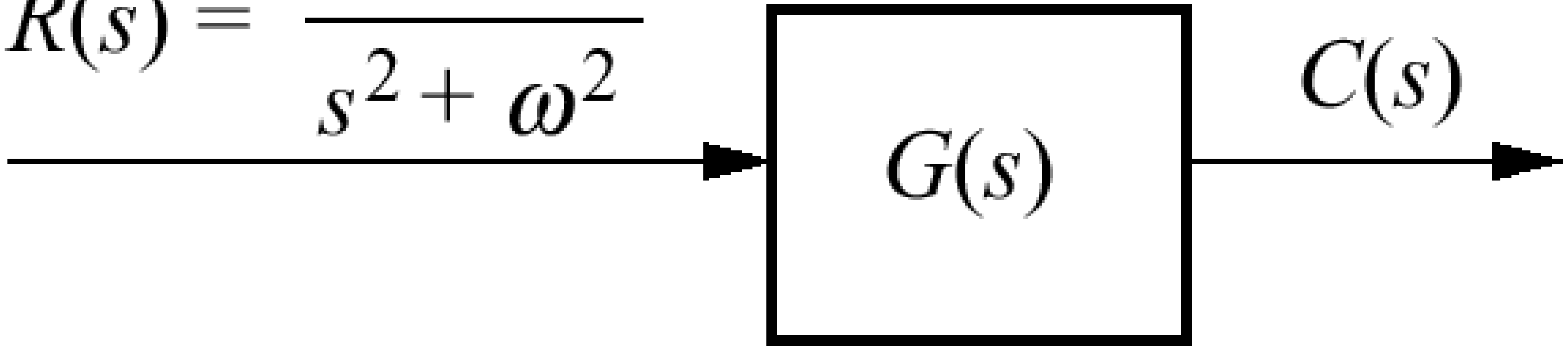
Frequency Response Techniques using Bode and Nyquist Plots

Concept of Frequency Response

Input sin(ωt) **System** **Response**

$$R(s) = \frac{\omega}{s^2 + \omega^2}$$



Concept of Frequency Response

$$C(s)=\frac{\omega}{s^2+\omega^2}G(s)$$

$$=\frac{K_1}{(s+j\omega)}+\frac{K_2}{(s-j\omega)}+Fraction\,terms\;for\,G(s)$$

Forced — **Steady-state**

Natural — **Dies out**

Concept of Frequency Response

$$K_1 = \frac{\omega}{(s - j\omega)} G(s)\Bigg|_{s=-j\omega} = \frac{1}{2} jG(-j\omega) = \frac{1}{2} M_G e^{-j(\phi_G - \pi/2)},$$

$$K_2 = \frac{\omega}{(s + j\omega)} G(s)\Bigg|_{s=j\omega} = -\frac{1}{2} jG(j\omega) = \frac{1}{2} M_G e^{j(\phi_G - \pi/2)}$$

where $M_G = |G(j\omega)|$

$\phi_G = angle\ of\ G(j\omega)$

Concept of Frequency Response

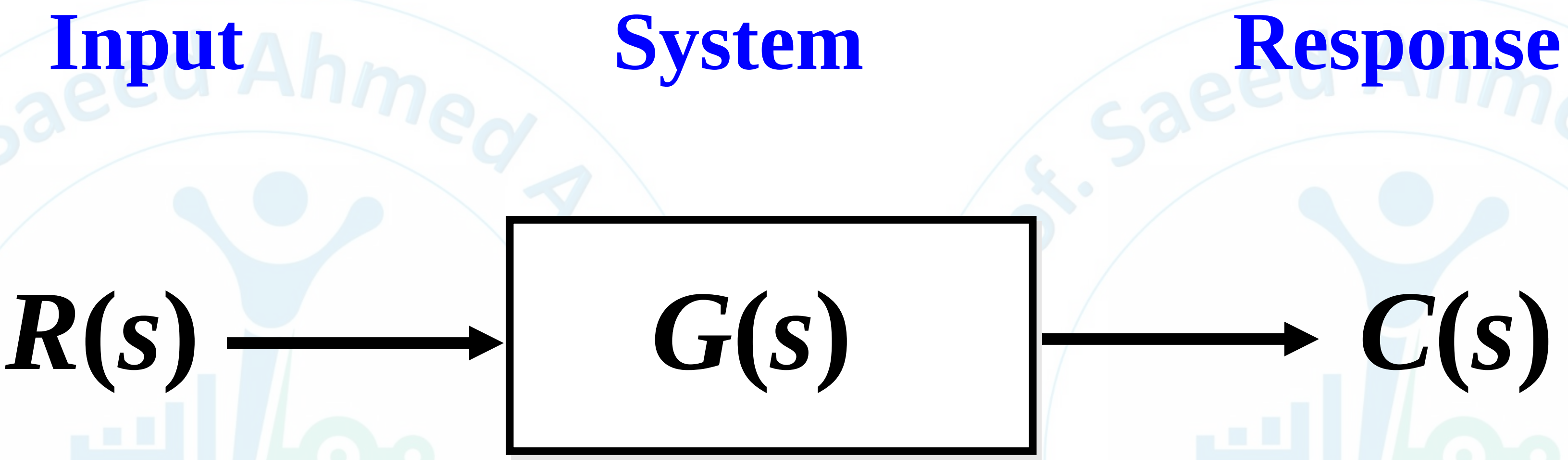
$$C_{ss}(s) = \frac{K_1}{(s+j\omega)} + \frac{K_2}{(s-j\omega)}$$

$$= \frac{1}{2} M_G \left[\frac{e^{-j(\phi_G - \pi/2)}}{(s+j\omega)} + \frac{e^{j(\phi_G - \pi/2)}}{(s-j\omega)} \right]$$

$$c_{ss}(t) = \frac{1}{2} M_G \left[e^{-j(\omega t + \phi_G - \pi/2)} + e^{j(\omega t + \phi_G - \pi/2)} \right]$$

$$= M_G \sin(\omega t + \phi_G)$$

Concept of Frequency Response



$$R(t) = \sin(\omega t) \quad \Rightarrow \quad c(t) = M_G \sin(\omega t + \varphi_G)$$

where $M_G = |G(j\omega)|$ **&** $\phi_G = \text{angle of } G(j\omega)$

Concept of Frequency Response

System Parameters

- Magnitude $M_G = |G(j\omega)|$
- Phase angle $\phi_G = angle\ of\ G(j\omega)$

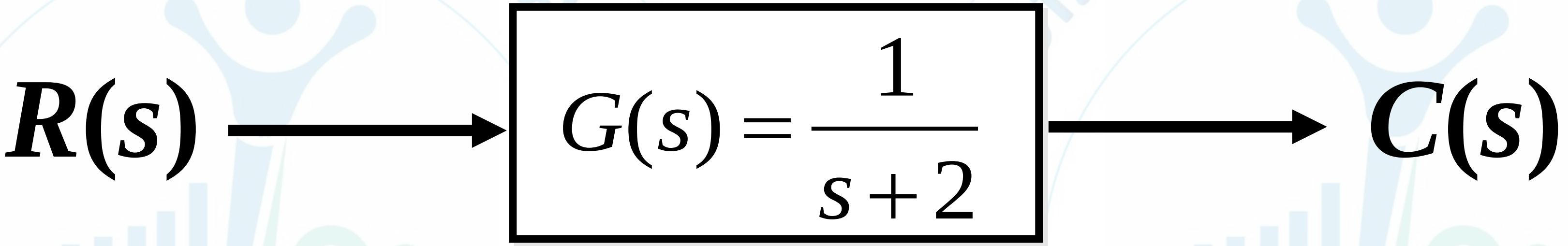
System Representation

Bode Diagram M_G & φ_G vs. ω

Nyquist Diagram M_G vs. φ_G for $0<\omega<\infty$

Concept of Frequency Response

Example 10.1



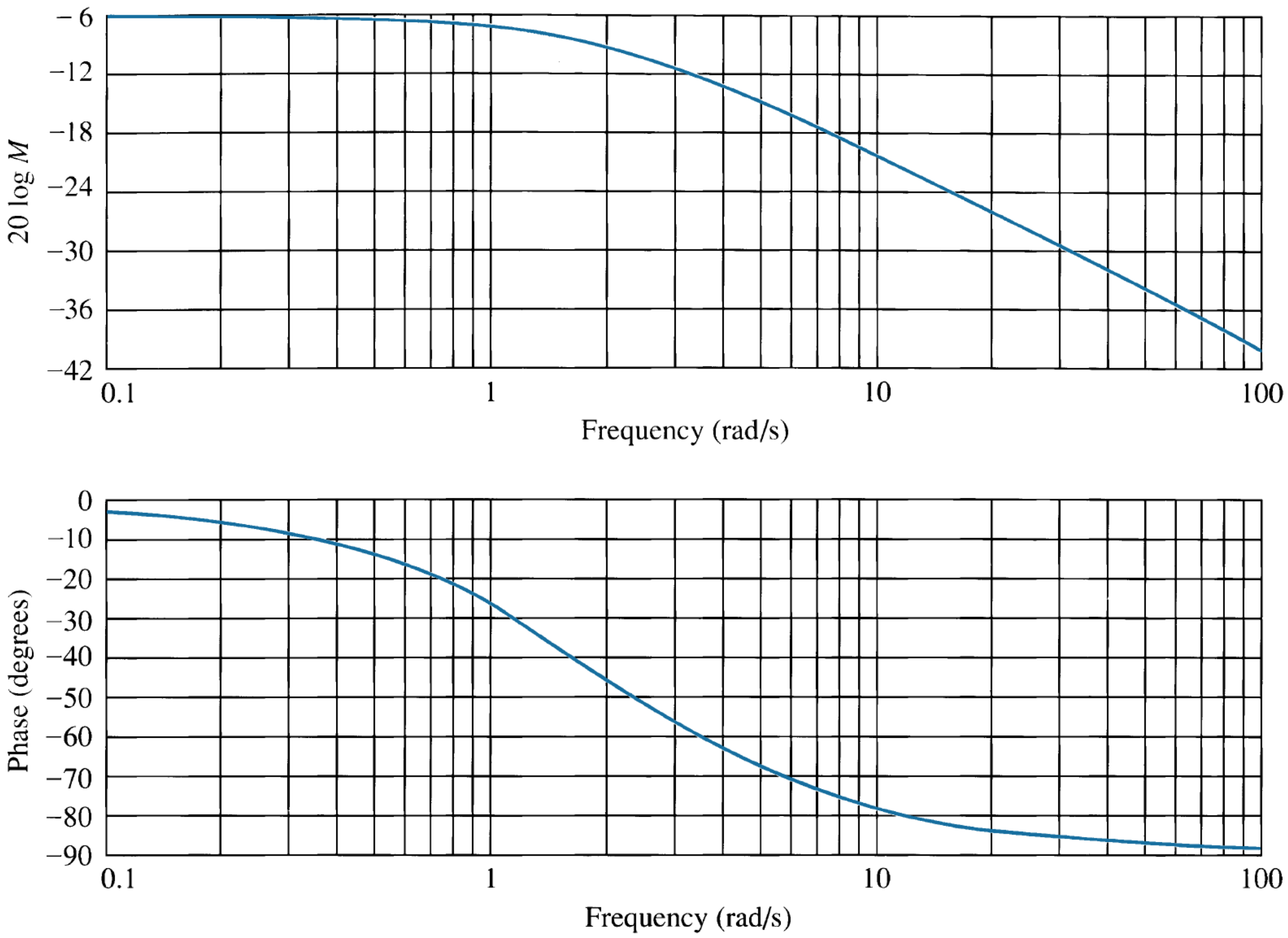
$$G(j\omega)=\frac{1}{j\omega+2}=\frac{2-j\omega}{\omega^2+4}=M_G e^{j\phi_G}$$

$$M_G=\frac{1}{\sqrt{\omega^2+4}},\quad and \quad \phi_G=-\tan^{-1}\left(\frac{\omega}{2}\right)$$

