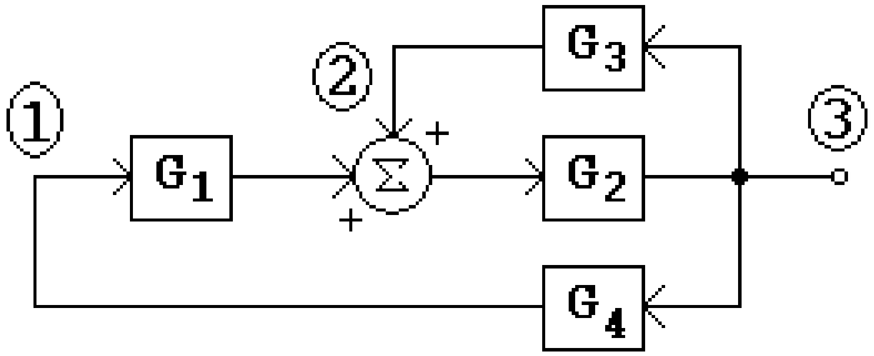
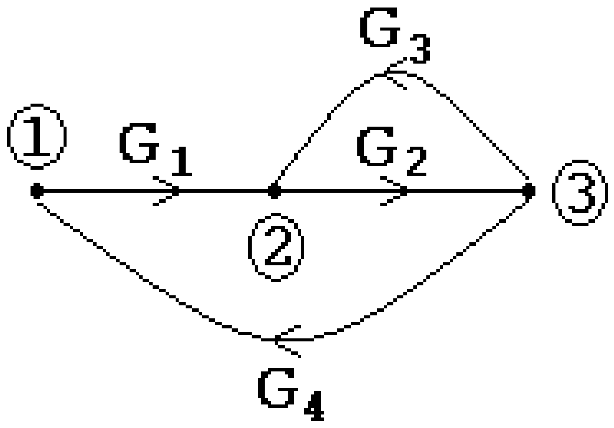
MENG366

Signal Flow Graph

Dr. Saeed Asiri
saeed@asiri.net

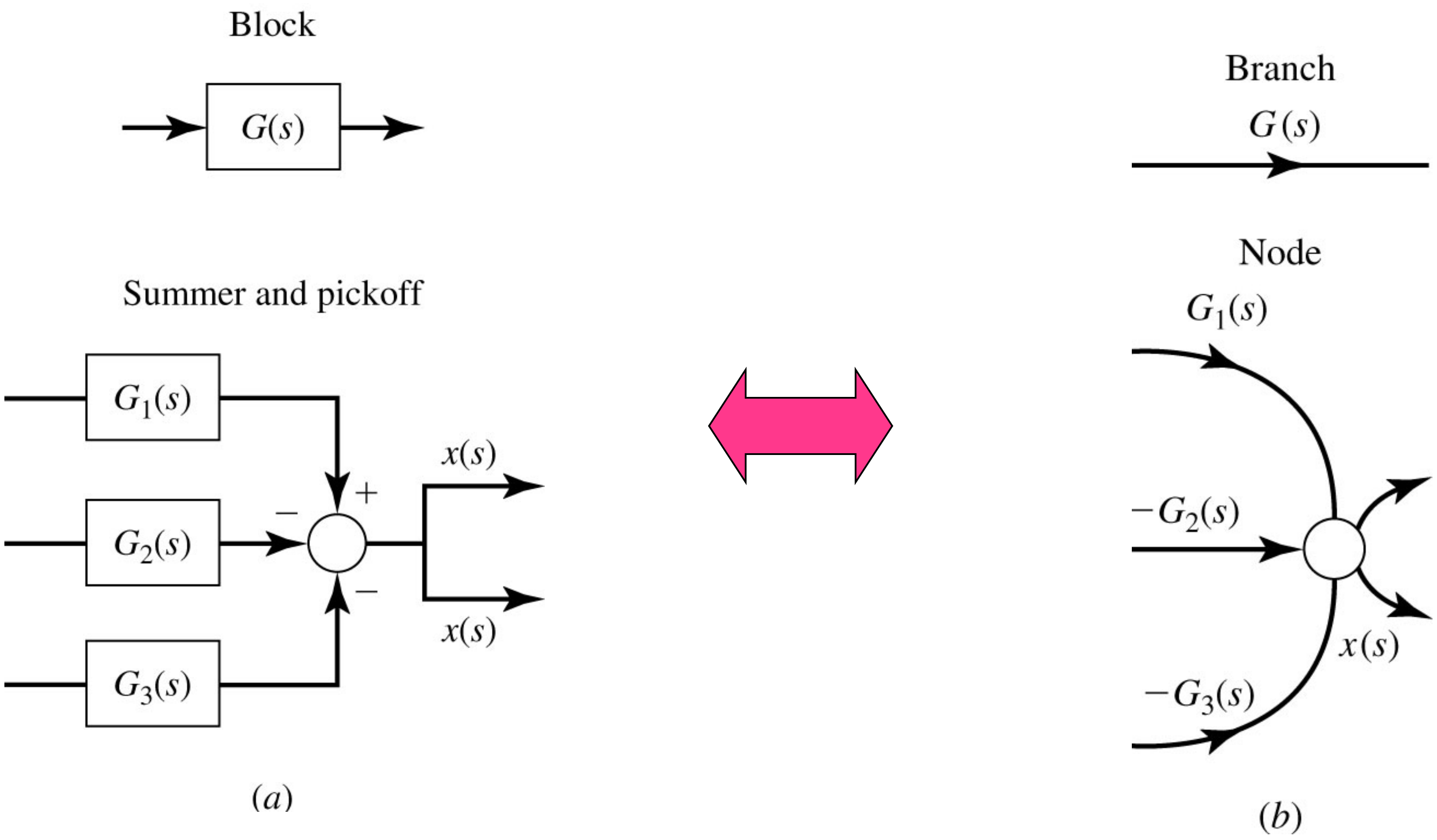
Signal Flow Graph



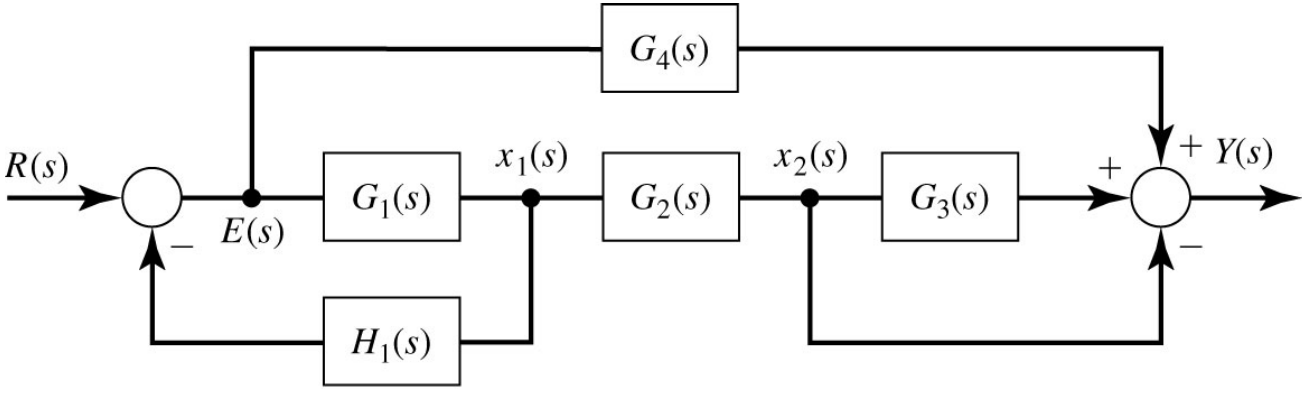


- Compact alternative ***notation to the block diagram***.
- It characterizes the system by a network of directed branches and associated transfer functions.
- The two ways of depicting signal are equivalent.

