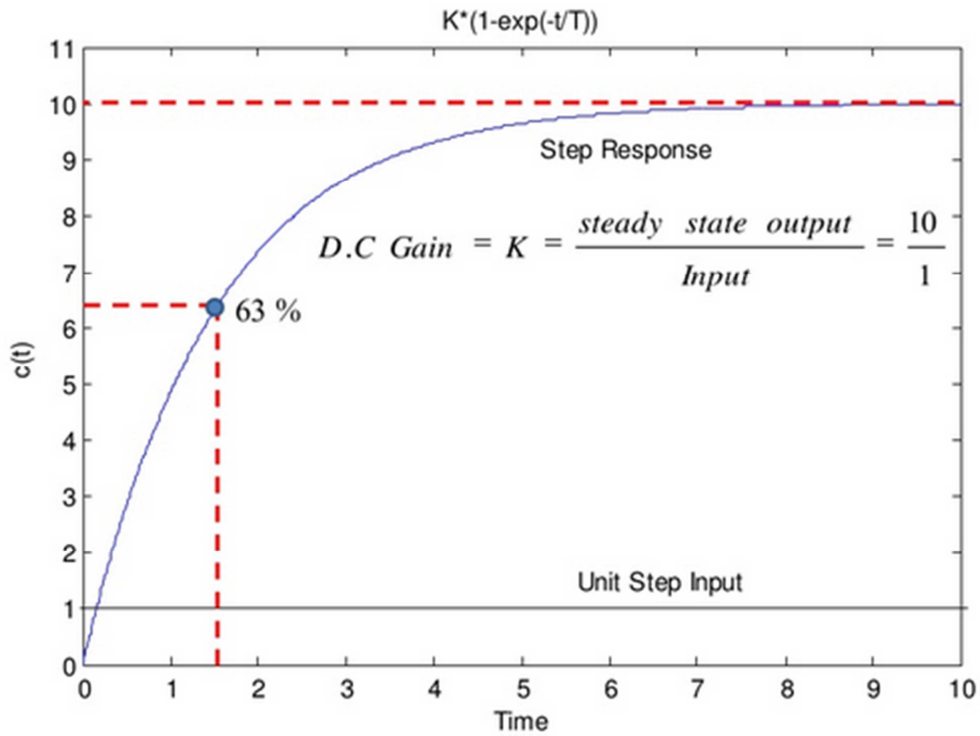


Figure 6: The step response specification of first order system

The step response of the first order system takes five time constants to reach its final value.