Concept of Frequency Response

Example 10.1

Bode Diagram

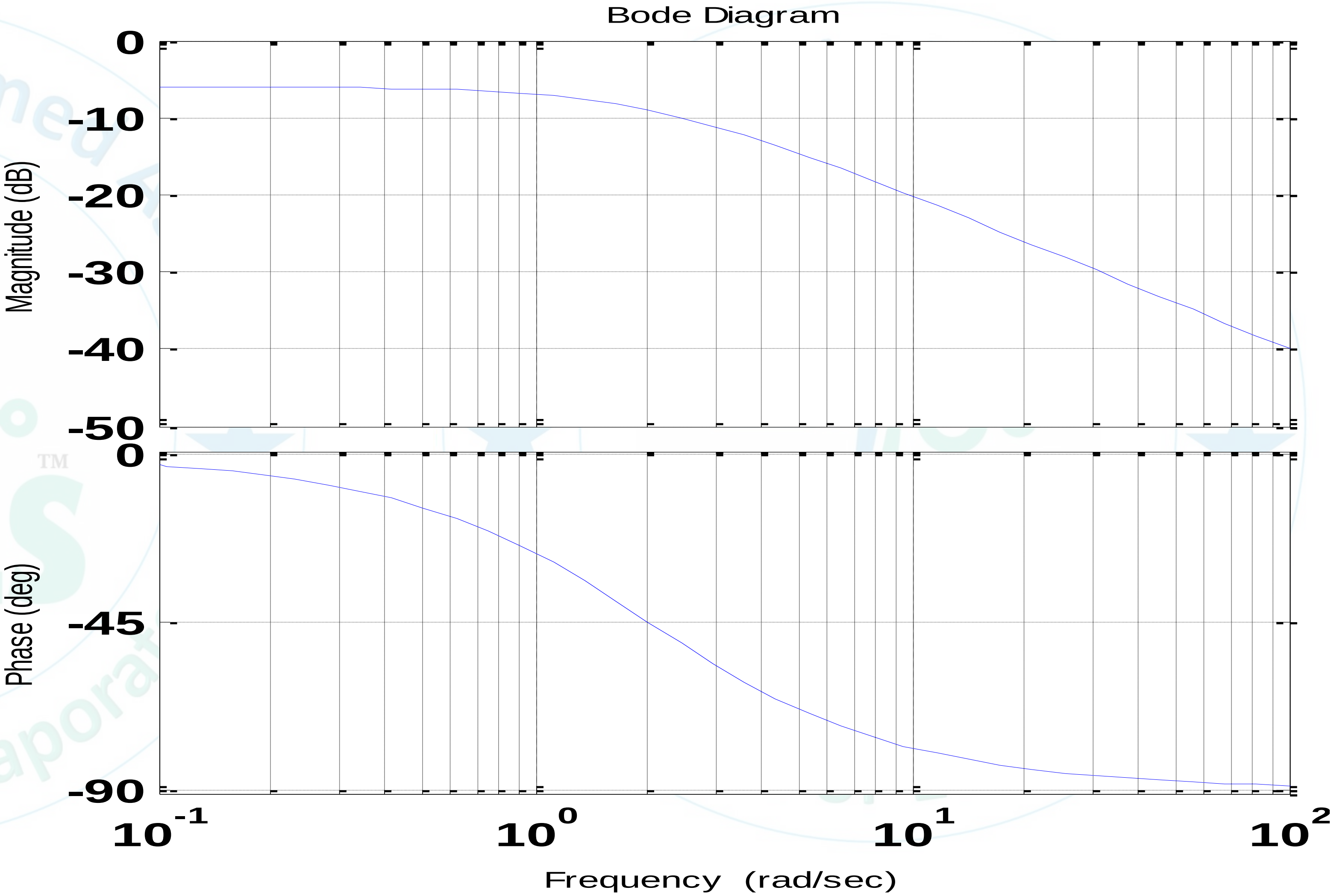


Frequency Response using Bode Plot

Example 10.1

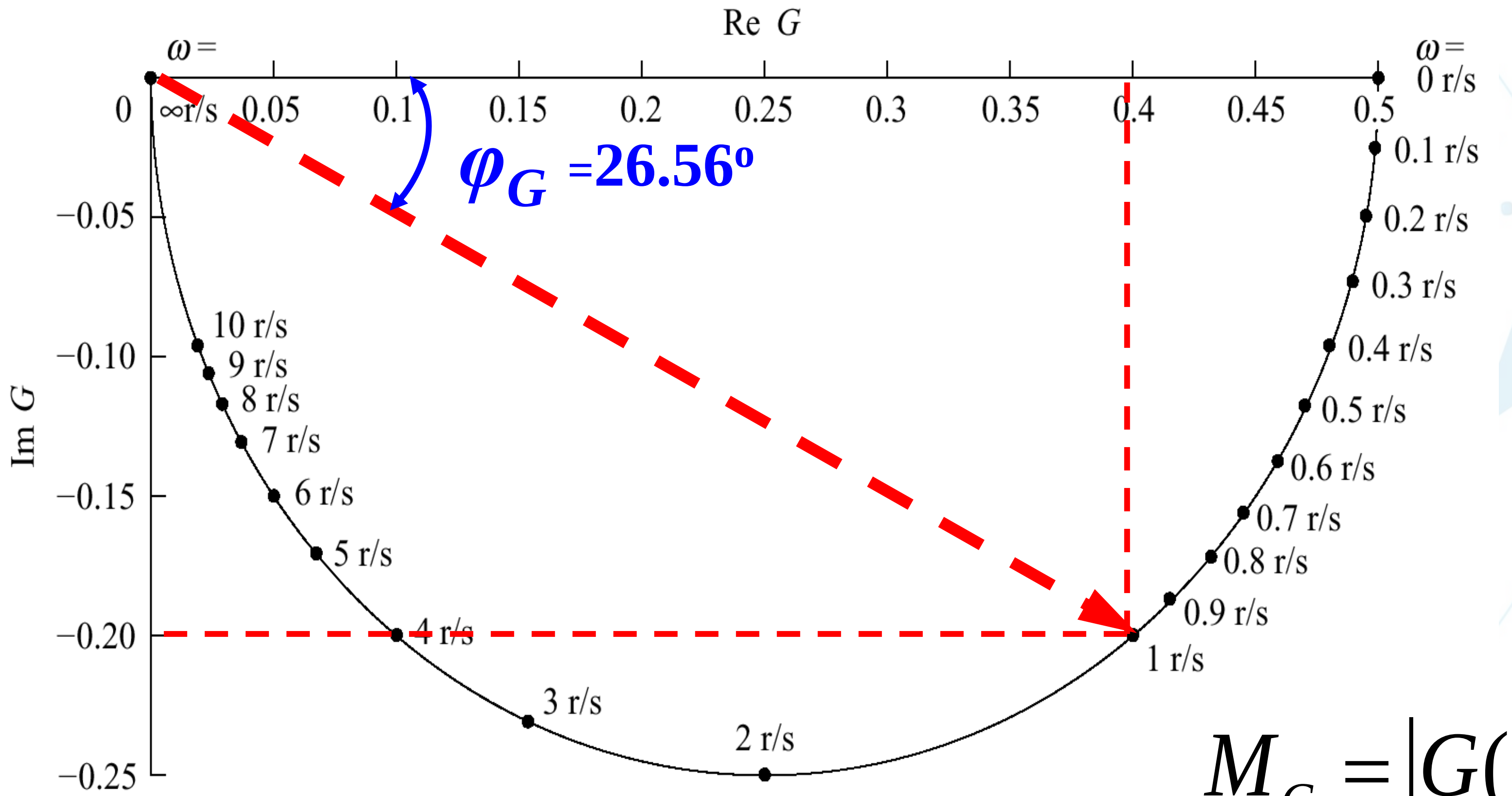
```
>> n=1;
>> d=[1  2];
>> bode(n,d)
>> grid
```