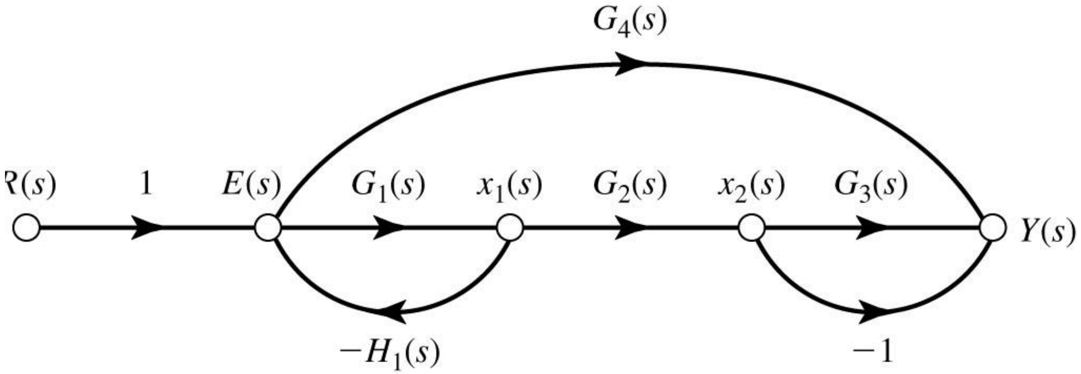
Signal Flow Graph



Signal Flow Graph



(a)



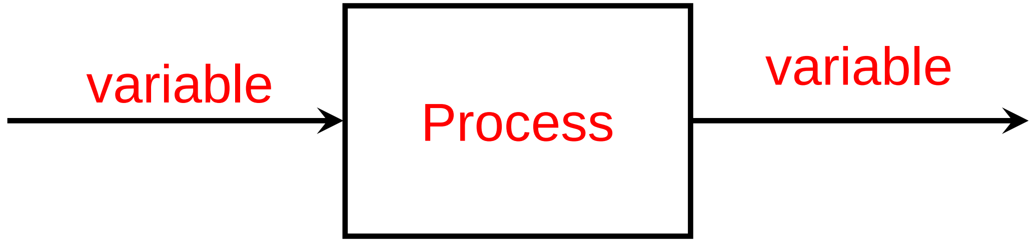
(b)

Block Diagram Models

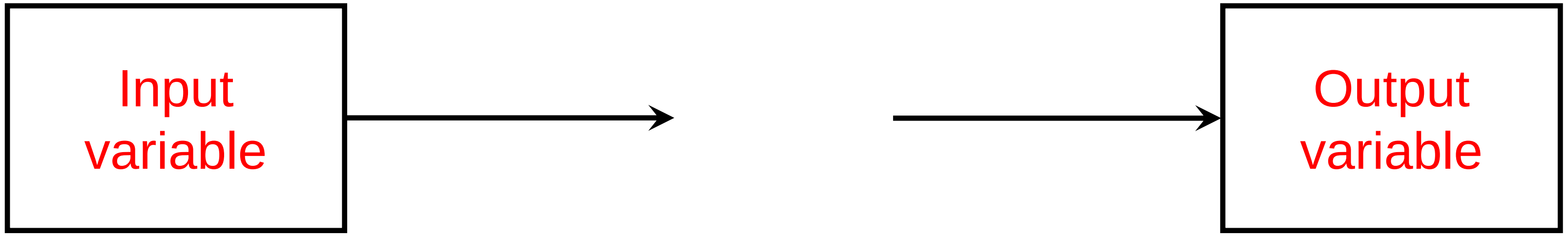
- Block diagrams are used as schematic representations of mathematical models
- The various pieces correspond to mathematical entities
- Can be rearranged to help simplify the equations used to model the system
- We will focus on one type of schematic diagram – feedback control systems

Block Diagram Models

- Processes are represented by the blocks in block diagrams:

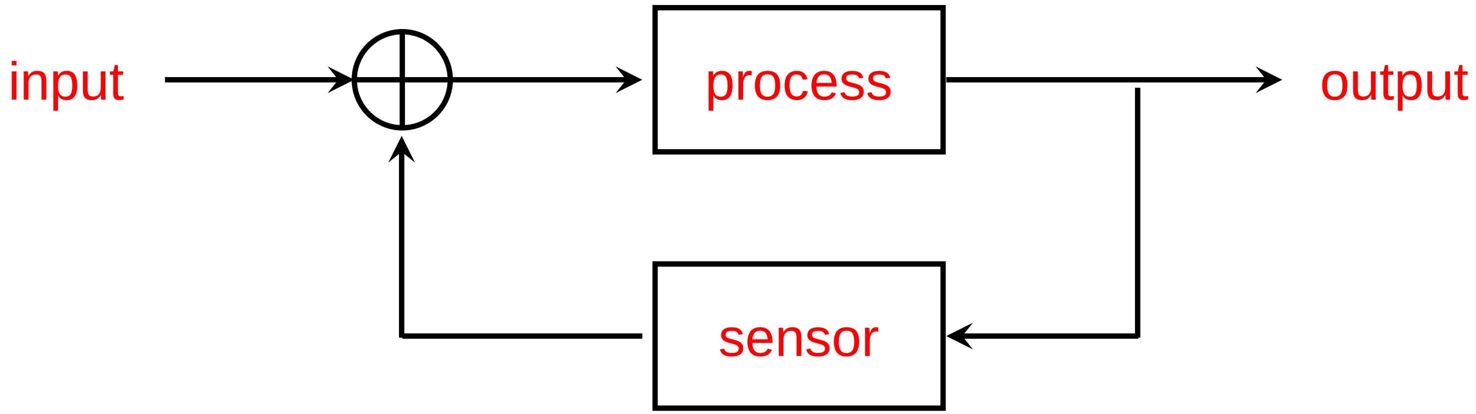


- Processes must have at least one input variable and at least one output variable
- Reclassify processes without input or output:

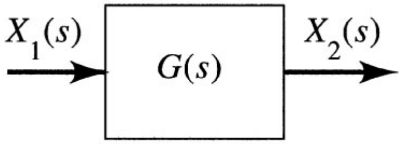


Block Diagram Models

- Many systems measure their output and use this measurement to control system behavior
- This is known as feedback control – the output is “fed back” into the system
- The summing junction is a special process that compares the input and the feedback
- Inputs to summing junction must have same units!