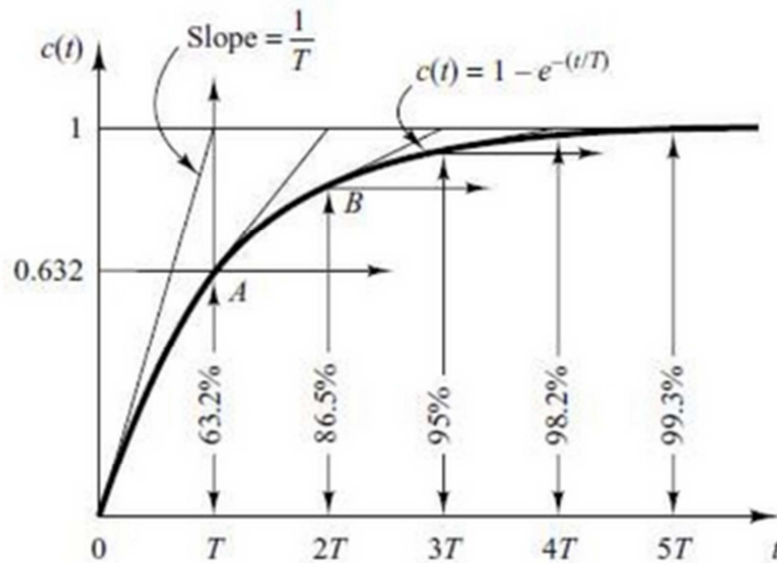


Figure 7: Step response

- $K = 10, T = 1, 3, 5, 7 \text{ s}$

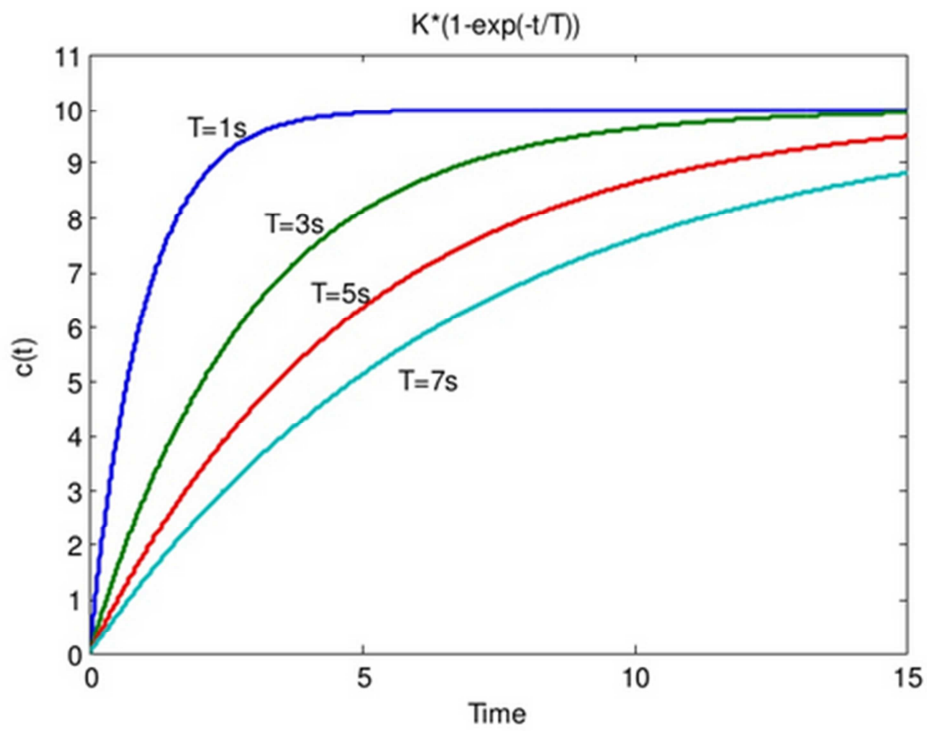


Figure 8: The step response at different value of T

- $K = 1, 3, 5, 10, T = 1 \text{ s}$

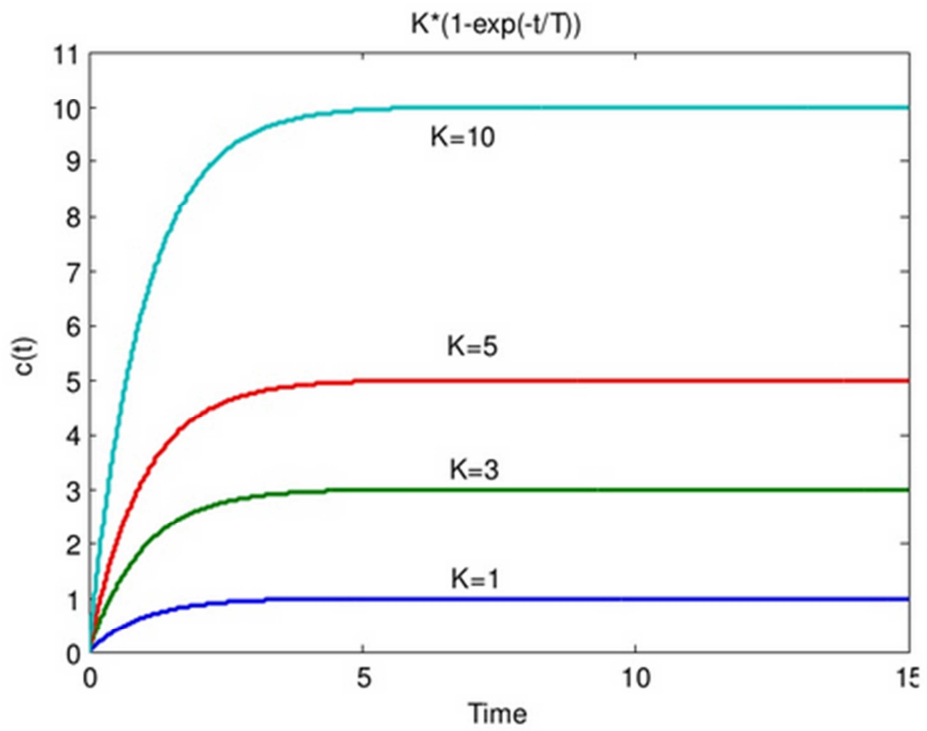


Figure 9: The step response at different value of K

3. First Order System with a Zero

The transfer function of the first order system with a zero can be represented using the form in equation (7)

$$G(s) = \frac{C(s)}{R(s)} = \frac{K(1 + \alpha s)}{Ts + 1} \quad (7)$$

- Zero of the system lie at $s = -1/\alpha$ and pole at $s = -1/T$
- Step response of the system would be:

$$C(s) = K \frac{1(1 + \alpha s)}{s(Ts + 1)}$$

The Laplace inverse transfer is

$$c(t) = K + \frac{K}{T}(\alpha - T) e^{-\frac{t}{T}} \quad (8)$$

- **Case 1: $T > \alpha$**

The shape of the step response is approximately same (with offset added by zero)

Example 5:

Consider the first order system given by

$$\frac{C(s)}{R(s)} = \frac{10(1 + 2s)}{3s + 1}$$

In this system:

1. $K = 10$
2. $T = 3$
3. $\alpha = 2$

$$c(t) = 10 + \frac{10}{3}(2 - 3)e^{-t/3}$$

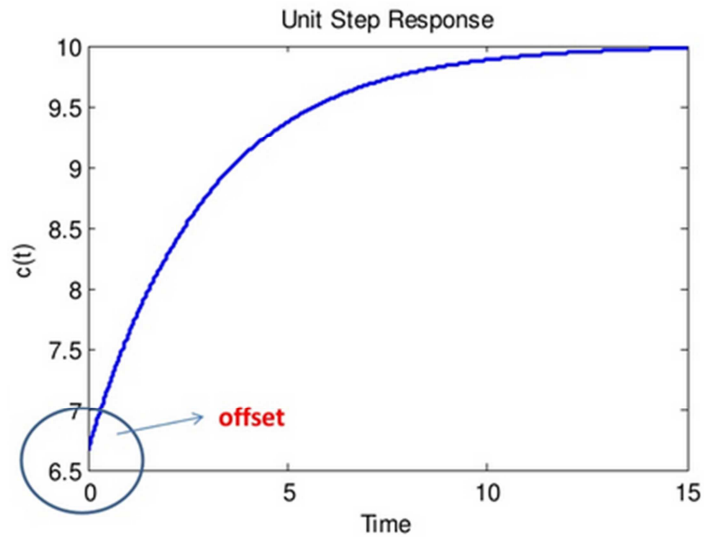


Figure 10: the step response of case 1 $T > \alpha$

$$\text{offset} = K + \frac{K}{T}(\alpha - T)$$

- **Case 2: $T < \alpha$**

Example 6:

Consider the first order system given by

$$\frac{C(s)}{R(s)} = \frac{10(1 + 2s)}{1.5s + 1}$$

In this system:

1. $K = 10$
2. $T = 1.5$
3. $\alpha = 2$

$$c(t) = 10 + \frac{10}{1.5}(2 - 1)e^{-t/1.5}$$

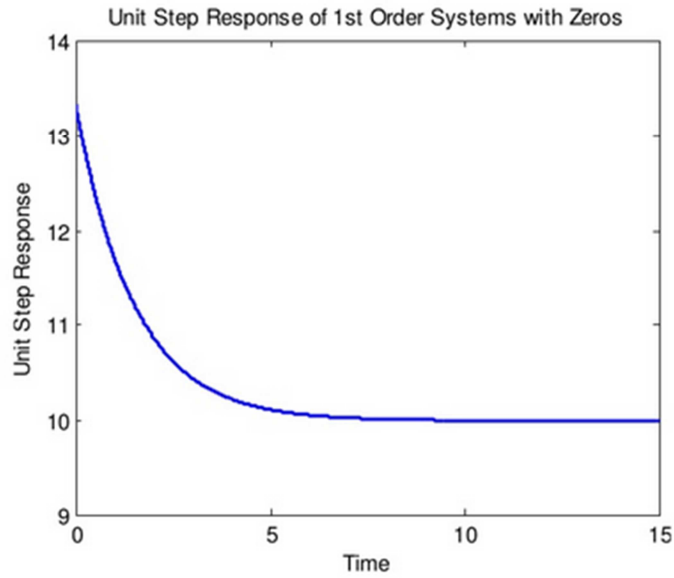


Figure 11: The step response for case $2 T < \alpha$

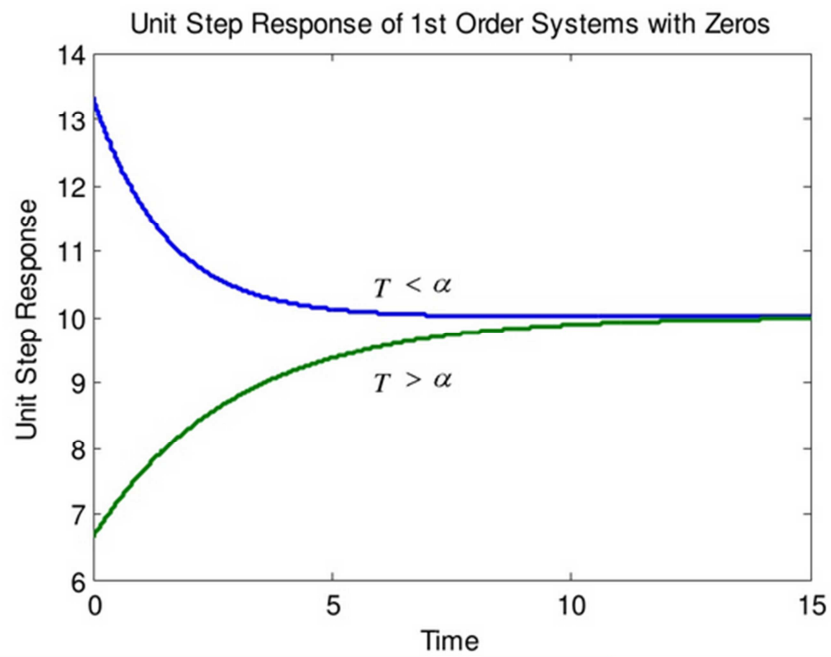


Figure 12: The step response of first order system with zero

4. First Order System with Delays

The first order system with delay time can have the following transfer function

$$\frac{C(s)}{R(s)} = \frac{K}{Ts + 1} e^{-st_d} \quad (9)$$

Where

t_d : is the delay time

The step response of this type of system is shown in figure 13



Figure 13: the step response of first order system with delay time

Example 6:

Consider the following first order system

$$G(s) = \frac{10}{s + 1} e^{-2s}$$

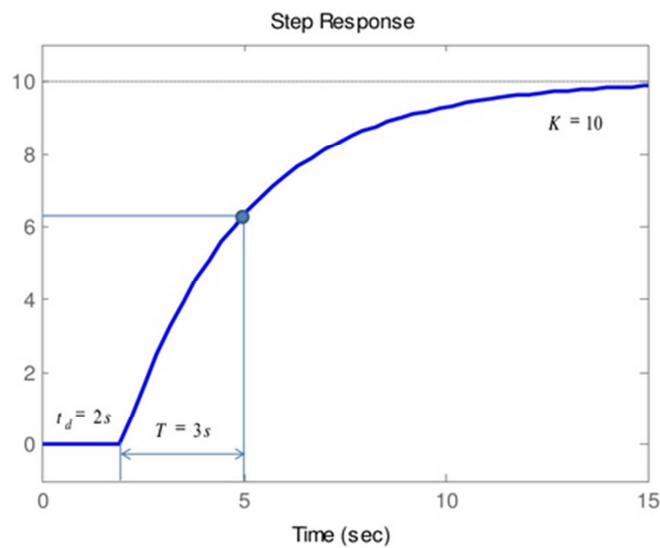


Figure 14: step response for system in Example 6

5. System Identification of Transfer Function of 1st Order Systems

The field of system identification uses statistical methods to build mathematical models of dynamical systems from measured data. System identification also includes the optimal design of experiments for efficiently generating informative data for fitting such models as well as model reduction.

A much more common approach is therefore to start from measurements of the behavior of the system and the external influences (inputs to the system) and try to determine a mathematical relation between them without going into the details of what is actually happening inside the system.

This approach is called system identification. Two types of models are common in the field of system identification:

- 1) **Grey box Model:** although the peculiarities of what is going on inside the system are not entirely known, a certain model based on both insight into the system and experimental data is constructed. This model does however still have a number of unknown free parameters which can be estimated using system identification.

One example, uses the Monod saturation model for microbial growth. The model contains a simple hyperbolic relationship between substrate concentration and growth rate, but this can be justified by molecules binding to a substrate without going into detail on the types of molecules or types of binding. Grey box modeling is also known as semi-physical modeling.

- 2) **Black box Model:** No prior model is available. Most system identification algorithms are of this type.

In science, computing, and engineering, a black box is a device, system or object which can be viewed in terms of its inputs and outputs (or transfer characteristics), without any knowledge of its internal workings. Its implementation is "opaque" (black). Almost anything might be referred to as a black box: a transistor, algorithm, or the human brain.



Figure 15: Black Box

5.1 System Identification of First order system

- Often it is not possible or practical to obtain a system's transfer function analytically.
- Perhaps the system is closed, and the component parts are not easily identifiable.
- The system's step response can lead to a representation even though the inner construction is not known.
- With a step input, we can measure the time constant and the steady-state value, from which the transfer function can be calculated.
- If we can identify T and K from laboratory testing we can obtain the transfer function of the system.

Example 7:

Assume the unit step response given in figure 16;

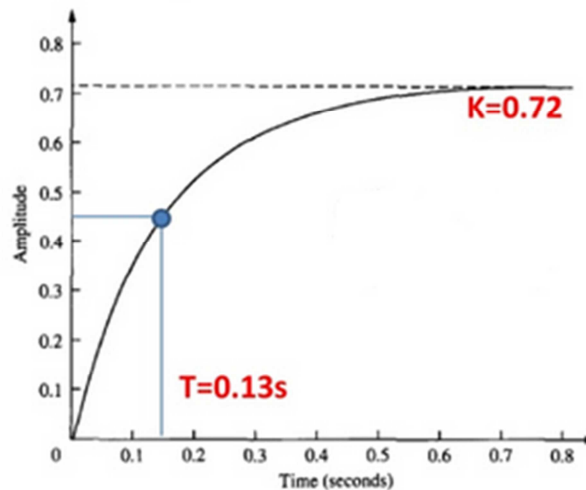


Figure 16: Unit step Response

- From the response, we can measure the **time constant T** , that is the time constant for the amplitude to reach 63% of its final value.

Since the final value is about 0.72 the time constant is evaluated for the curve reaches

$$0.63 \times 0.72 = 0.45, \text{ then } T = 0.13$$

- K is simply the steady state value.
- The transfer function is obtained as:

$$\frac{C(s)}{R(s)} = \frac{0.72}{0.13s + 1} = \frac{5.5}{s + 7.7}$$

6. Simulation of First order system using Simulink

In this section we study a open loop and closed loop system for case a first order system with delay and show the parameter of first order system.

Note: Maybe there more than blocks representation but we discuss and use the most model simulate the practical experiments as shown in the following example

Example 8:

Consider the first order system with delay given by the following transfer function

$$G(s) = \frac{2}{3s + 1} e^{-1s}$$

As shown in the previous section

$$K = 2 \quad (\text{DC gain})$$

$$T = 3 \quad (\text{Time Constant})$$

$t_d = 1$ (delay time)

We can simulate this system in Simulink using the basic block diagrams (Transfer Fcn, gain , sum and Transport Delay)

- **Open Loop System**

We assume the system in unit step input.