Hendrik Bode
1905 -1982



Frequency Response using Nyquist Plot

Example 10.1



Note: r/s = rad/s

$$M_G = |G(j\omega)|$$

$$M_G = 0.44$$

Frequency Response using Nyquist Plot

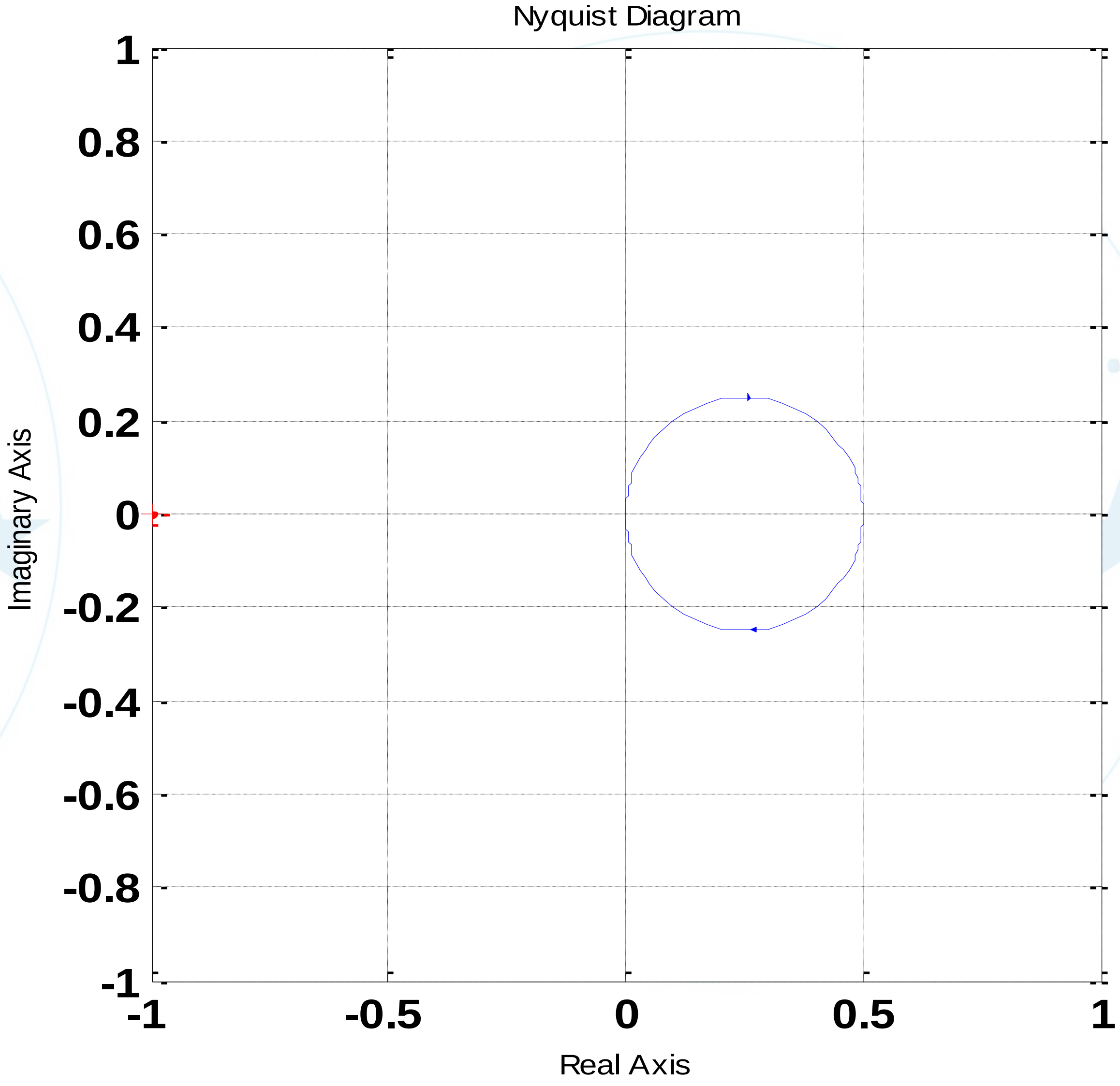
Example 10.1

```
>> n=1;
>> d=[1  2];
>> nyquist(n,d)
```

Harry Nyquist

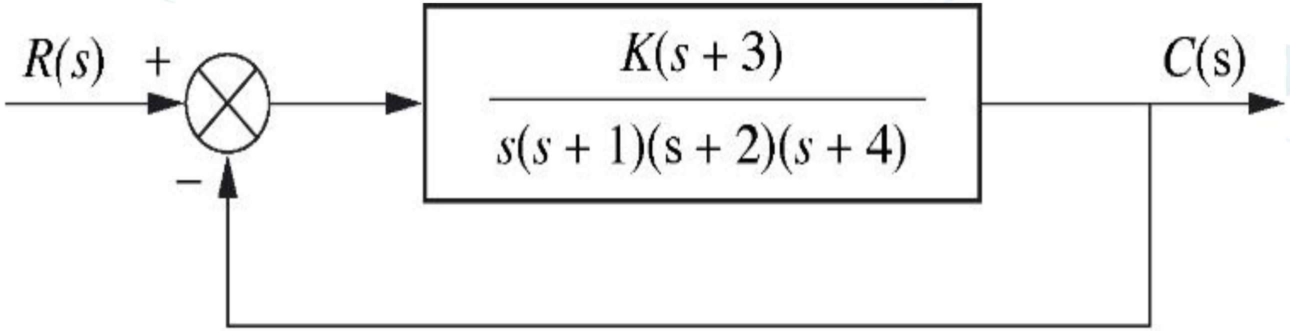
b. Feb. 7, 1889, Nilsby, Sweden

d. April 4, 1976, Harlingen, Texas, U.S.



Nyquist Stability

Example



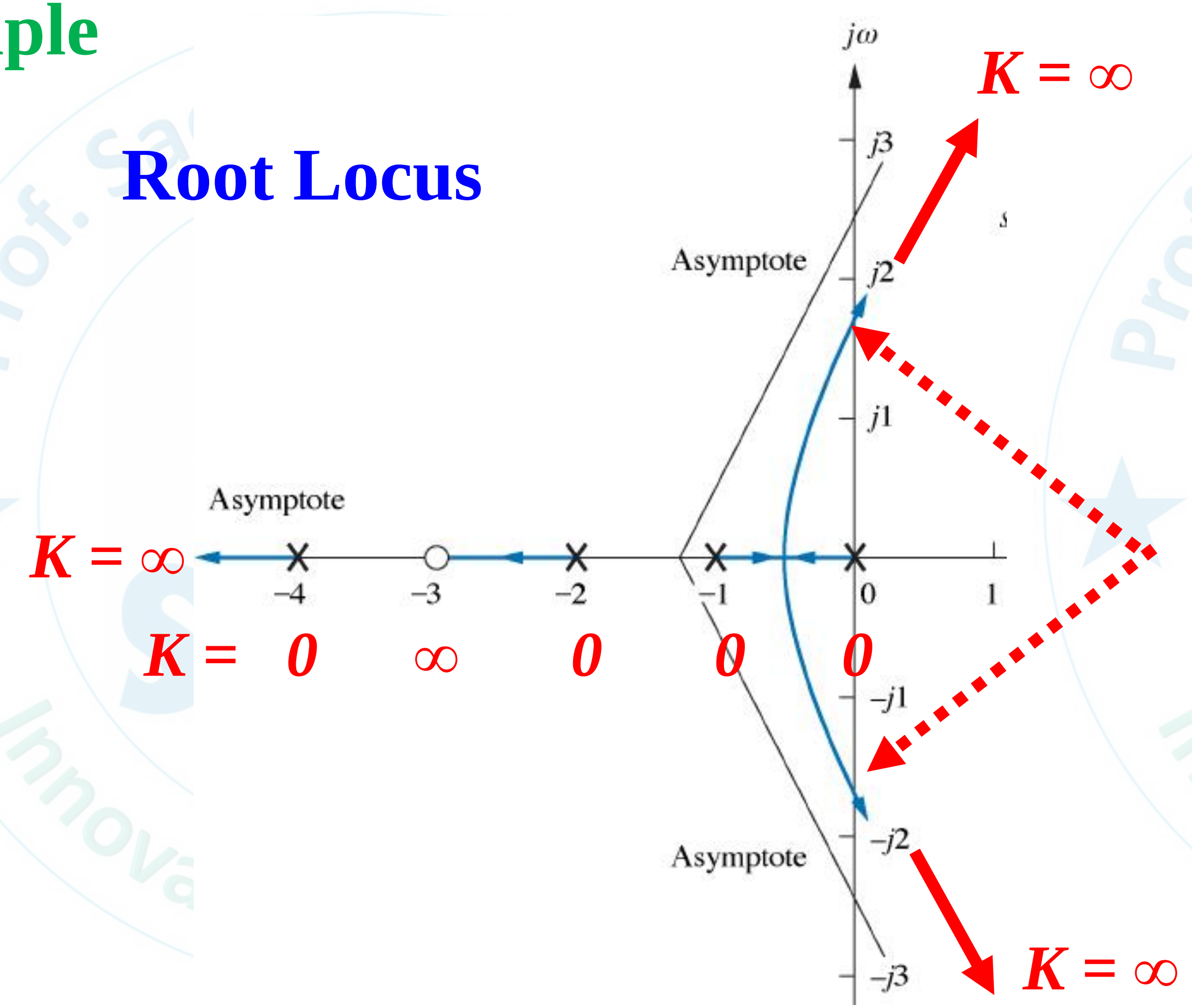
$$T = \frac{K(s+3)}{s^4 + 7s^3 + 14s^2 + (8+K)s + 3K}$$

s^4	1	14	$3K$
s^3	7	$8+K$	
s^2	$90-K$	$21K$	
s^1	$\dfrac{-K^2-65K+720}{90-K}$		
s^0	$21K$		