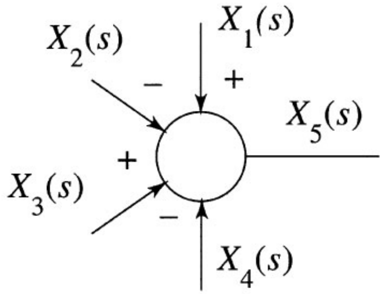


Block Diagram Models



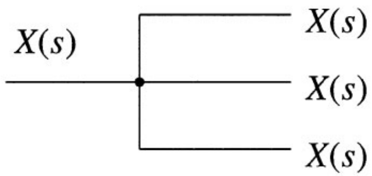
$X_2(s) = G(s)X_1(s)$

(a) ***block***

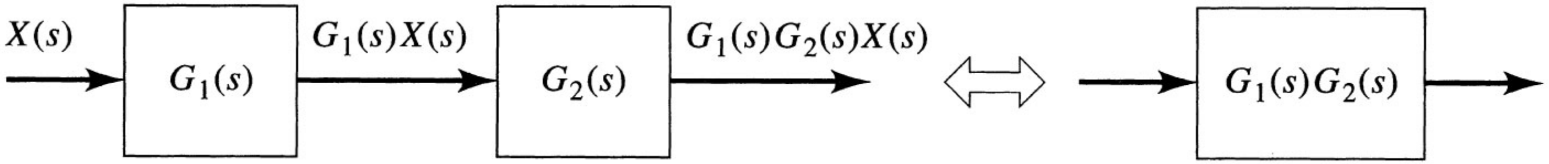


$X_5(s) = X_1(s) - X_2(s) + X_3(s) - X_4(s)$

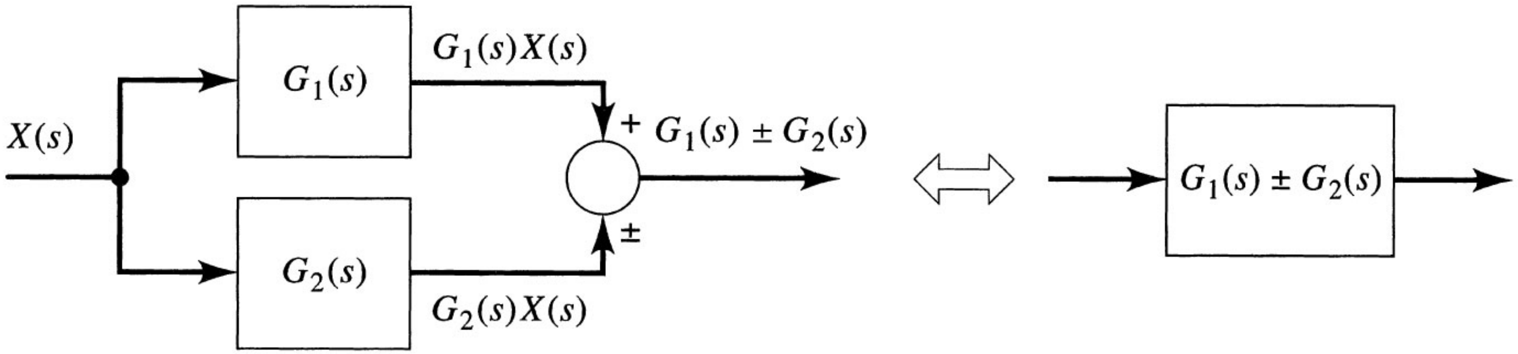
(b) ***summer***



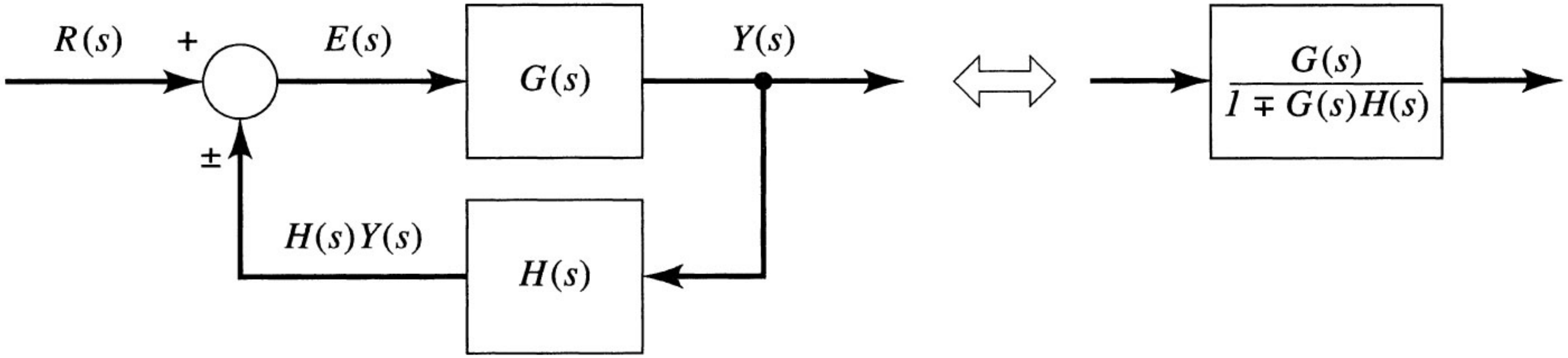
(c) ***pickoff point***



(a)



Block Diagram Models



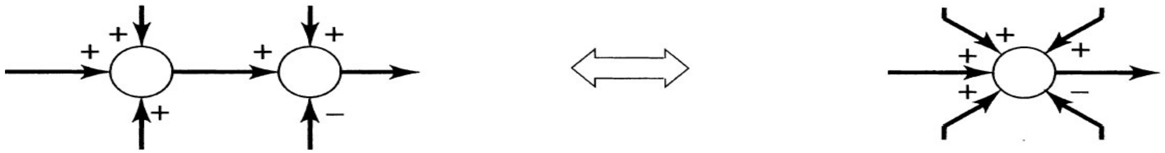
$$Y(s) = G(s)E(s)$$

$$E(s) = R(s) \pm H(s)Y(s)$$

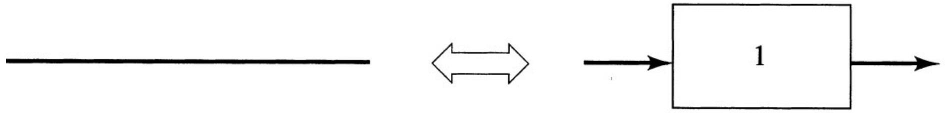
$$Y(s) = G(s)[R(s) \pm H(s)Y(s)] = G(s)R(s) \pm G(s)H(s)Y(s)$$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$

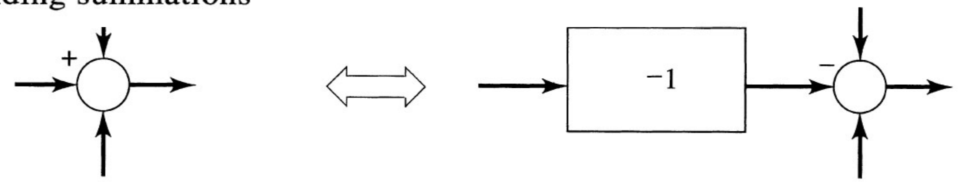
Block Diagram Models



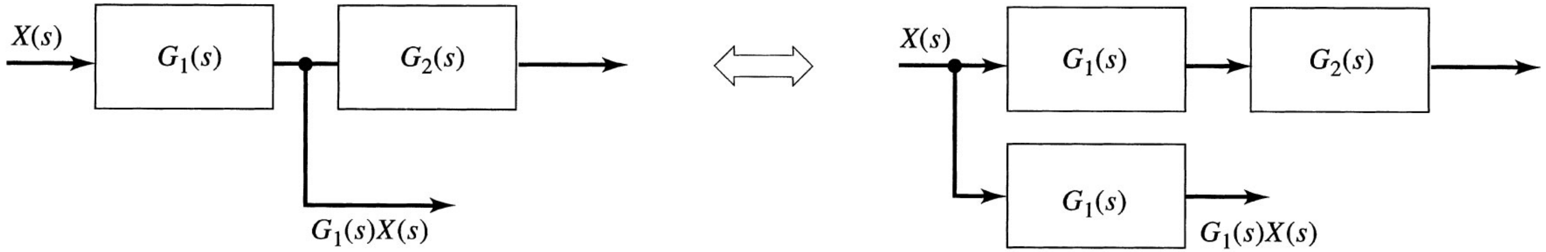
(*e*) Combining or expanding summations



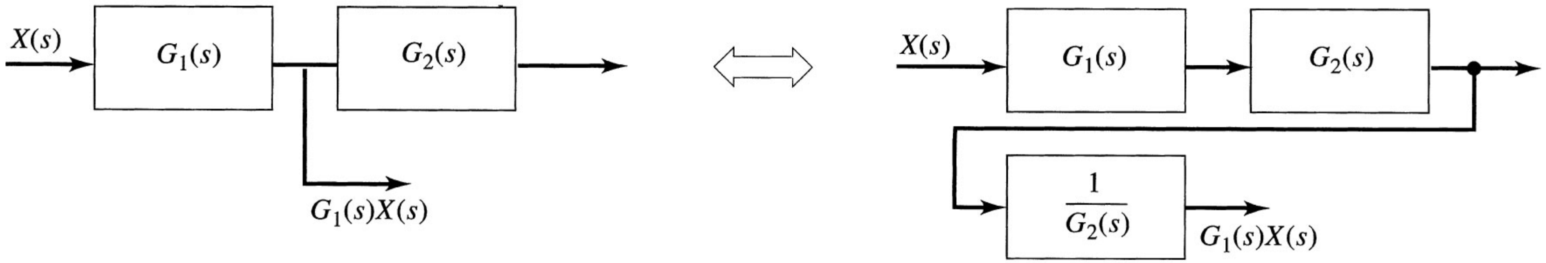
(*a*) Insertion or removal of unity gain