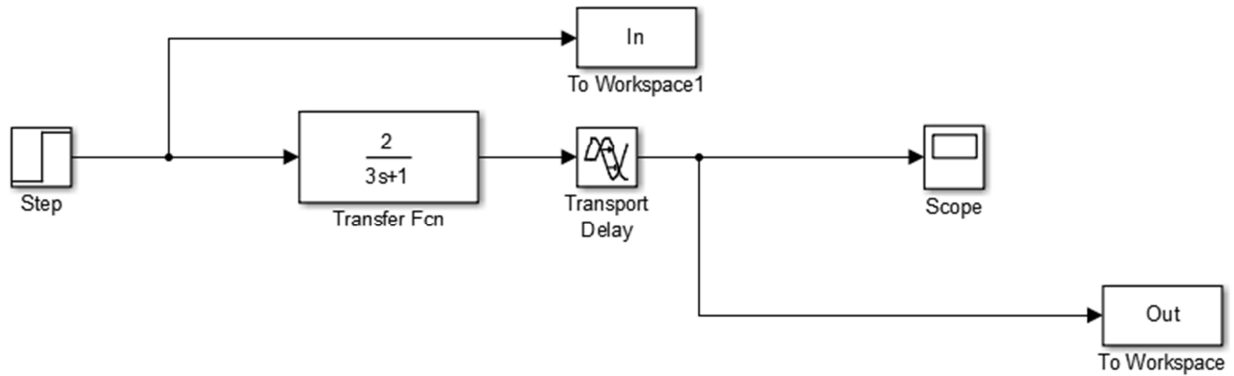


Figure 17: Open loop of system in example 8

The result of step response and the parameter is shown in figure 18

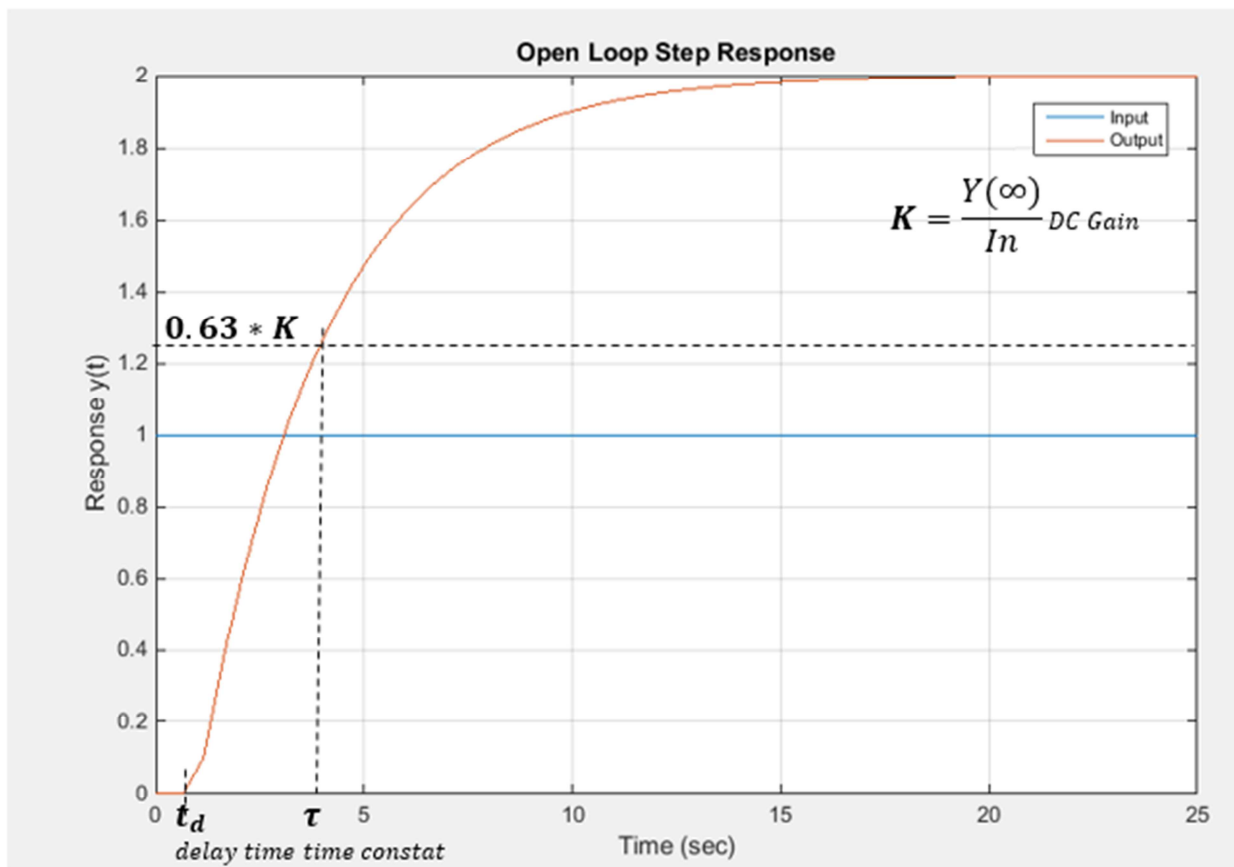


Figure 18: Open loop Response

So if the experimental data for first order system is known we can obtain the transfer function by

determine the parameter (K, τ, t_d) as discussed in the system identification example.

- **Closed loop Response**

The block diagram representation of closed loop system is shown in the figure 19

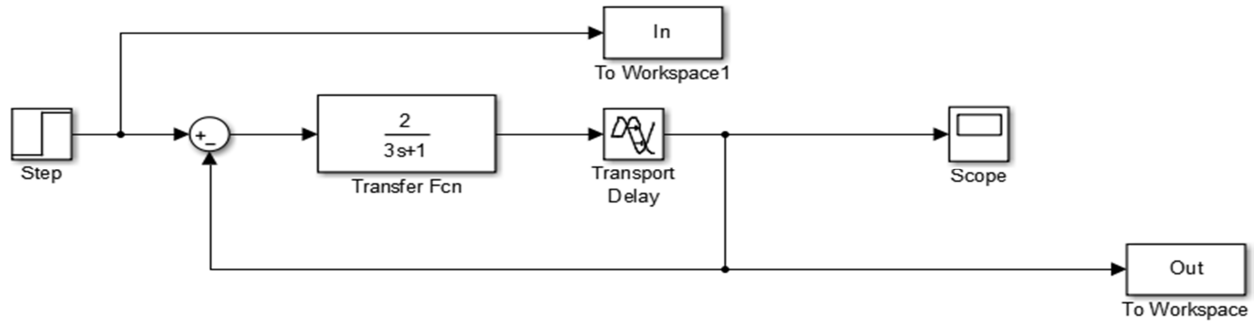


Figure 19: Closed Loop Block Diagram

The step response is shown in figure 20

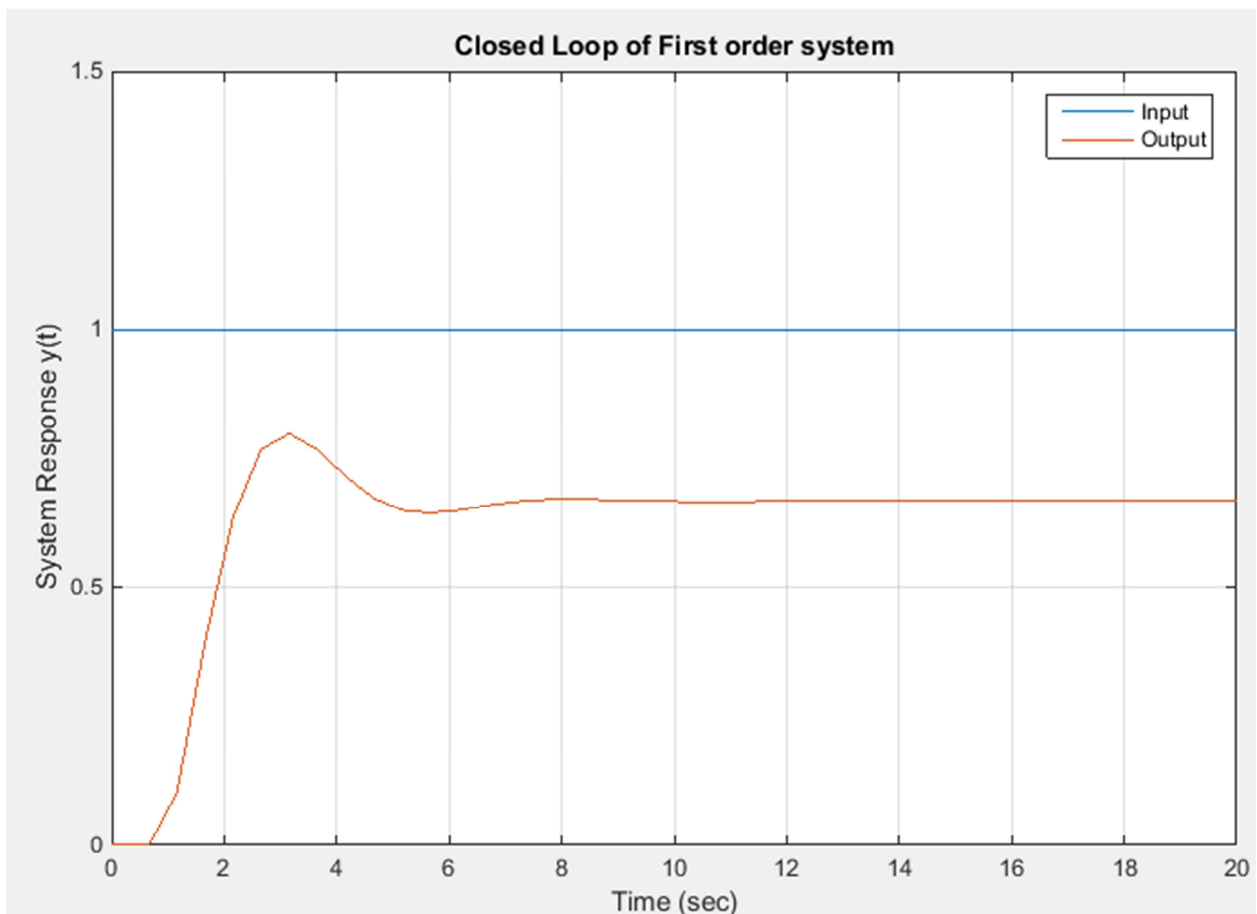


Figure 20: Step Response of closed loop

Depend on the two responses of system, we observe to determine the transfer function of the

system we must get the parameter from the open loop characteristic because in this case we obtain the response and the behavior of the system without any modification or addition effect.