Nyquist Stability

Example

Root Locus



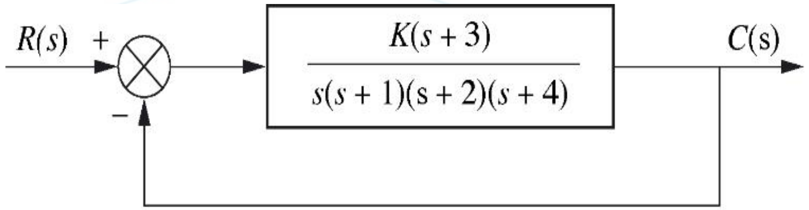
From Routh Table

$$-K^2 - 65K + 720 = 0$$

$$\rightarrow K = 9.65$$

Nyquist Stability

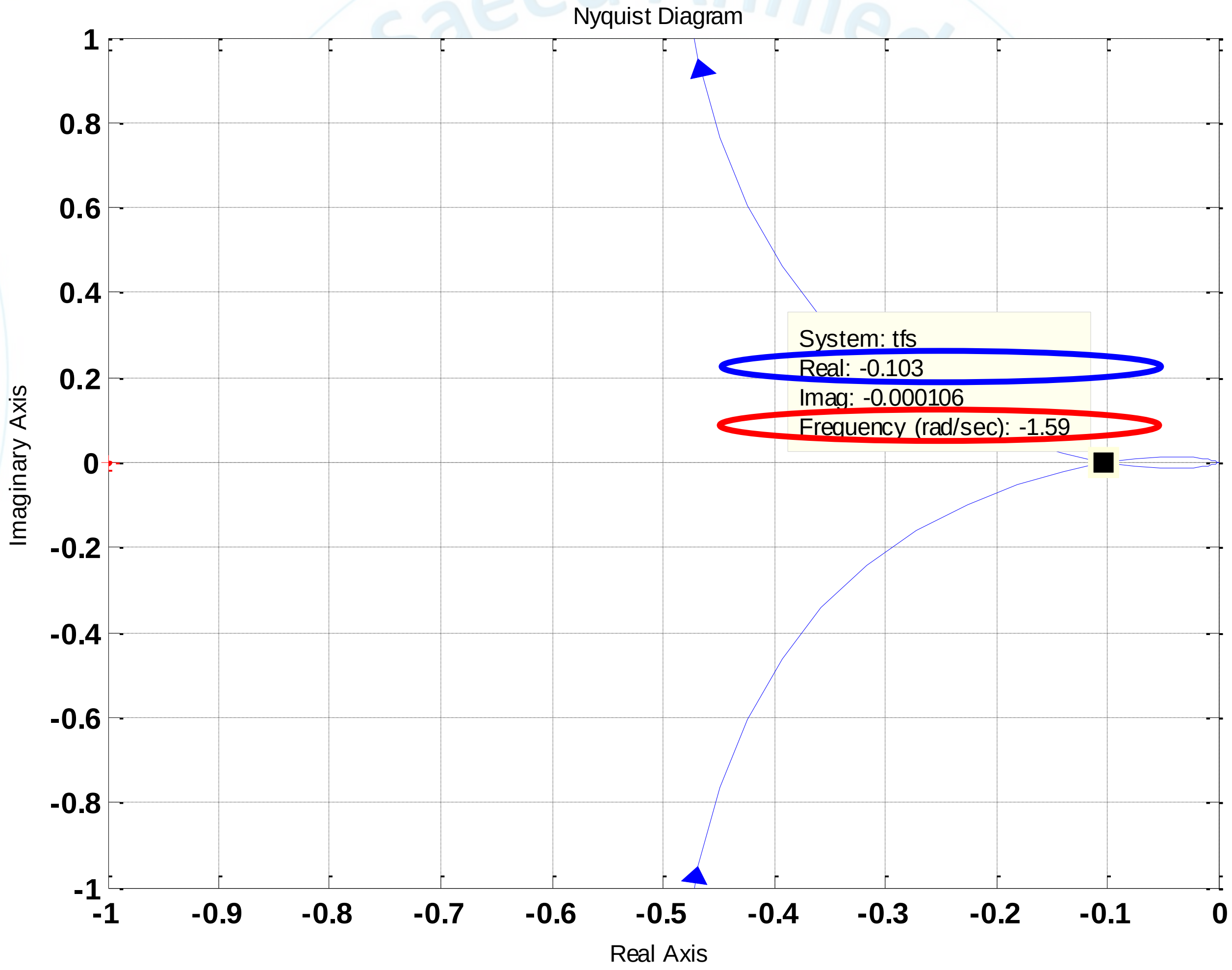
Exampl




```
>> n=[1 3];
>> tfs=zpk([-3],[0 -1 -2 -4],1);
>> nyquist(tfs)
```