(*b*) Changing a summer sign



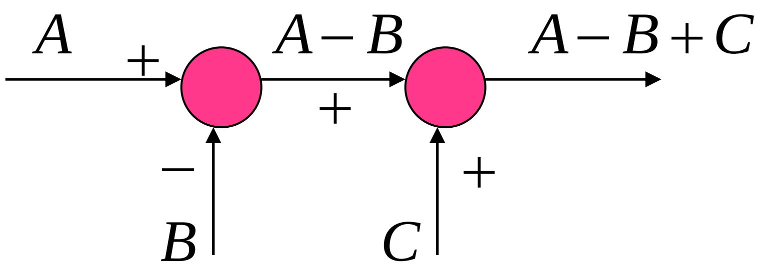
(*c*) Moving a pickoff point back

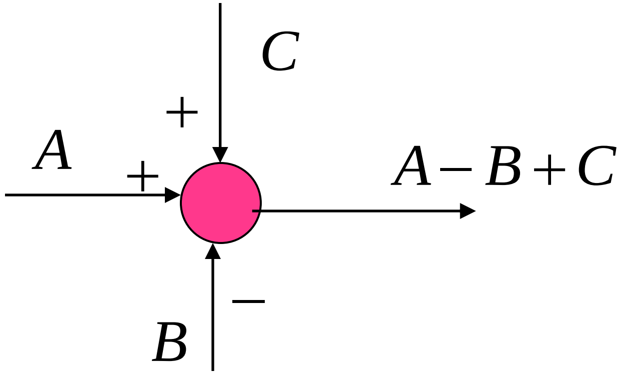


(*d*) Moving a pickoff point forward

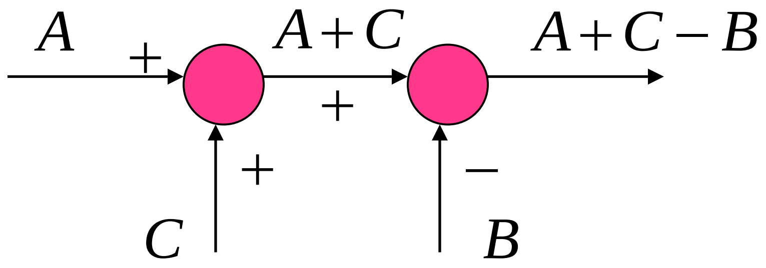
Block Diagram Models

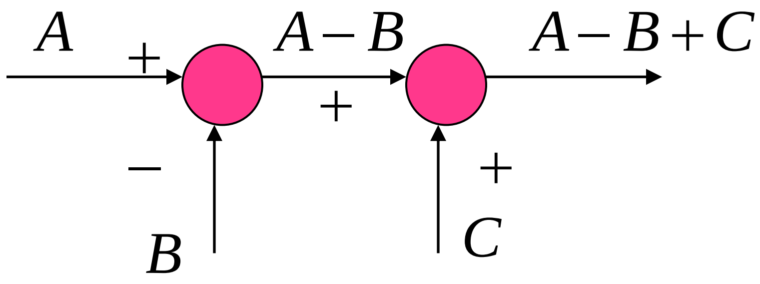
original





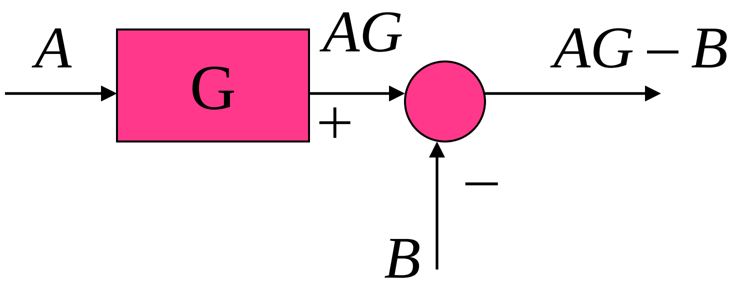
equivalent

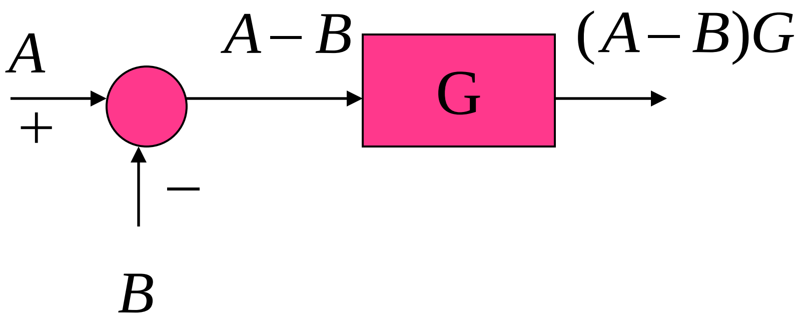




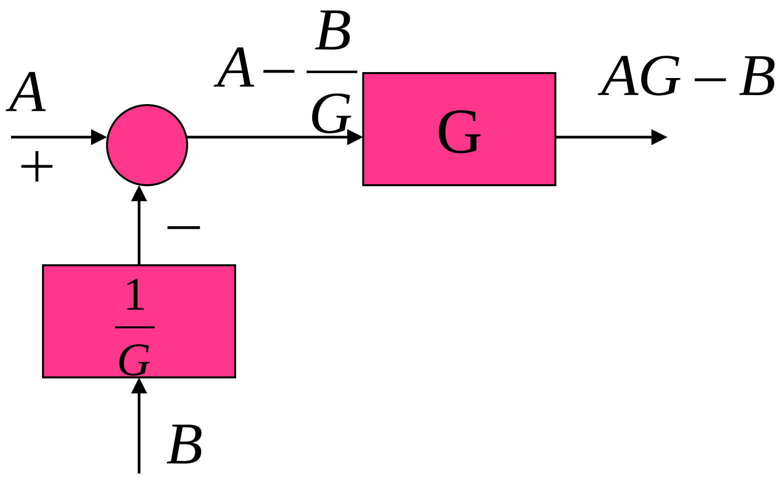
Block Diagram Models

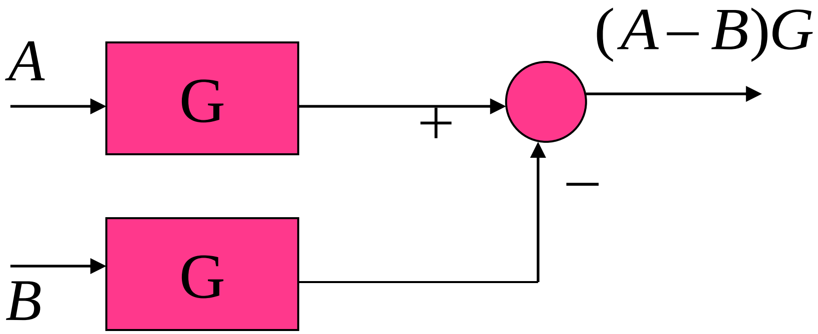
original