System Identification Toolbox:

System Identification Toolbox provides MATLAB® functions, Simulink® blocks, and an app for constructing mathematical models of dynamic systems from measured input-output data.

It lets you create and use models of dynamic systems not easily modeled from first principles or specifications.

You can use time-domain and frequency-domain input-output data to identify continuous-time and discrete-time transfer functions, process models, and state-space models. The toolbox also provides algorithms for embedded online parameter estimation.

System Identification Toolbox lets you create models from measured input-output data. You can:

1. Analyze and process data
2. Determine suitable model structure and order, and estimate model parameters
3. Validate model accuracy

System identification is open using the command (*ident*)

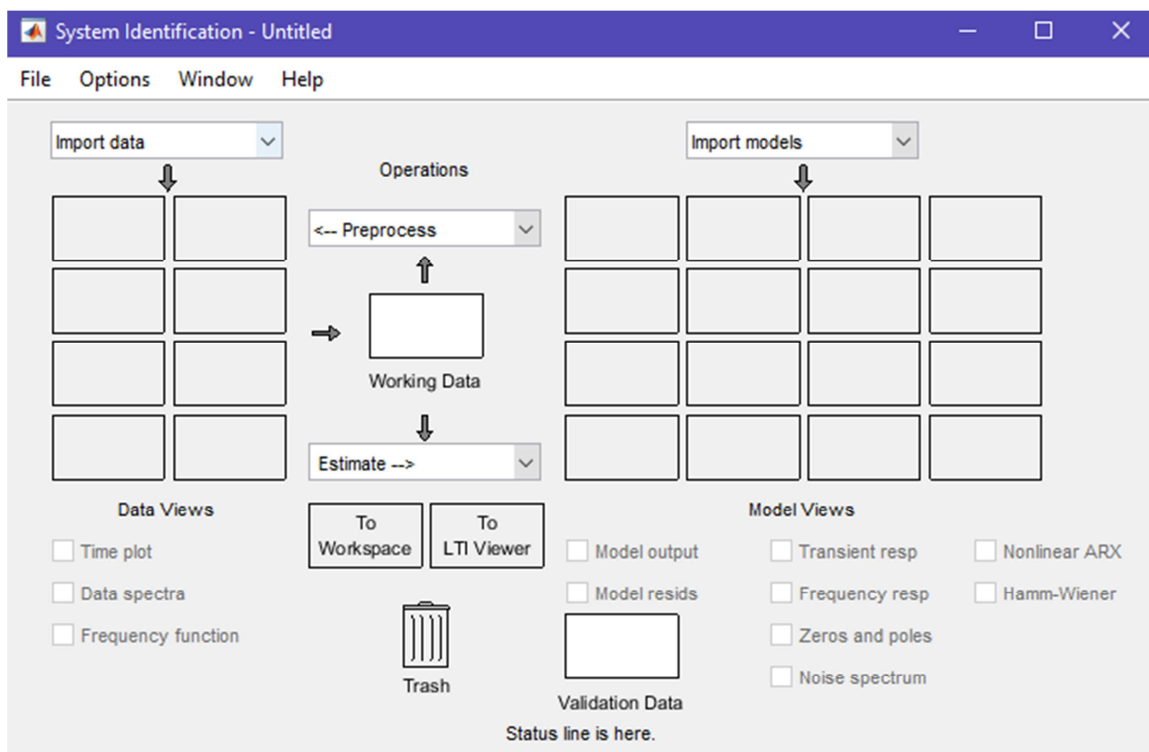


FIGURE 5: SYSTEM IDENTIFICATION TOOLBOX

1. Importing Data

When [preparing data for identifying models](#), you need to specify information such as input-output channel names, sampling time, and intersample behavior. The toolbox lets you attach this

information to the data, which facilitates visualization of data, domain conversion, and various preprocessing tasks.

The procedure of importing data as shown in figure 2:

- 1) Select the time domain data
- 2) Write the variable of input and Output

For example, if the **(Input: Vin, Output: Vc)**

- 3) Write the Starting time, and Sample time and this depend on the experimental data, for example **(Starting time :0, Sample time :0.1)**

Name ▲	Value
lc	501x1 double
t	501x1 double
tout	509x1 double
Vc	501x1 double
Vin	501x1 double
VL	501x1 double

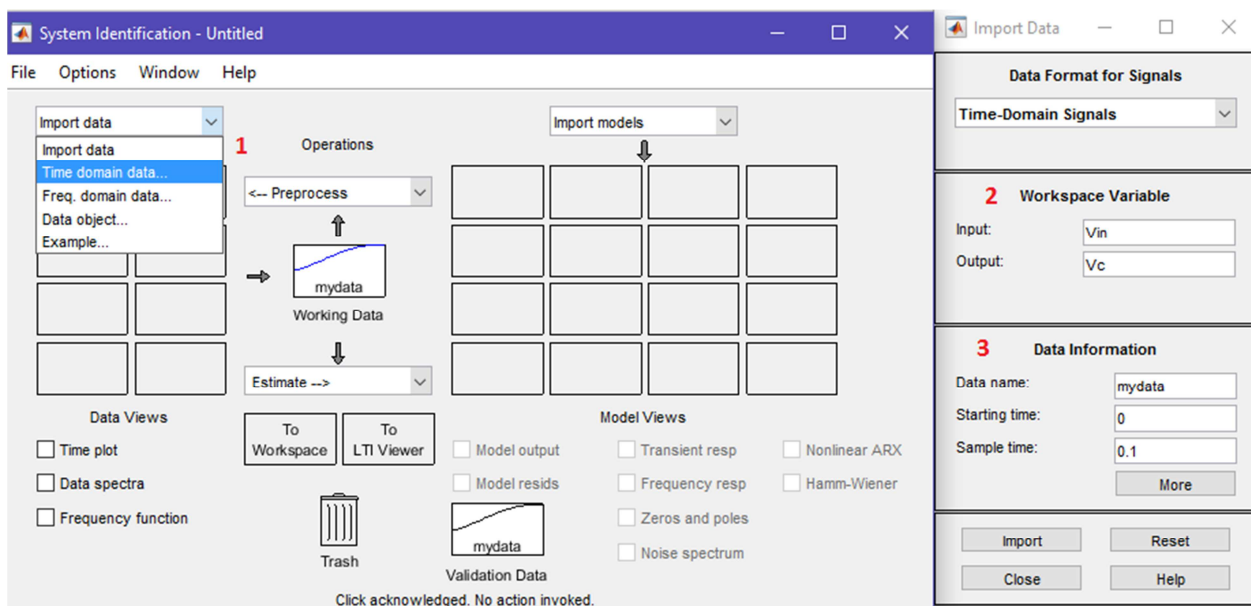


Figure 6: Importing Data

Note: This Testing Data is attached in **Data_test1.mat**

- 4) Click on Import the data will stored in model as figure 3
- 5) We can plot the data by check the box (Time Plot)