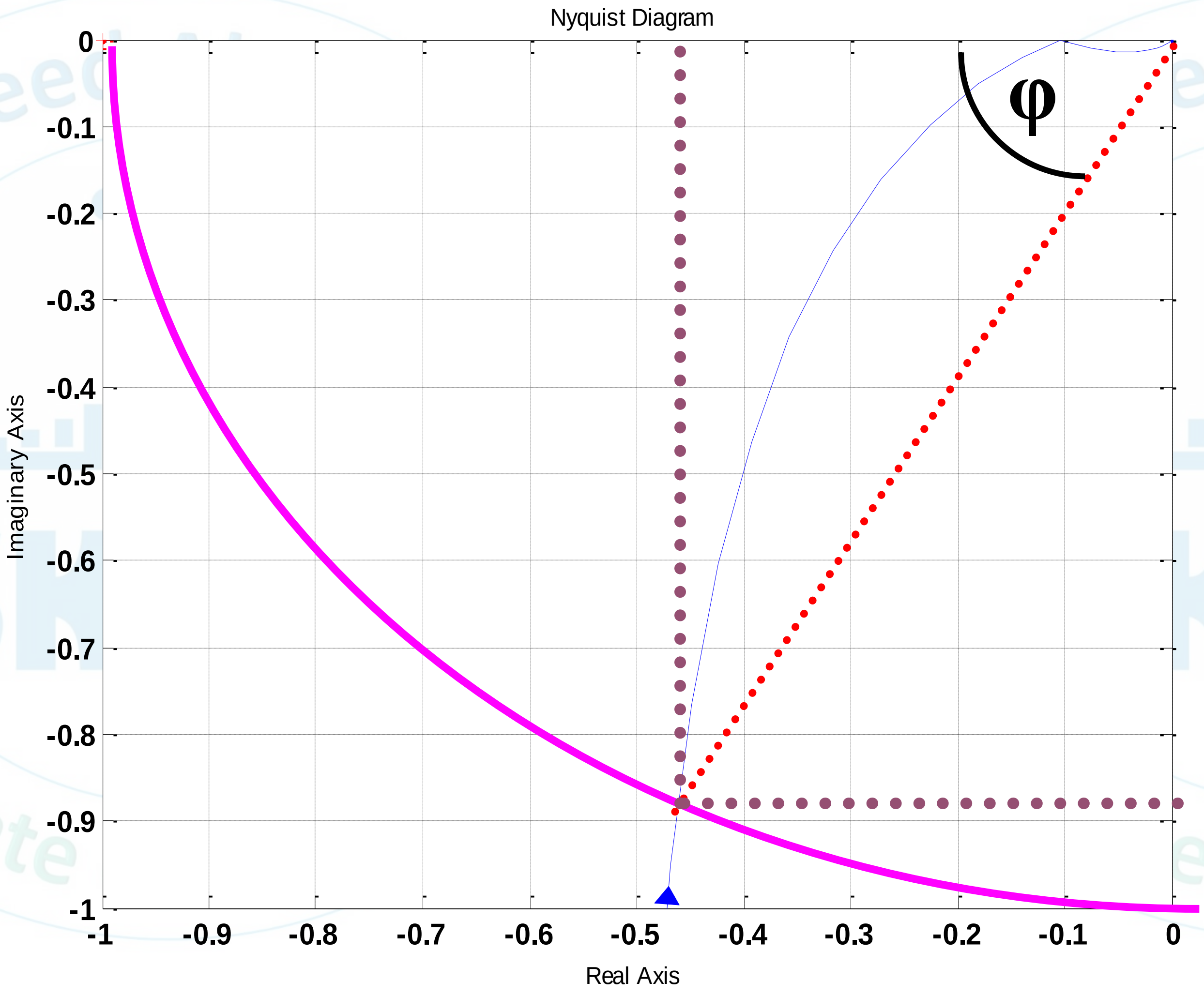
Gain Margin

$GM = 1/0.103 = 9.7$



Nyquist Stability

Example



$\varphi = \tan^{-1}(0.88/0.47)$

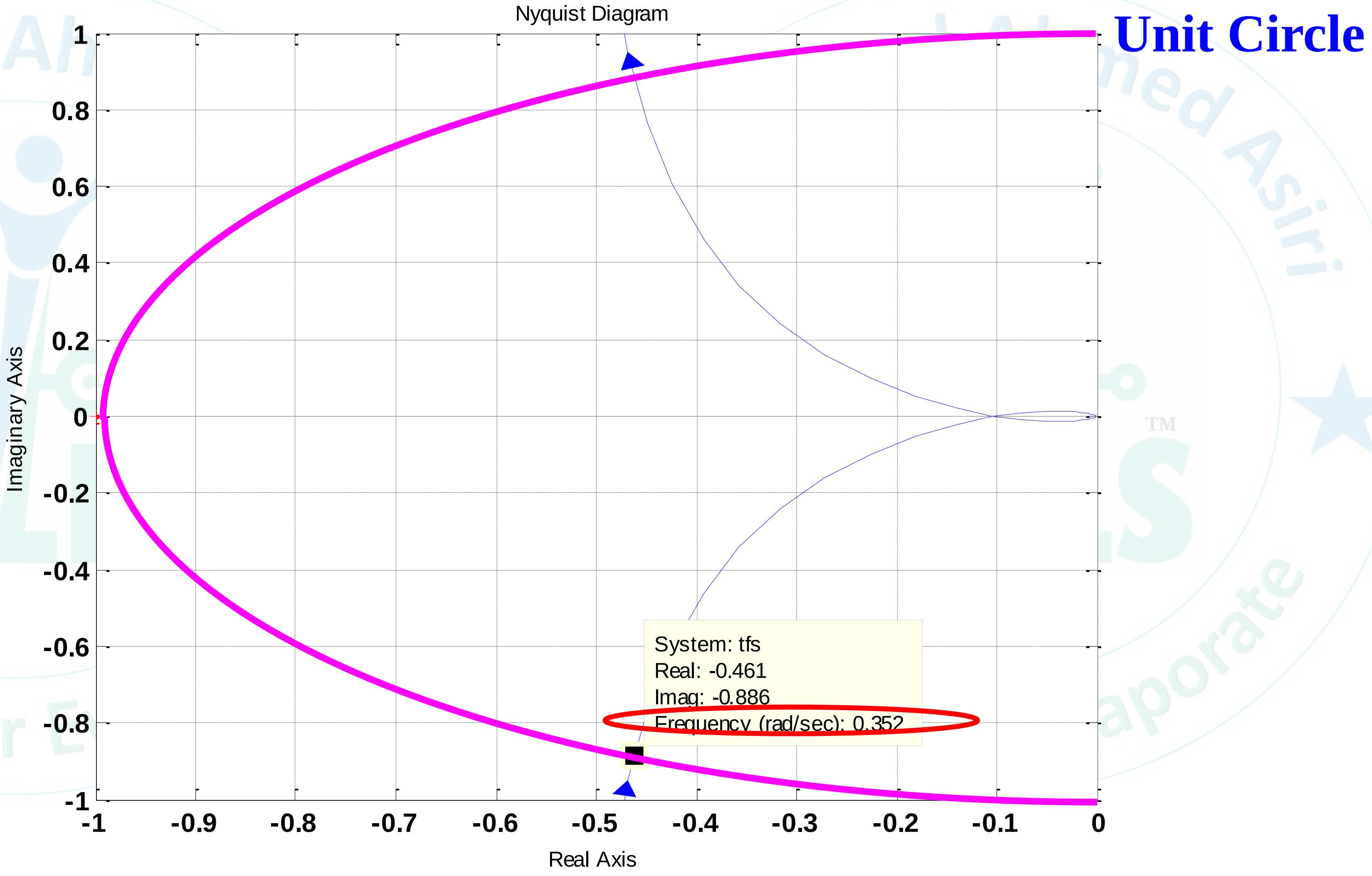
$= 61.89^o$

Phase Margin

$\varphi = 61.89^o$

Nyquist Stability

Example



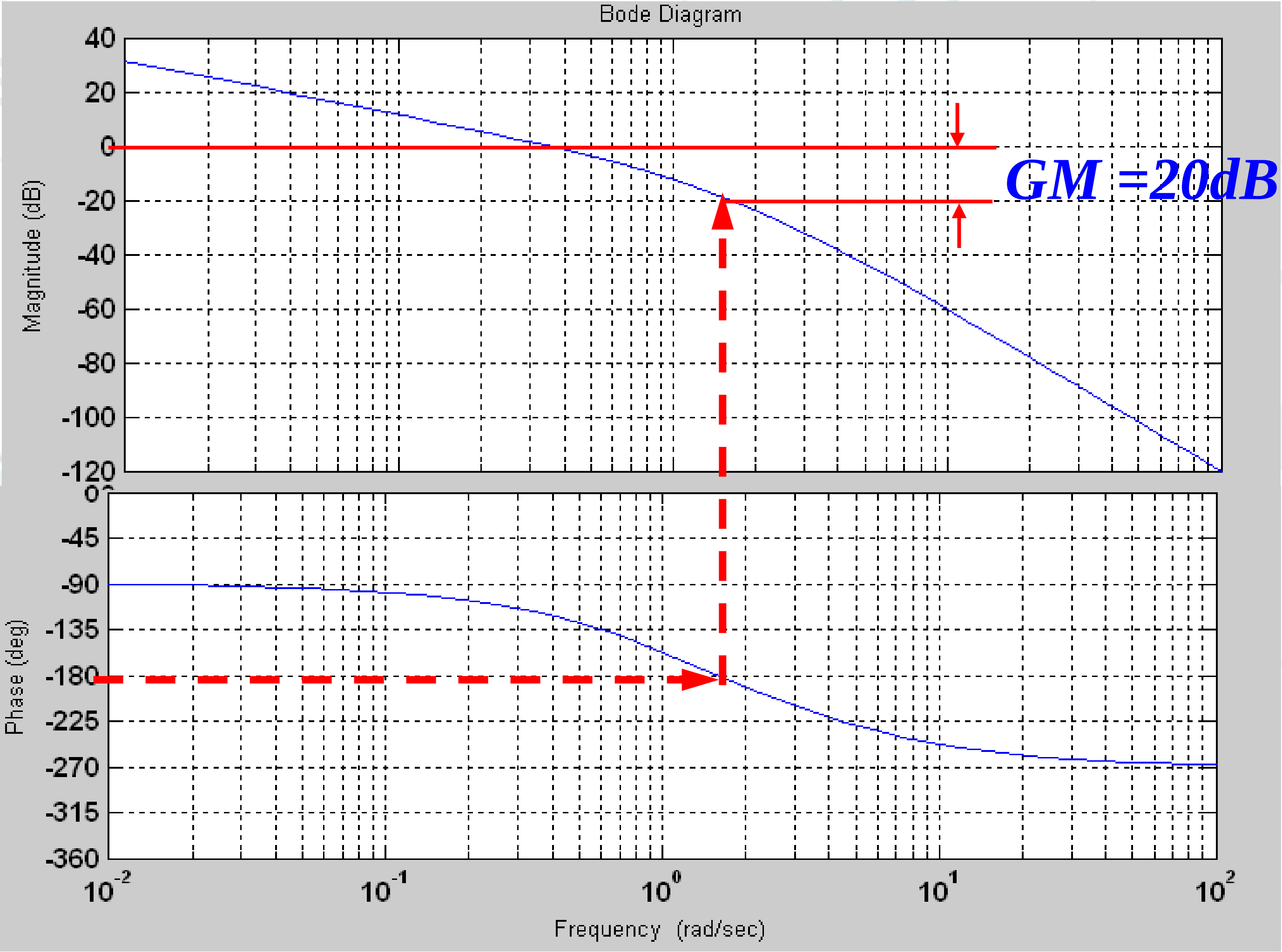
Nyquist Stability

Example

```
>> [Gm,Pm,wg,wp]=margin(tfs)

Gm =    9.6456

Pm =   62.4587

wg =    1.5877

wp =    0.3497
```