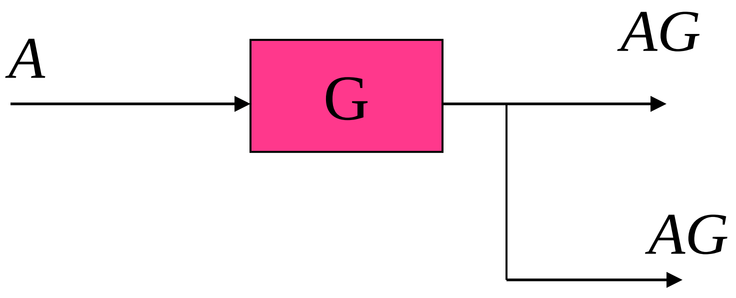
equivalent

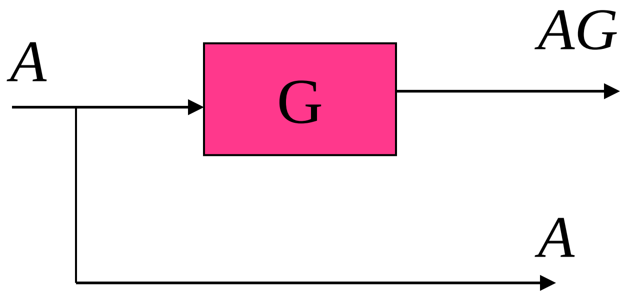




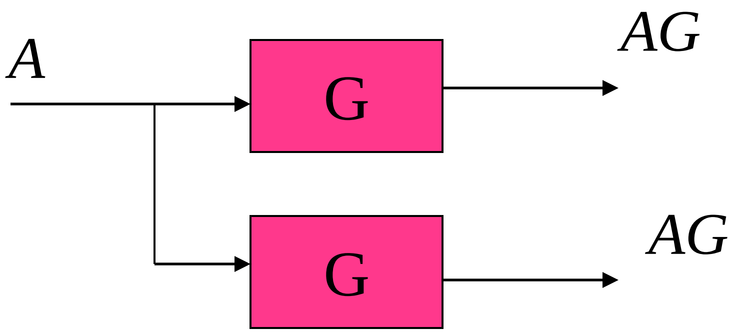
Block Diagram Models

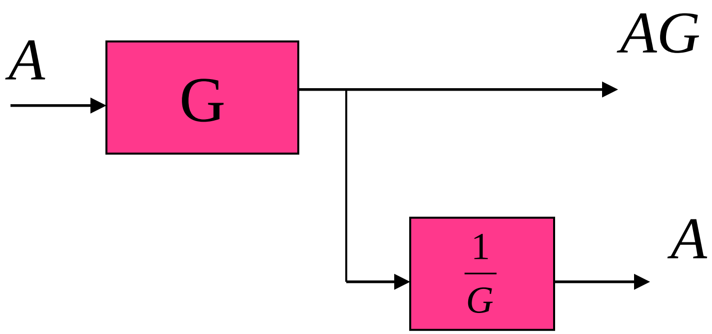
original





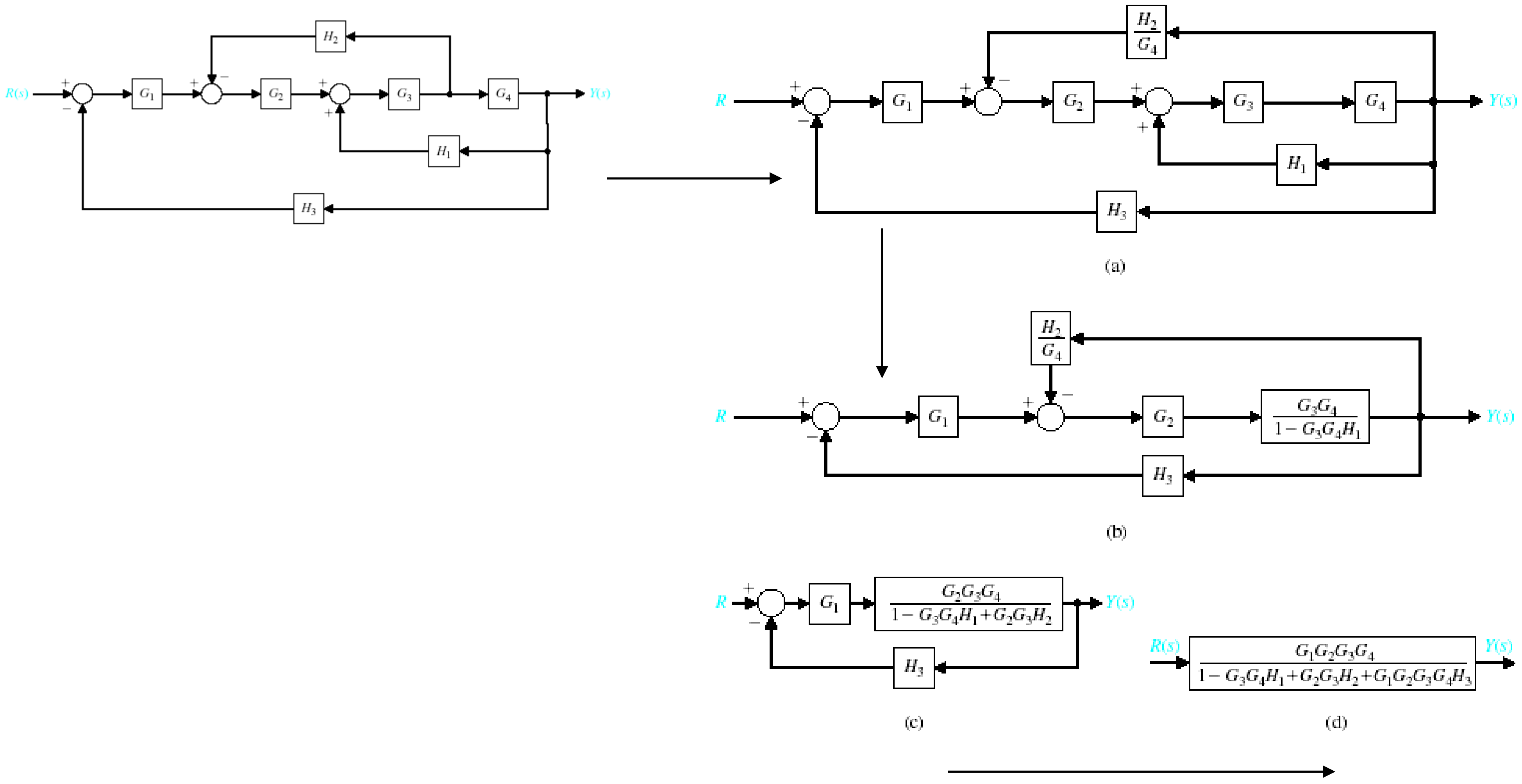
equivalent





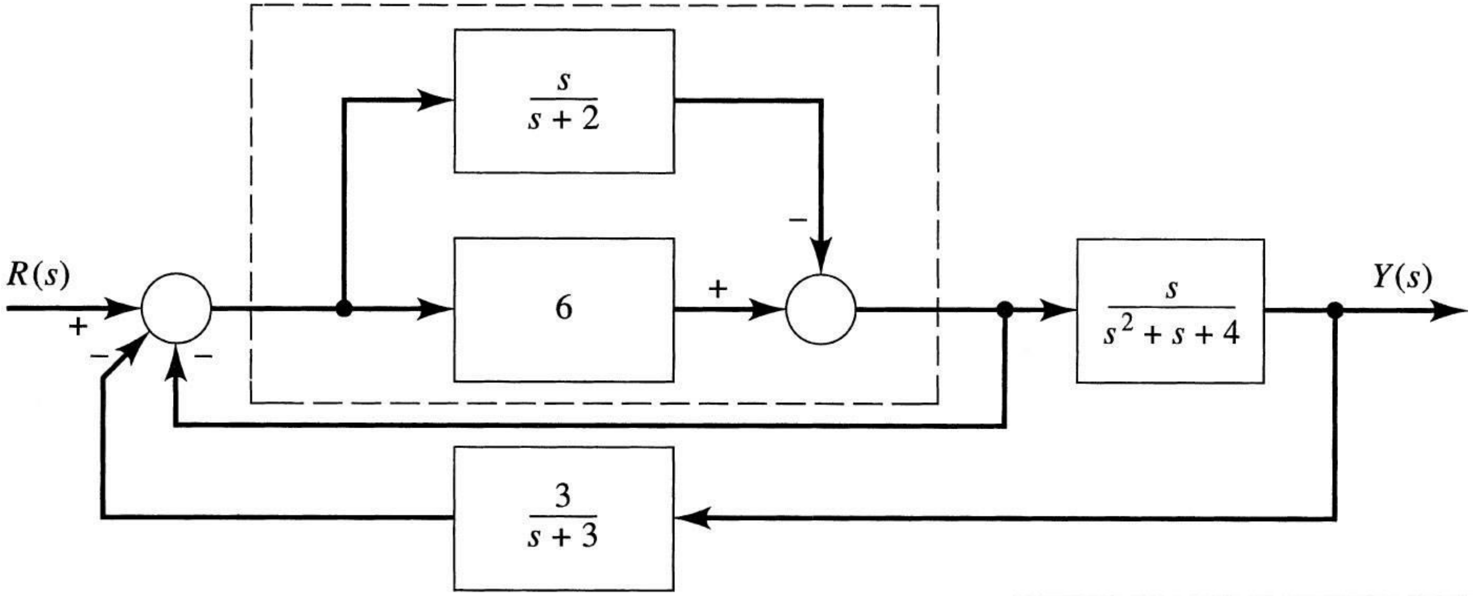
Block Diagram Models

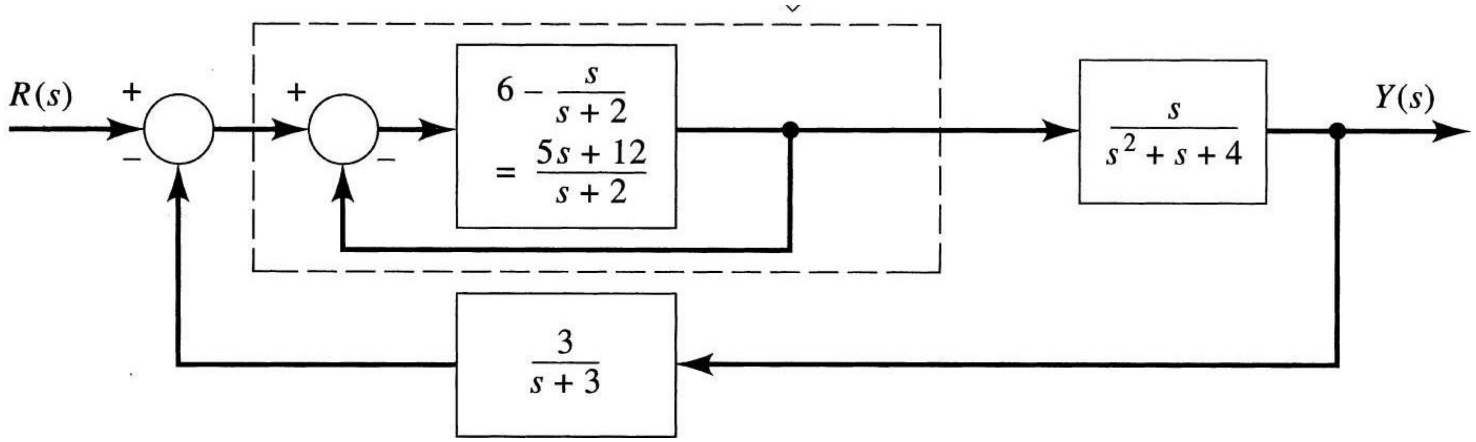
Example 1



Block Diagram Models

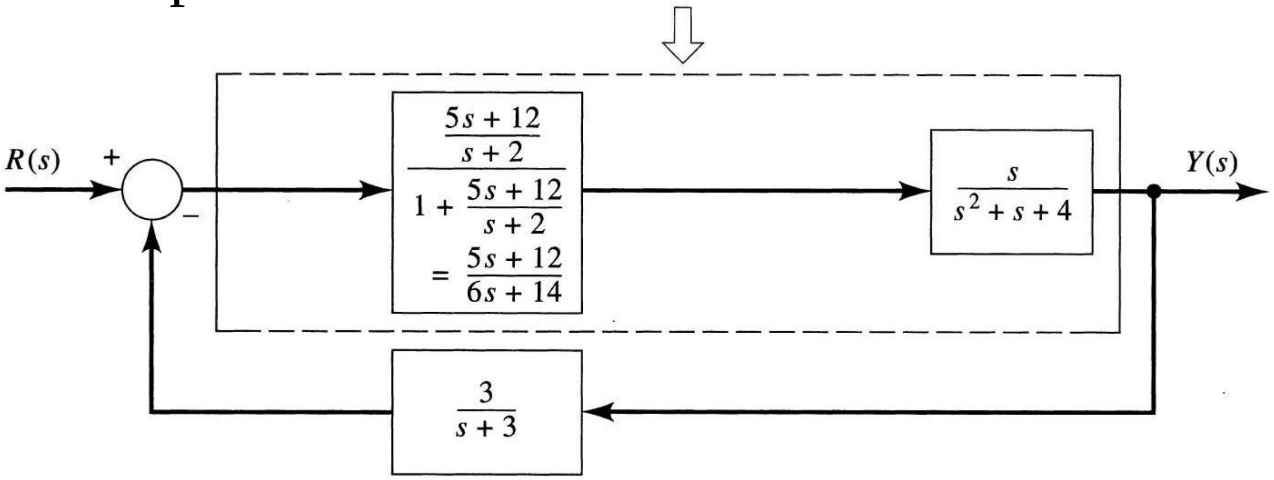
Example 2

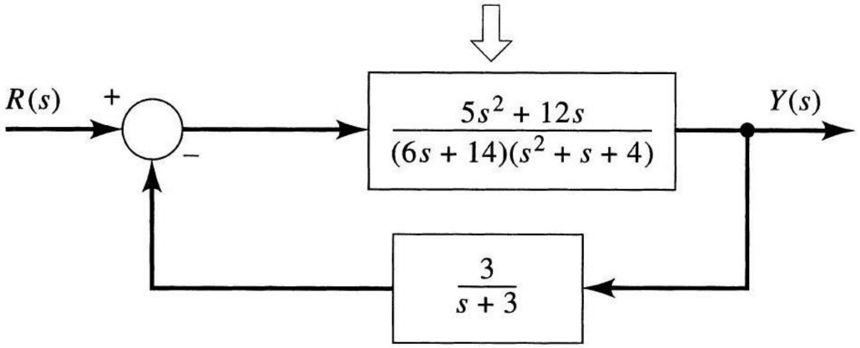


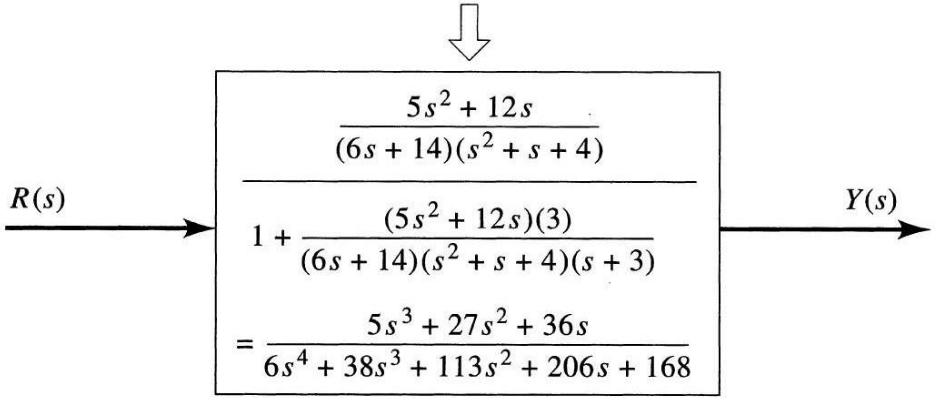