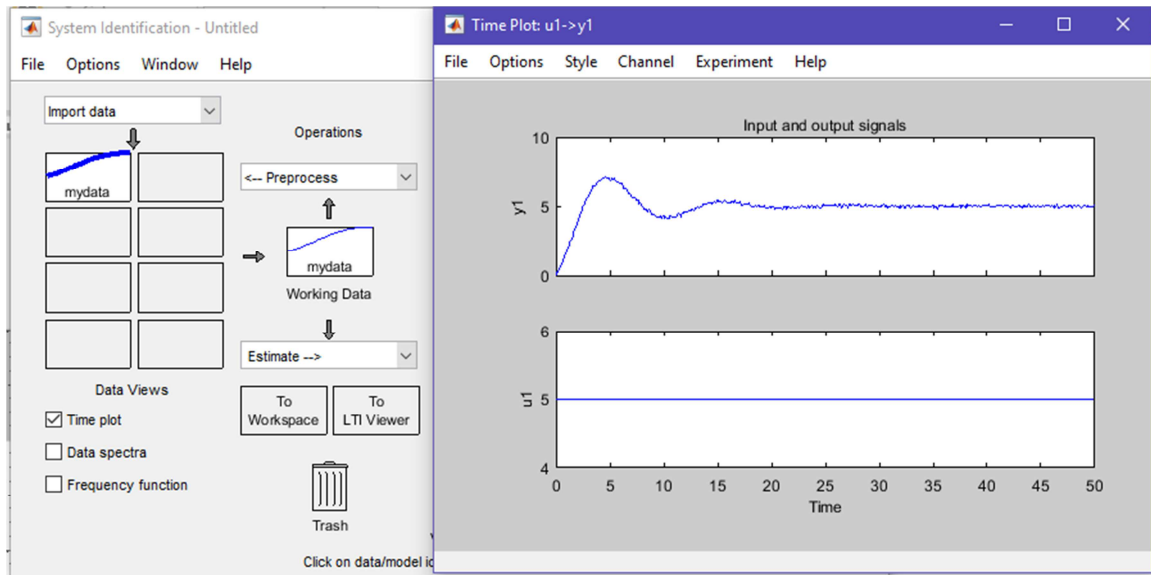


Figure 7: Imported Data

2. Estimating Model Parameters

Parametric models, such as transfer functions or state-space models, use a small number of parameters to capture system dynamics. System Identification Toolbox estimates model parameters and their uncertainties from time-response and frequency-response data. You can analyze these models using time-response and frequency-response plots, such as step, impulse, Bode plots, and pole-zero maps.

As shown in the figure 4, we can choose an estimated model from menu,

Note: the working data block must contain the data that we need to find estimated model for it,

For example, if we choose Transfer model, the new screen will appear as shown in figure 5, form this screen:

- 1) determine the number of poles and number of zeros of estimated transfer function,
- 2) Estimate the transfer function

Note:

- 1) Always started estimation by small number of poles and zeros and increase it by one and repeat the estimation.
- 2) There is no specific rules to select the optimal number of poles and zeros, you need trial and error or experience.

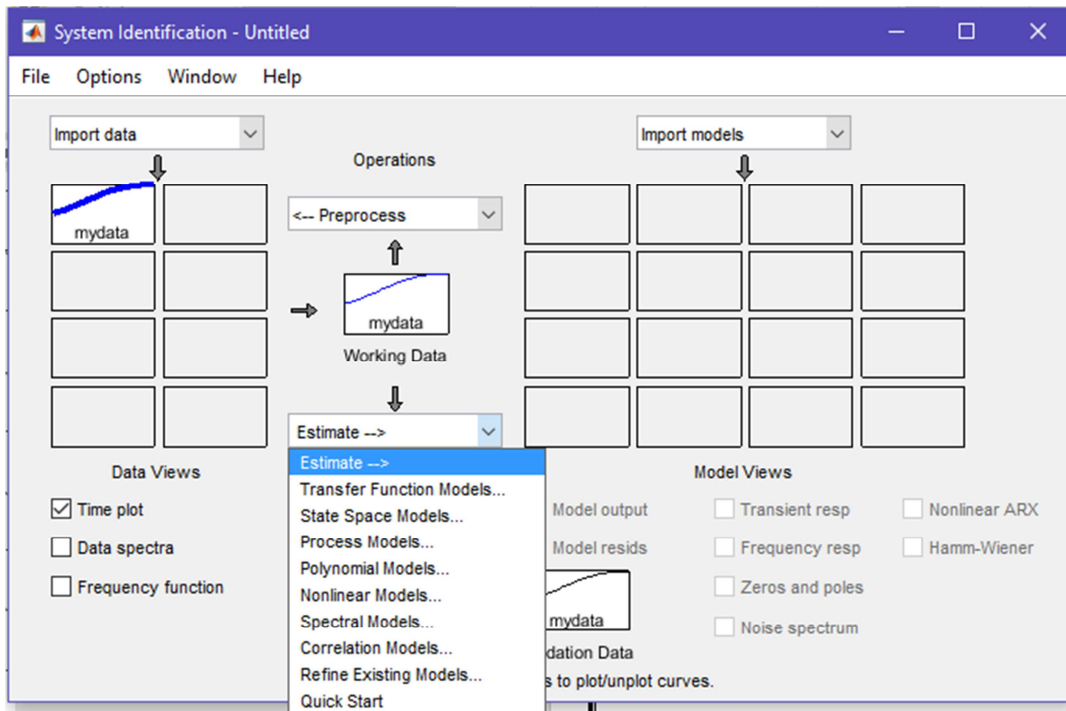


Figure 8: Estimate Model Selection

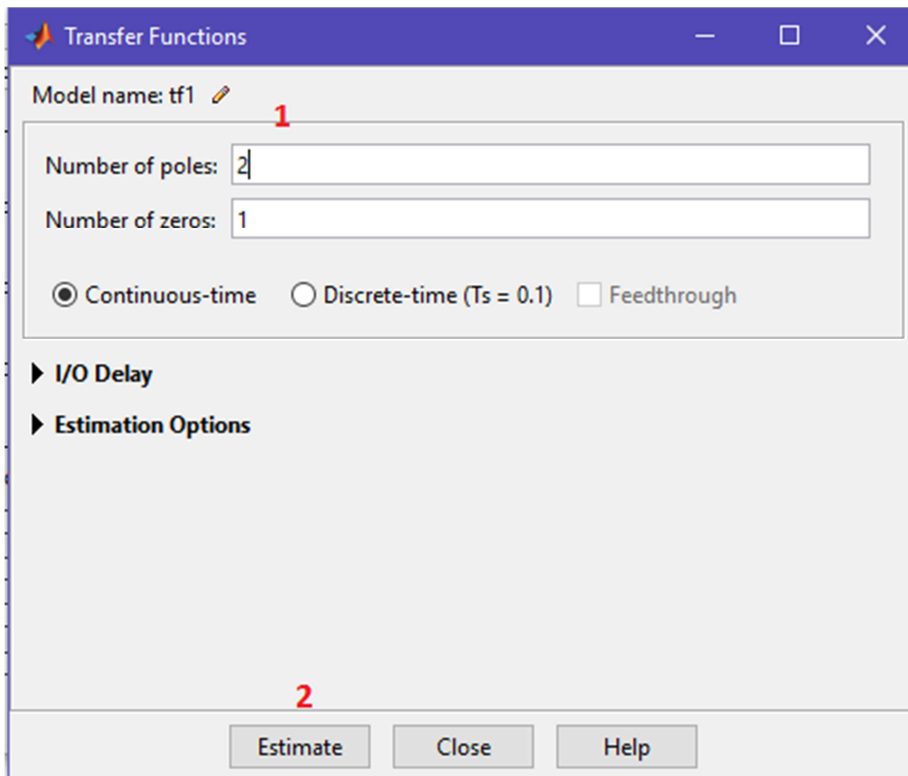


Figure 9: Estimate Transfer Function

3. Validation Model

For a given set of input data, the toolbox computes the output of the identified model and lets you compare that output with the measured output from a real system.