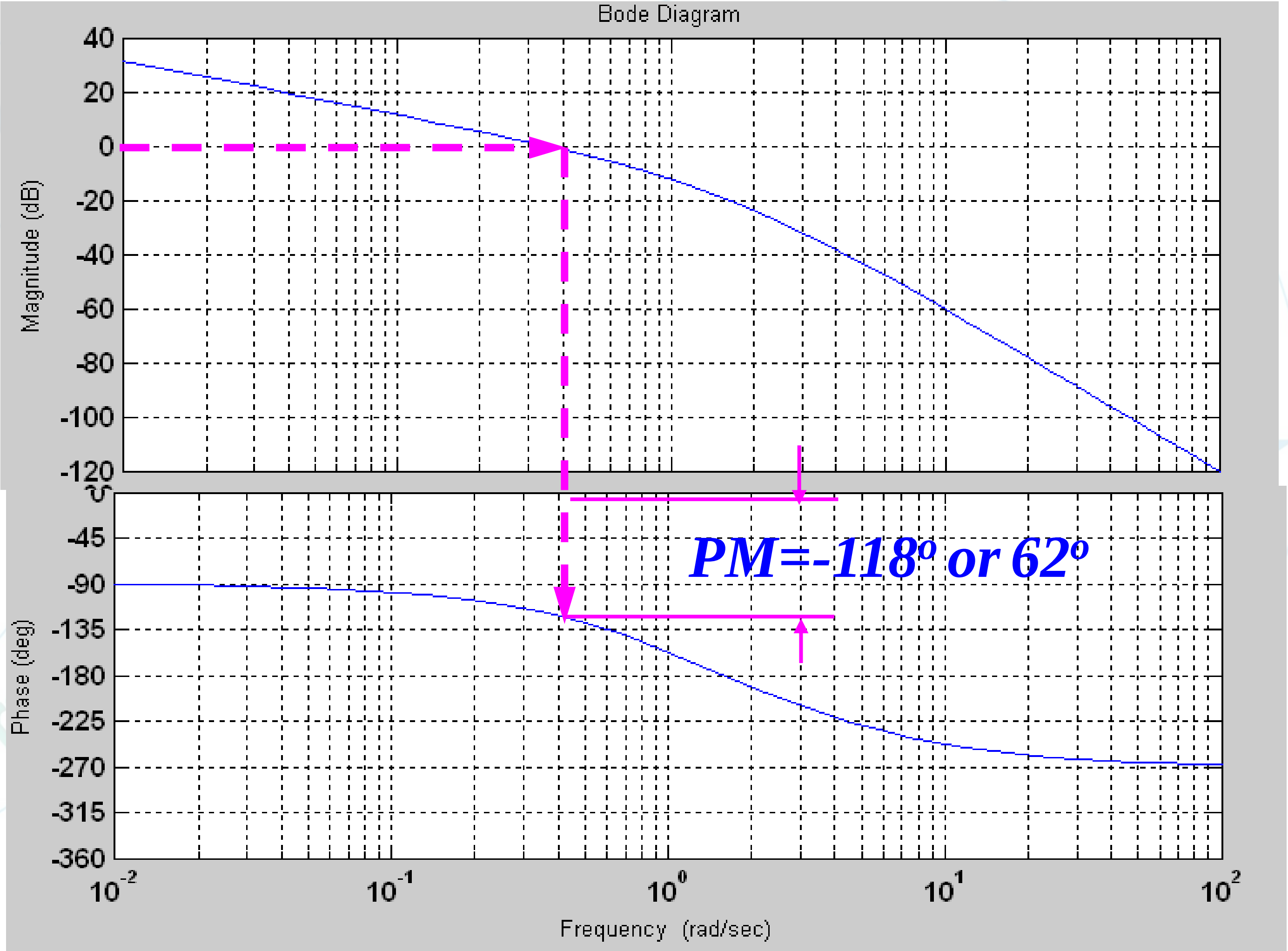
Nyquist Stability

Example



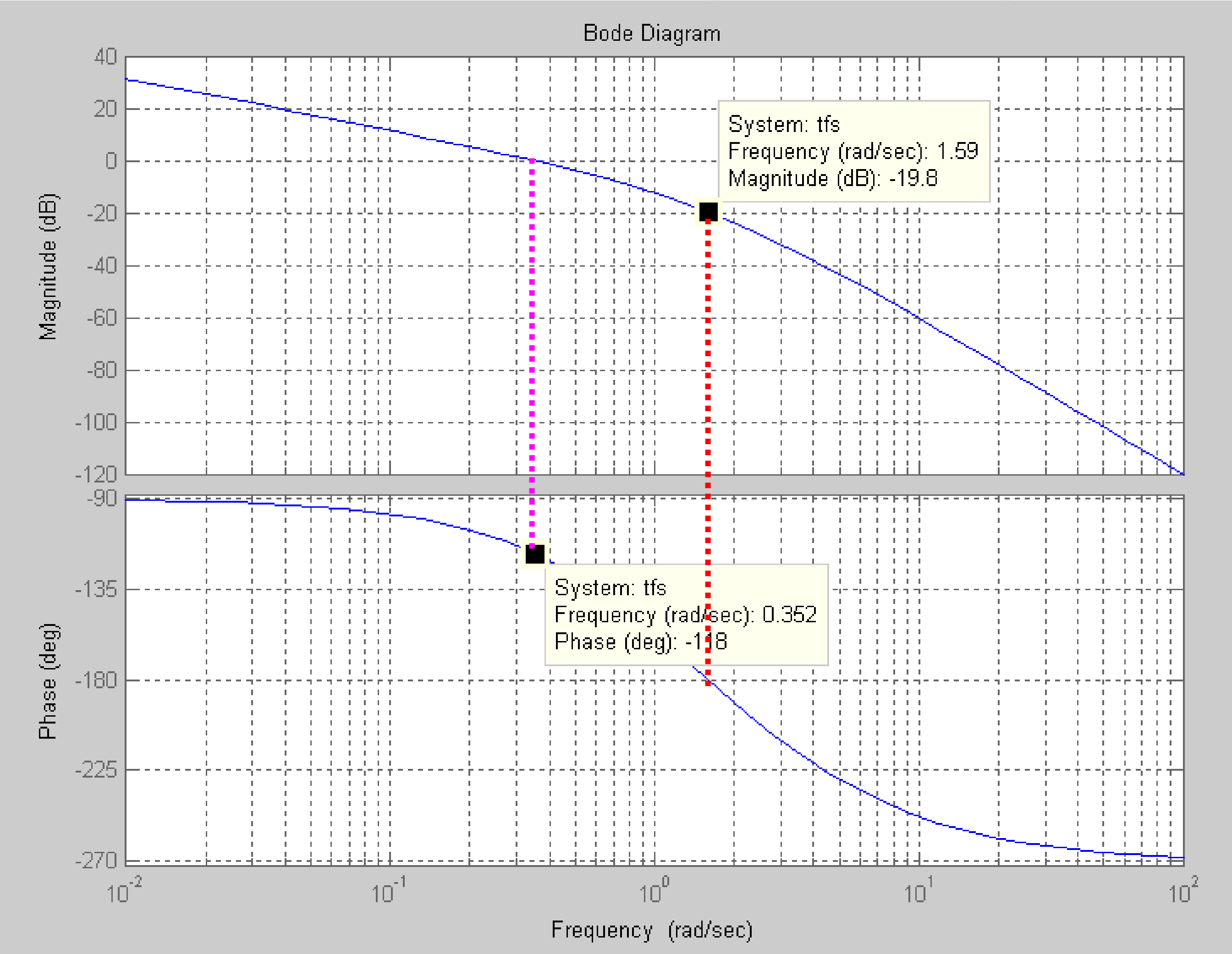
Nyquist Stability

Example

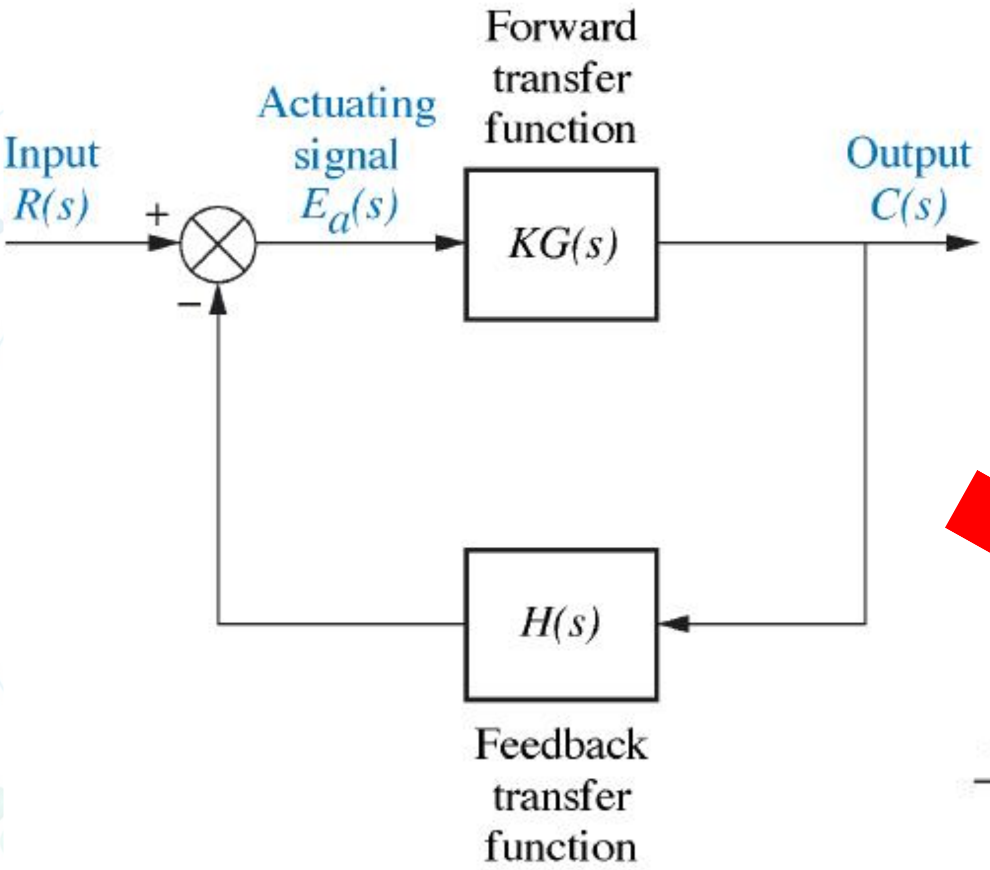


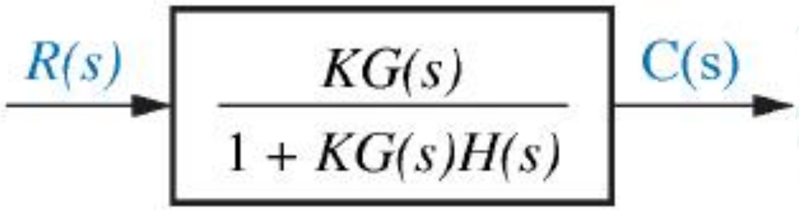
Nyquist Stability

Example



Nyquist Stability





Nyquist Stability

When $1+KG(s)H(s)=0$

System output → ∞

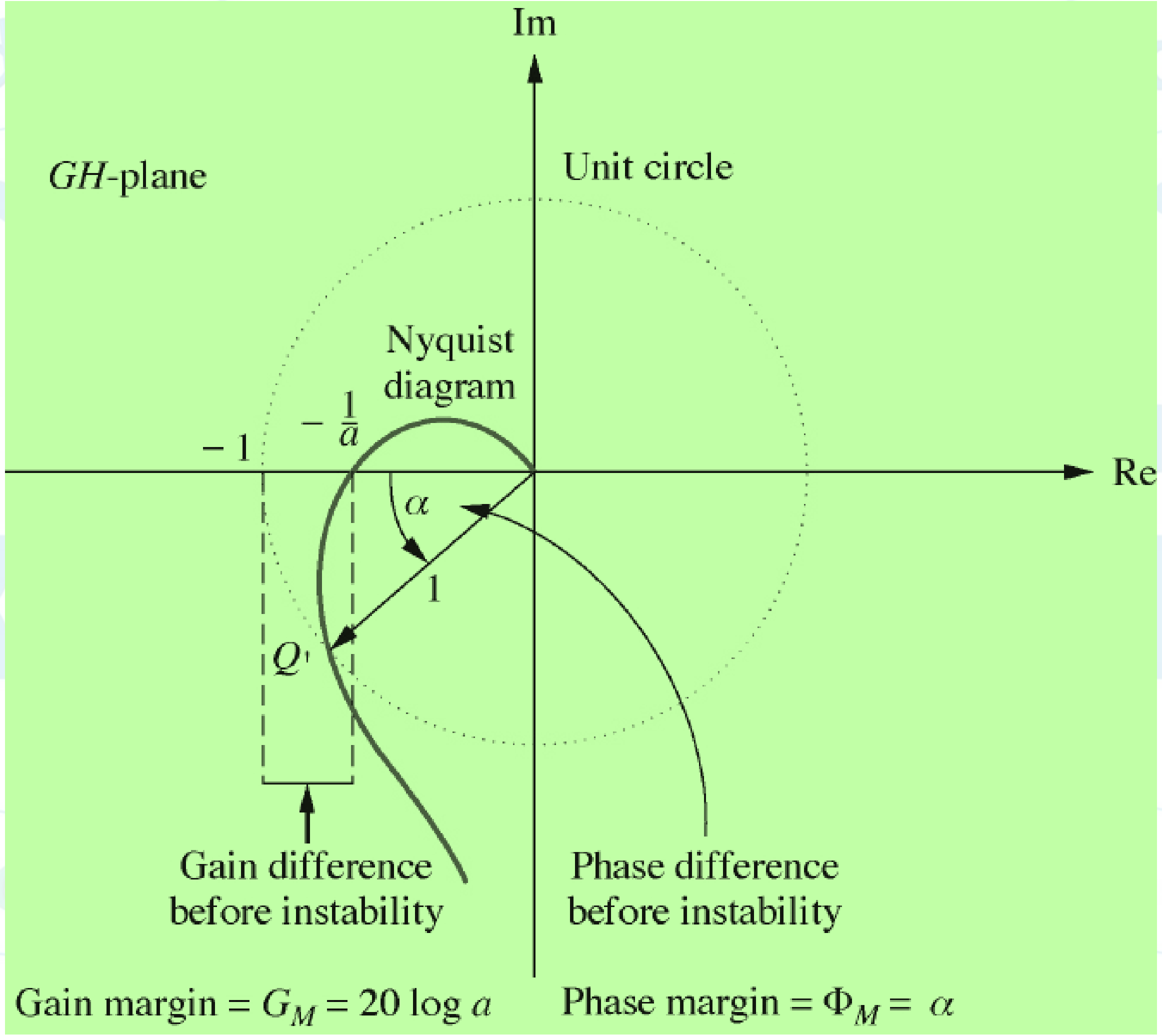
a. Magnitude Property $KG(s)H(s)=-1,$

or $K=\dfrac{1}{|G(s)H(s)|}$ **Gain Margin**

b. Phase angle Property

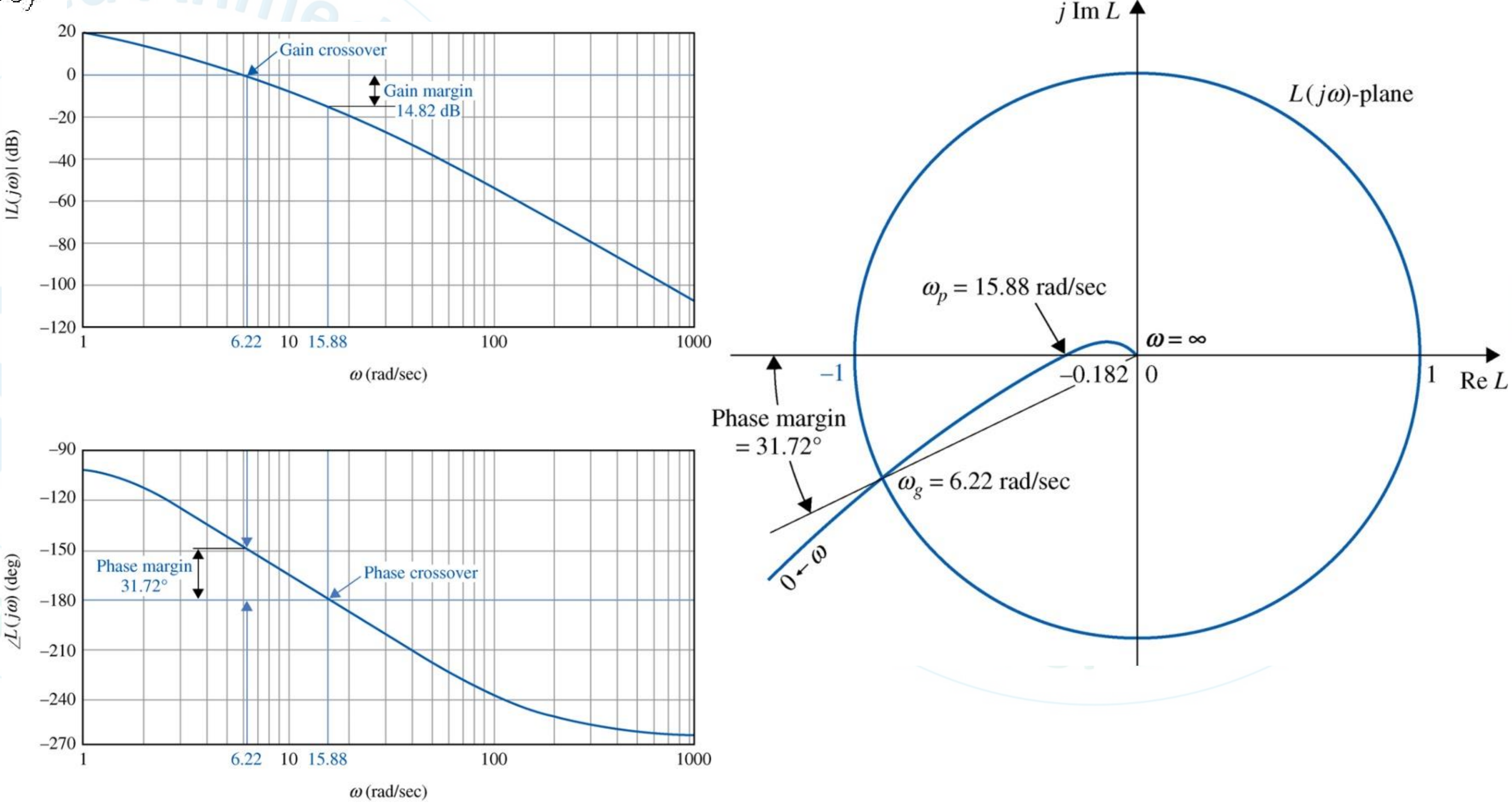
$$\angle[G(s)H(s)]=\pi$$

Nyquist Stability



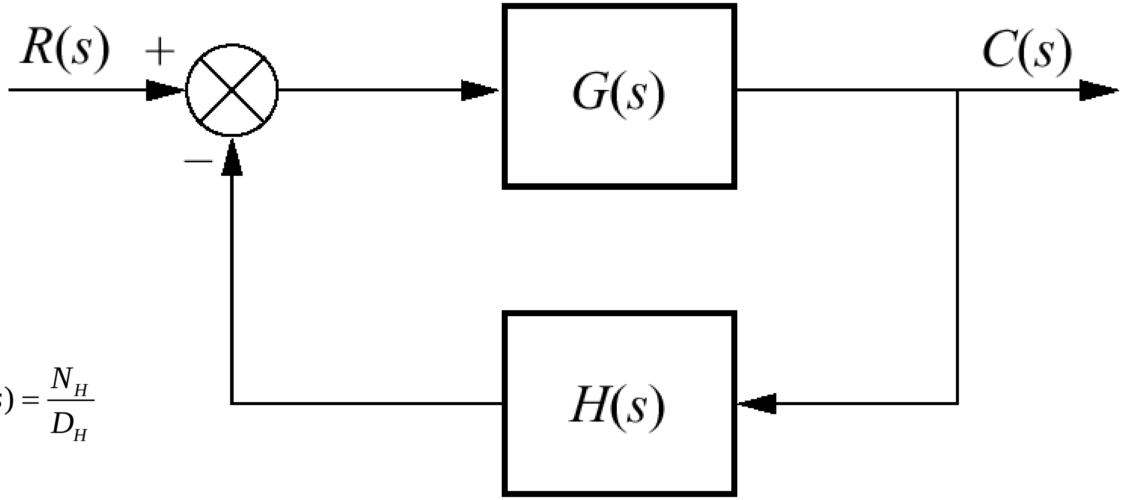
Nyquist Stability

$$L(s) = \frac{2500}{s(s+5)(s+50)}.$$



Nyquist Stability

Defines number of closed-loop poles in the right half-plane

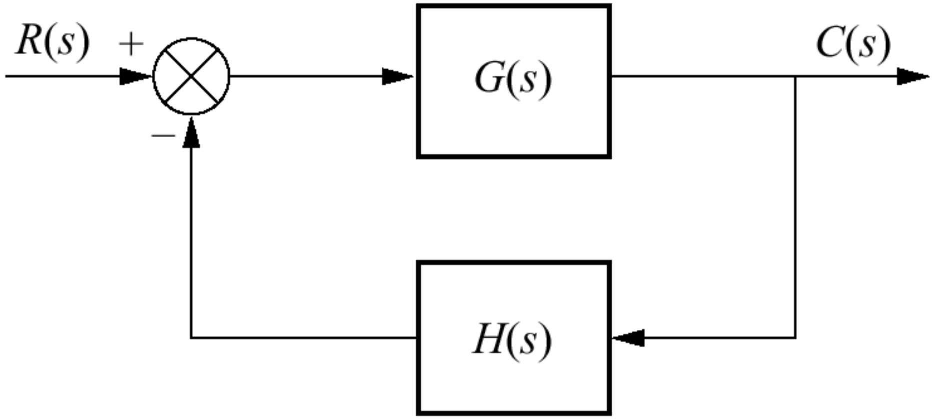


$$H(s) = \frac{N_H}{D_H}$$

$$G(s) = \frac{N_G}{D_G} \quad \&$$

$$G(s)H(s) = \frac{N_G}{D_G}\frac{N_H}{D_H}$$

Nyquist Stability



$$G(s)H(s)=\frac{N_G}{D_G}\frac{N_H}{D_H}$$

$$1+G(s)H(s)=\frac{N_GN_H+D_GD_H}{D_GD_H}$$

$$T(s)=\frac{G(s)}{1+G(s)H(s)}=\frac{N_GD_H}{D_GD_H+N_GN_H}$$

Nyquist Stability

Relationships between:

Poles of 1+$G(s)H(s)$ & Poles of $G(s)H(s)$

$$G(s)H(s) = \frac{N_G}{D_G}\frac{N_H}{D_H}$$

$$1 + G(s)H(s) = \frac{N_G N_H + D_G D_H}{D_G D_H}$$

Zeros of 1+$G(s)H(s)$ & Poles of $T(s)$