Block Diagram Models

Example 2 cont.

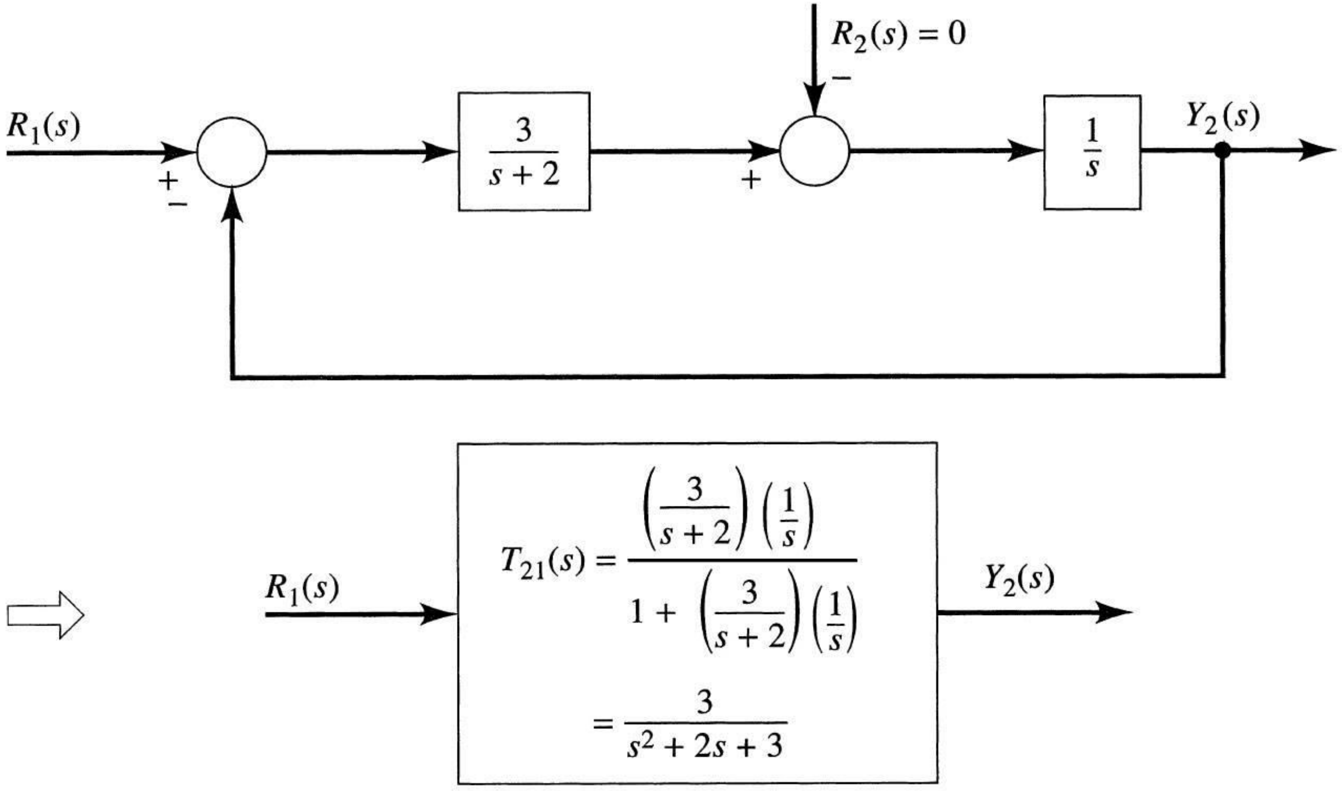






Block Diagram Models

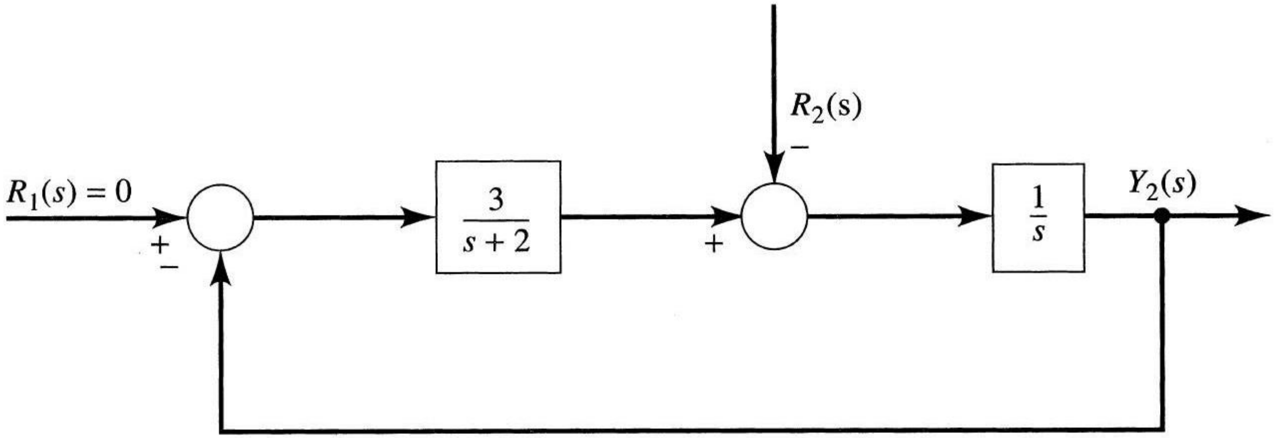
Example 3

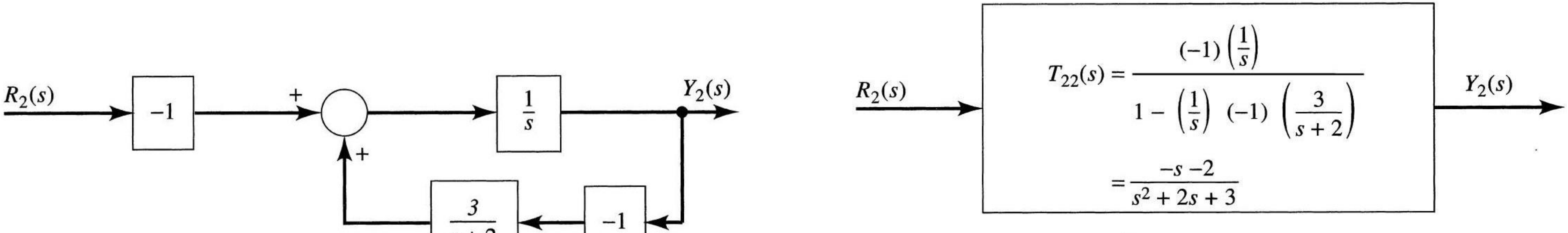


(a)

Block Diagram Models

Example 3 cont.





$$T_{22}(s) = \frac{(-1)\left(\frac{1}{s}\right)}{1 - \left(\frac{1}{s}\right)(-1)\left(\frac{3}{s+2}\right)} = \frac{-s-2}{s^2+2s+3}$$

(b)

Introduction to **Signal Flow**

- Alternative method to block diagram representation, developed by Samuel Jefferson Mason.