After estimated the model, we can compare the estimated output with measured output and determine the fitness value, as shown in figure 6

The validation Data, must be matching with Importing data, otherwise the validation will be incorrect

Check the box (Model Output) and the response will appear as shown in figure 7

Best fits: is measure the error ratio between estimated and measured output.

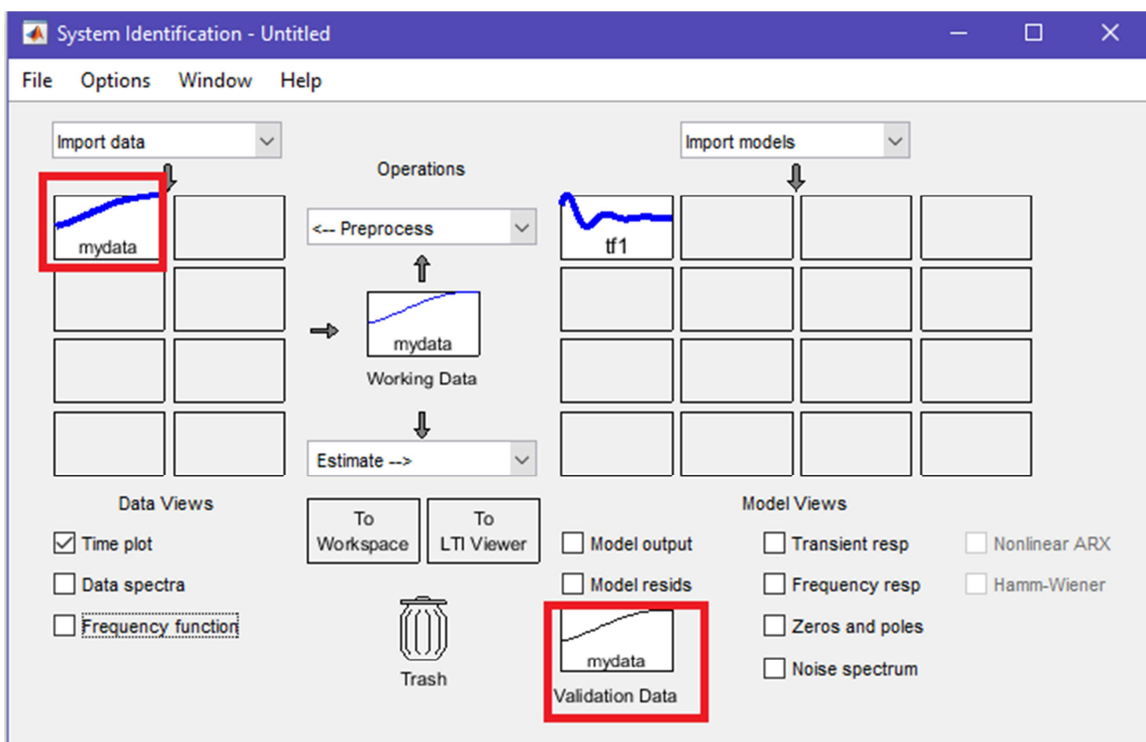
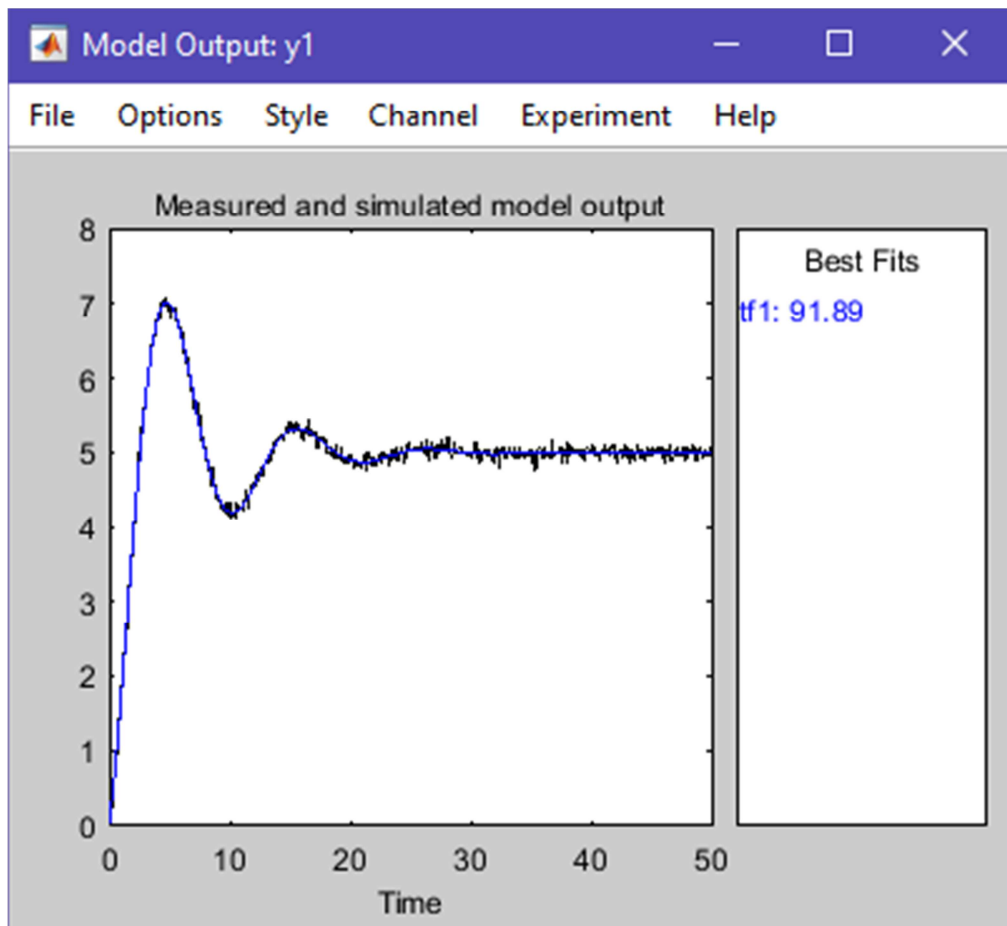
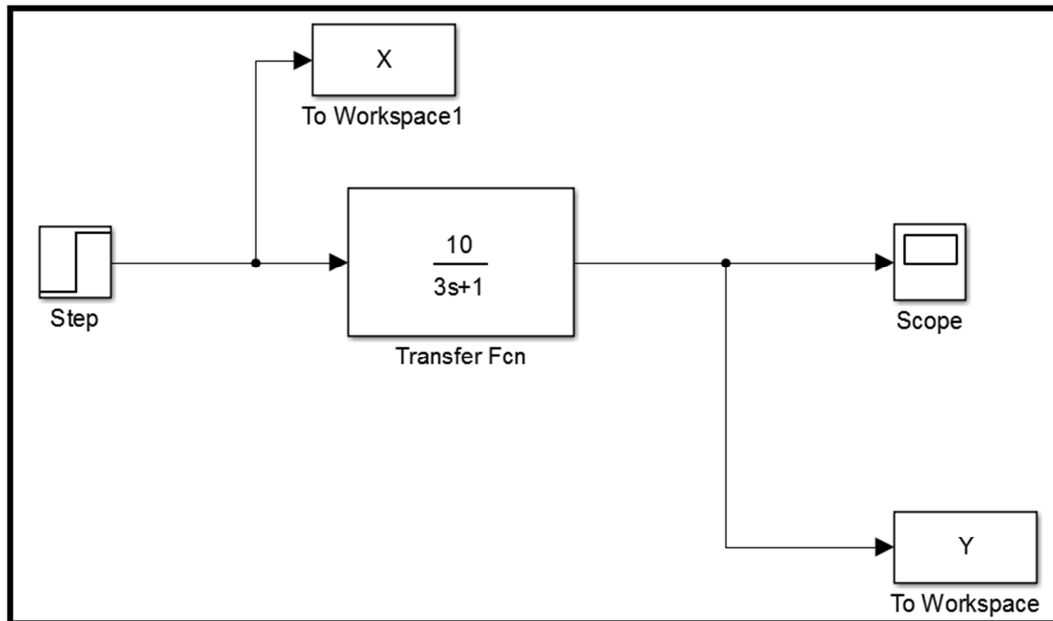