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{N_G D_H}{D_G D_H + N_G N_H}$$

Nyquist Diagram and Analysis

Nyquist Criterion

- Let us first introduce the most **important equation** when dealing with the Nyquist criterion:

$$Z = N + P$$

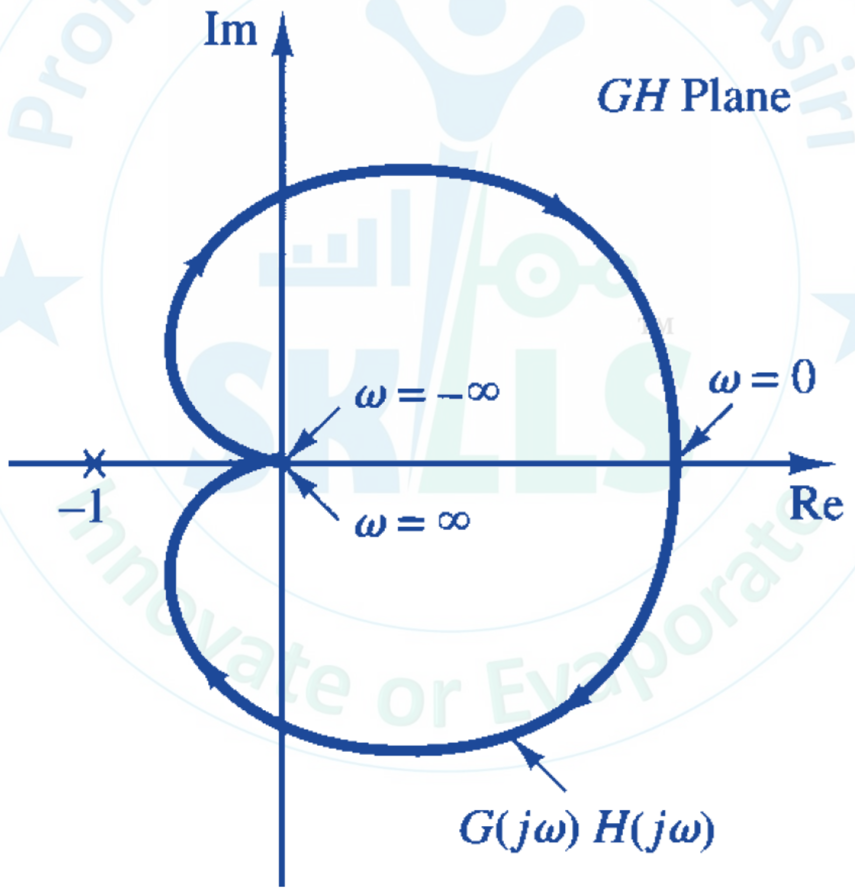
- Where:
  - **Z** is the number of zeros of the characteristic equation (and therefore the number of poles of the closed-loop transfer function) that are in the right-half of the s plane.
  - **N** is the number of encirclements of the (-1, 0) point.
  - **P** is the number of poles of the open-loop characteristic equation that are in the right-half of the s plane.

Nyquist Diagram and Analysis

Example

Investigate the stability of $GH(s) = \frac{K}{(1+T_1 s)(1+T_2 s)}$ using Nyquist Stability Criteria

Since P = 0, so we need N=Z=0 for stability.
As shown in the figure, there is no encirclement for -1. Hence, the closed loop system is stable for any positive value of K, T_1, T_2

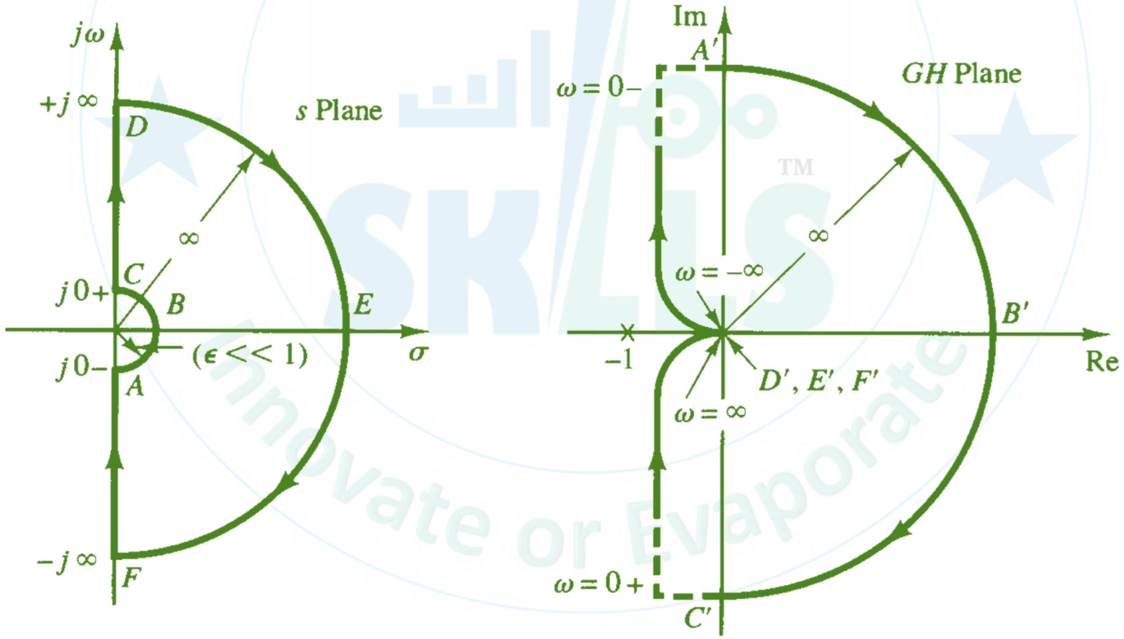


Nyquist Diagram and Analysis

Example

Investigate the stability of $G(s) = K/[s(1 + Ts)]$ using Nyquist Stability Criteria

Since P = 0, so we need N=Z=0 for stability.
As shown in the figure, there is no encirclement for -1. Hence, the closed loop system is stable.