- Advantage: the availability of a flow graph gain formula, also called Mason's gain formula.

- A signal-flow graph consists of a network in which nodes are connected by directed branches.

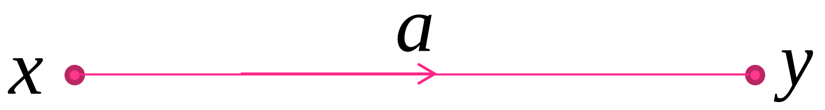
- It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

Fundamentals of Signal Flow Graphs

- Consider a simple equation below and draw its signal flow graph:

$$y = ax$$

- The signal flow graph of the equation is shown below;

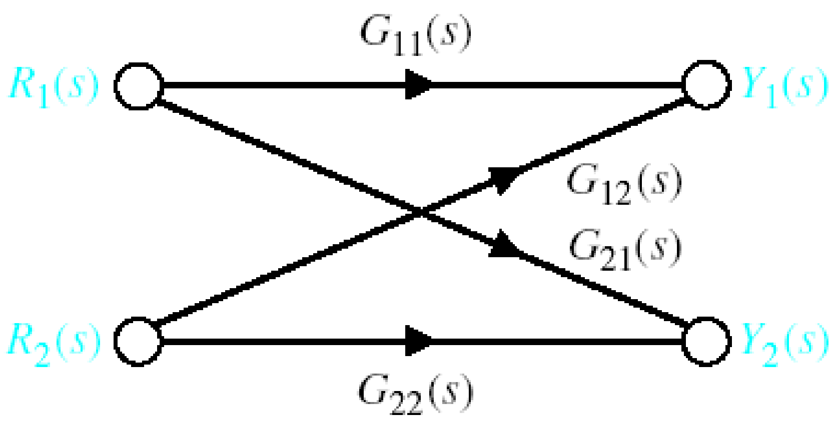


- Every variable in a signal flow graph is designed by a **Node**.
- Every transmission function in a signal flow graph is designed by a **Branch**.
- Branches are always **unidirectional**.
- The arrow in the branch denotes the **direction** of the signal flow.

Signal-Flow Graph Models

$$Y_1(s) = G_{11}(s) \cdot R_1(s) + G_{12}(s) \cdot R_2(s)$$

$$Y_2(s) = G_{21}(s) \cdot R_1(s) + G_{22}(s) \cdot R_2(s)$$

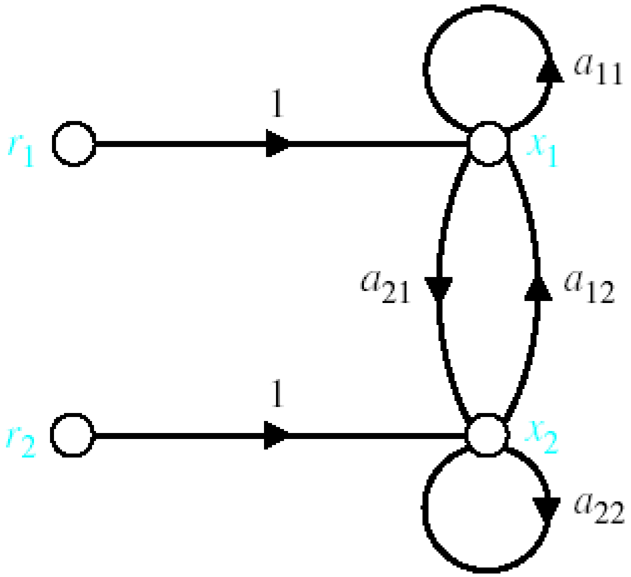


Signal-Flow Graph Models

r_1 *and* r_2 are inputs and x_1 and x_2 are outputs

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + r_1 = x_1$$

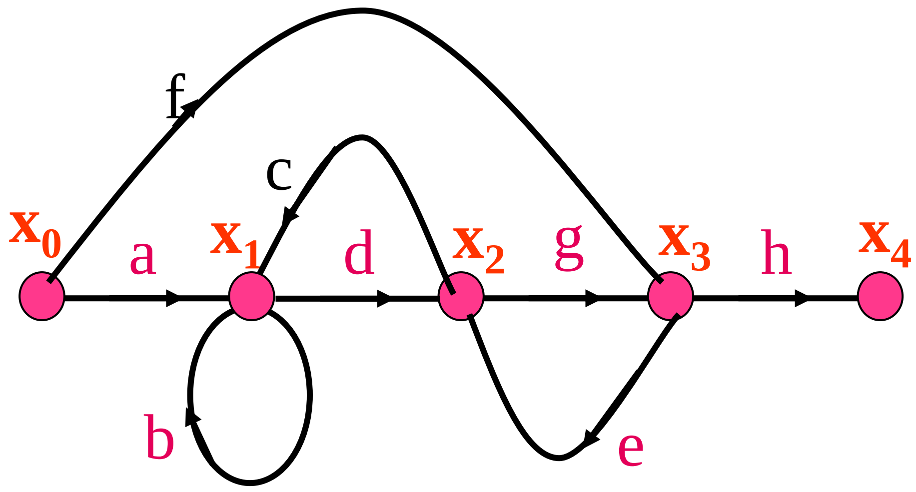
$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + r_2 = x_2$$



Signal-Flow Graph Models

x_o is input and x_4 is output

$$x_1 = ax_0 + bx_1 + cx_2$$
$$x_2 = dx_1 + ex_3$$
$$x_3 = fx_0 + gx_2$$
$$x_4 = hx_3$$

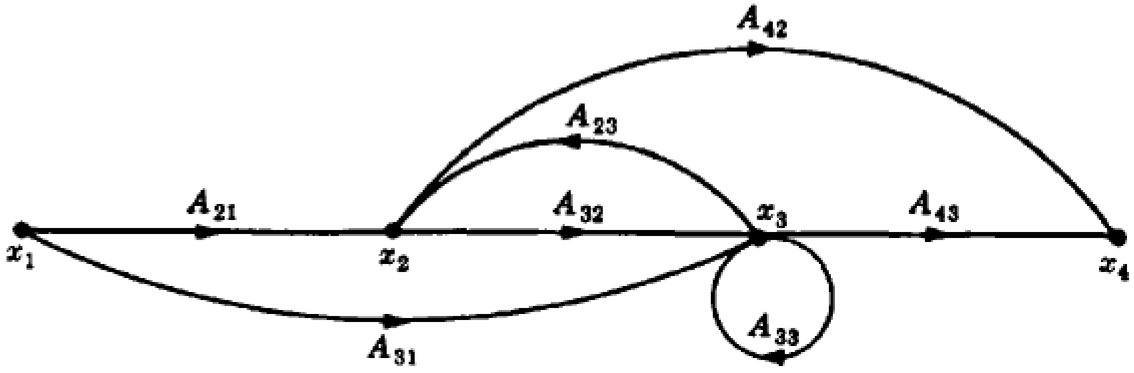


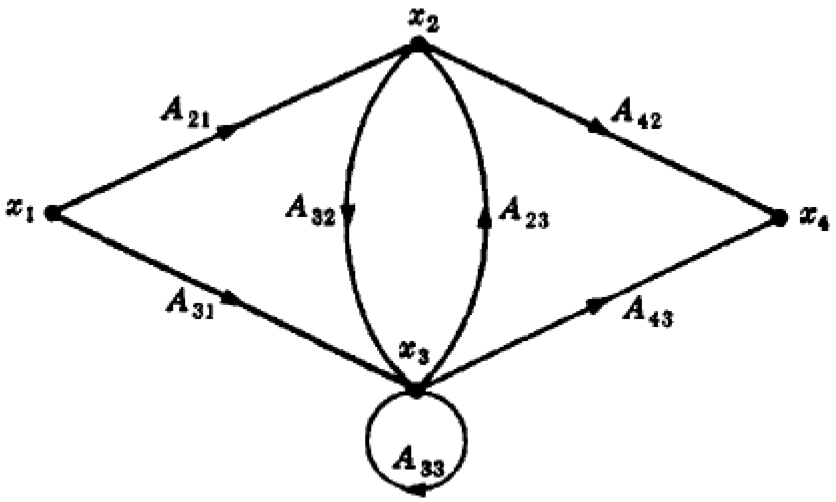
Signal-Flow Graph Models

Construct the signal flow graph for the following set of simultaneous equations.

$$x_2 = A_{21}x_1 + A_{23}x_3 \qquad x_3 = A_{31}x_1 + A_{32}x_2 + A_{33}x_3 \qquad x_4 = A_{42}x_2 + A_{43}x_3$$

- There are four variables in the equations (i.e., x_1,x_2,x_3,and x_4) therefore four nodes are required to construct the signal flow graph.
- Arrange these four nodes from left to right and connect them with the associated branches.