Figure 10: Importing model and validation data

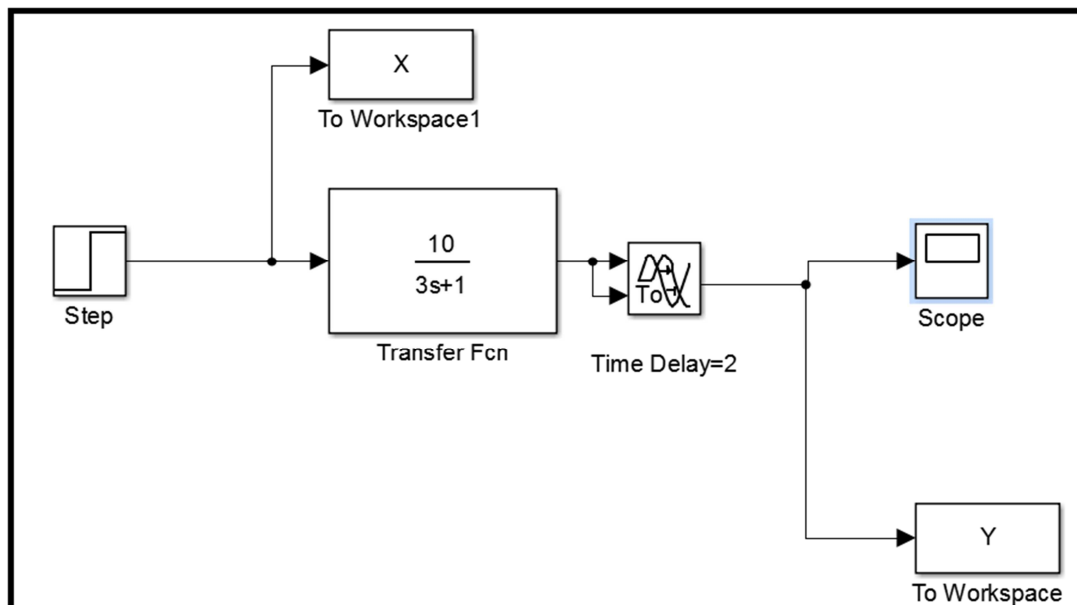


Exercises:

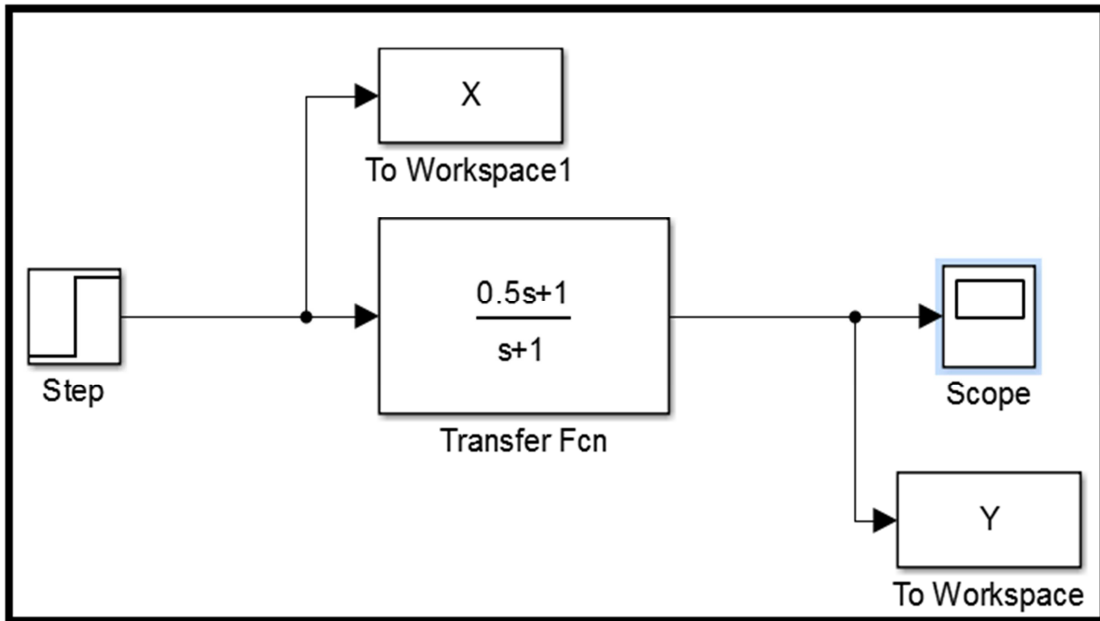
Build a Simulink model for the following systems:



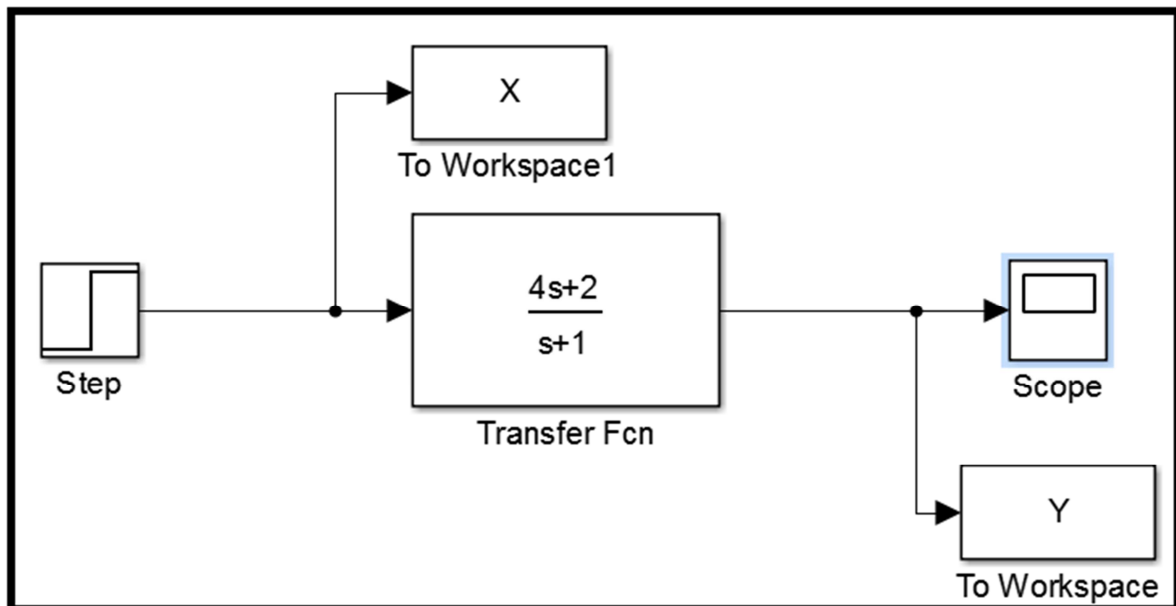
System A



System B



System C



System D

1. From the response on the scope find the transfer function for each system by calculations.
2. From X and Y data in the workspace estimate the transfer function for each system using system identification toolbox in matlab.