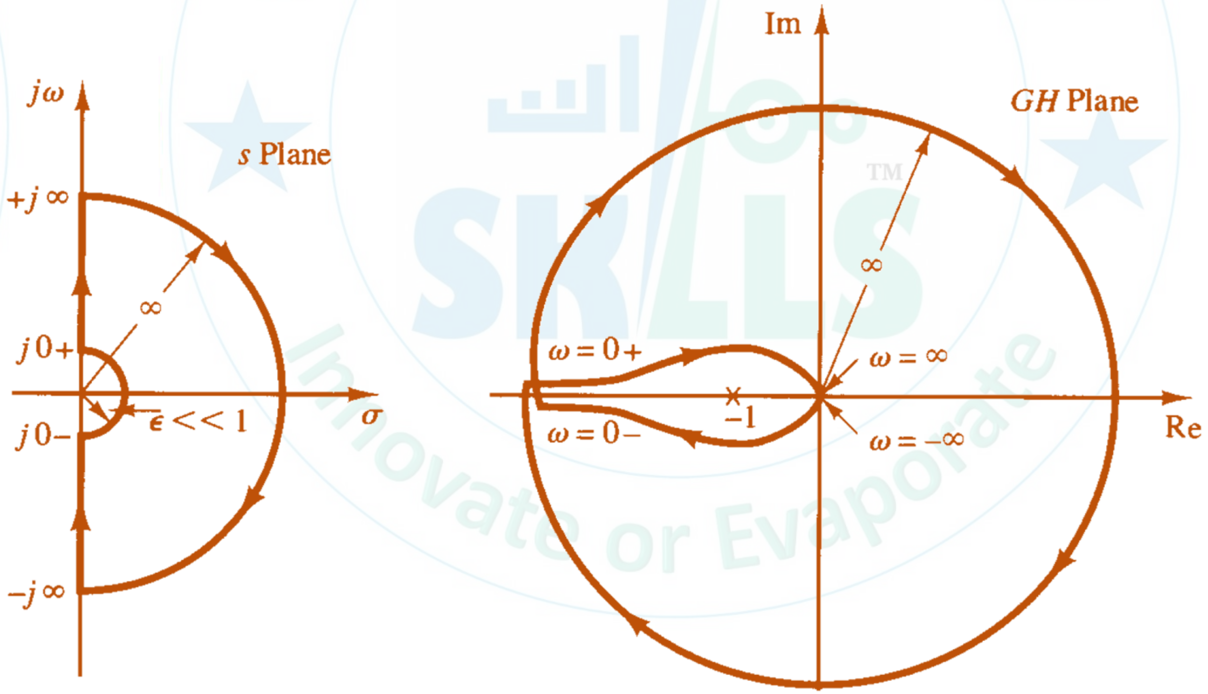


Nyquist Diagram and Analysis

Example

Investigate the stability of $G(s) = K / [s^2 (1 + T s)]$ using Nyquist Stability Criteria

Since P = 0 for positive *T*, so we need N=Z=0 for stability.
But as shown in the figure, there is two clockwise encirclements for -1 so N = 2.
Hence, the closed loop system is unstable.

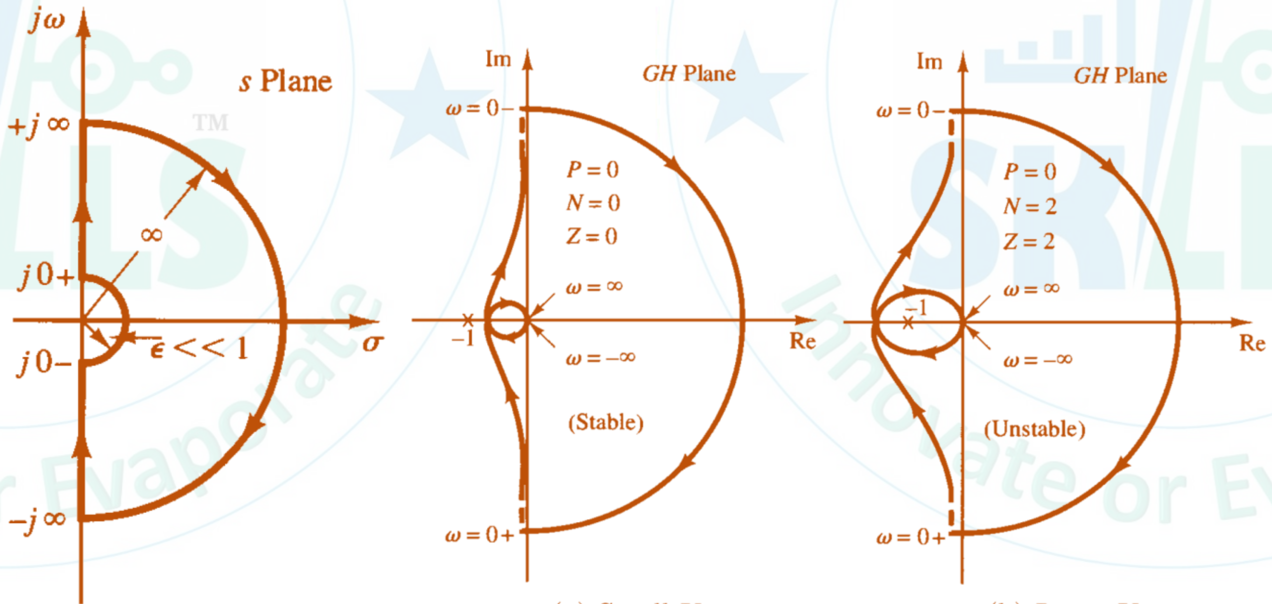


Nyquist Diagram and Analysis

Example

Investigate the stability of $GH(s) = \dfrac{K}{s(1+T_1 s)(1+T_2 s)}$ using Nyquist Stability Criteria

Since P = 0 for positive T_1 and T_2, so we need N=Z=0 for stability.



(a) Small K (b) Large K

Nyquist Diagram and Analysis

Example

Consider the following system:

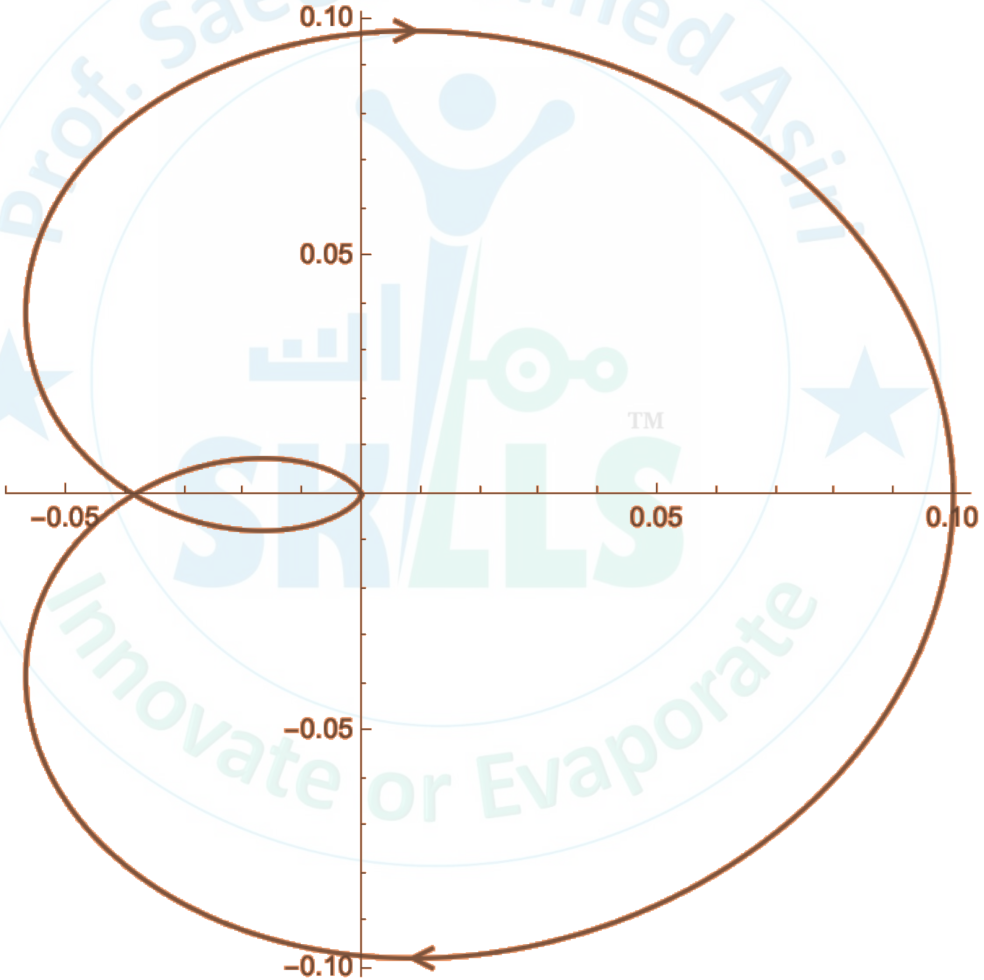
$$G(s) = \frac{1}{(s+2)(s^2+2s+5)}$$

Suppose this is in unity feedback with a constant gain controller K. In other words, we have a negative feedback loop where the forward gain is $KG(s)$ and the loop gain is also $KG(s)$.

Investigate the stability of the system using

a) Routh Array

b) Nyquist Plot



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