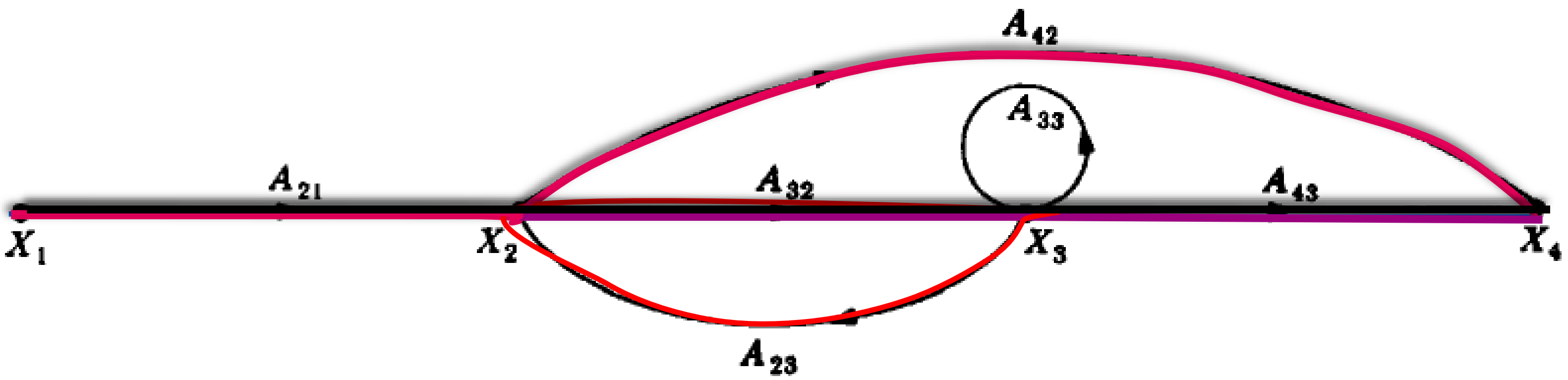


- Another way to arrange this graph is shown in the figure.

Signal-Flow Graph Models

Terminologies:

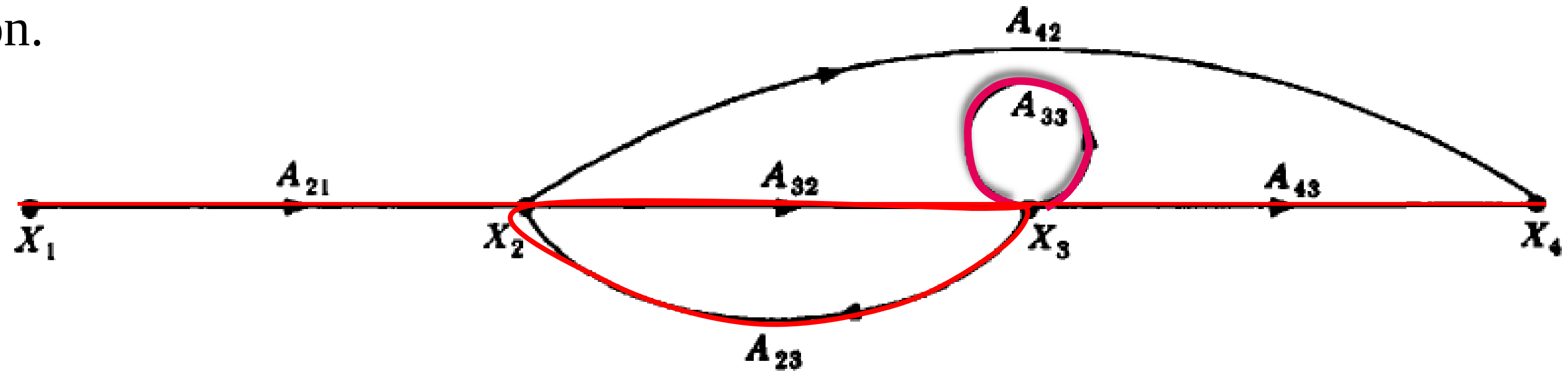
- An **input node** or source contain only the outgoing branches. i.e., X_1
- An **output node** or sink contain only the incoming branches. i.e., X_4
- A **path** is a continuous, unidirectional succession of branches along which no node is passed more than ones. i.e., X_1 to X_2 to X_3 to X_4 X_1 to X_2 to X_4 X_2 to X_3 to X_4
- A **forward path** is a path from the input node to the output node. i.e., X_1 to X_2 to X_3 to X_4, and X_1 to X_2 to X_4, are forward paths.
- A **feedback path** or feedback loop is a path which originates and terminates on the same node. i.e.; X_2 to X_3 and back to X_2 is a feedback path.



Signal-Flow Graph Models

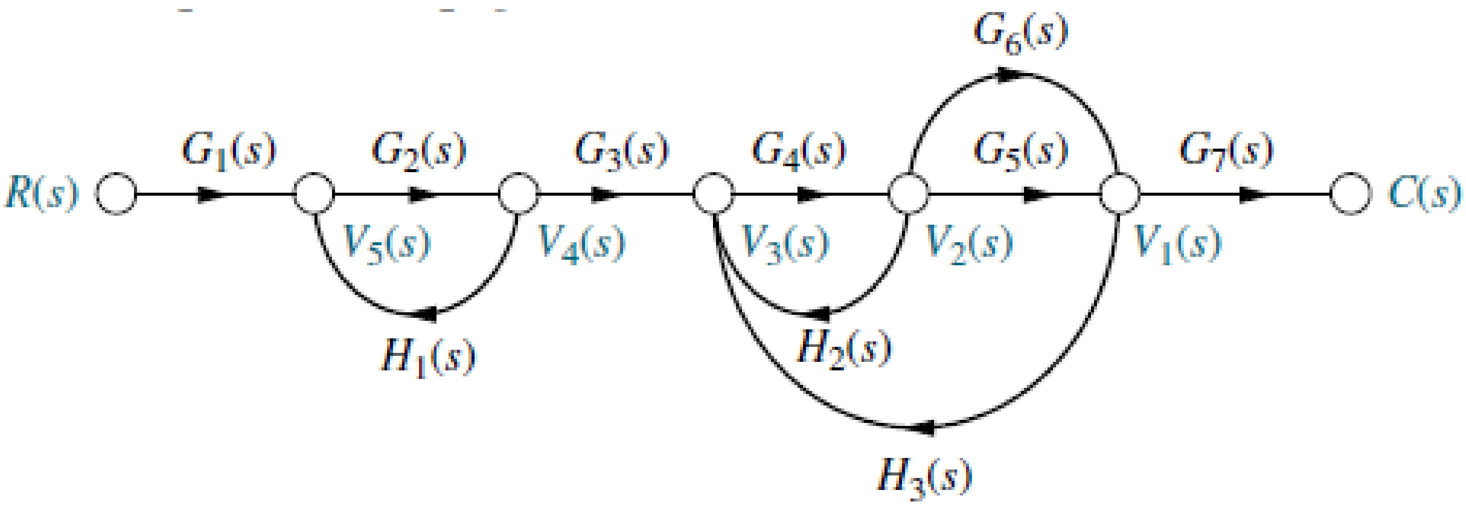
Terminologies:

- A **self-loop** is a feedback loop consisting of a single branch. i.e.; A_{33} is a self loop.
- The **gain** of a branch is the transmission function of that branch.
- The **path gain** is the product of branch gains encountered in traversing a path. i.e. the gain of forwards path X_1 to X_2 to X_3 to X_4 is $A_{21}A_{32}A_{43}$
- The **loop gain** is the product of the branch gains of the loop. i.e., the loop gain of the feedback loop from X_2 to X_3 and back to X_2 is $A_{32}A_{23}$.
- Two loops, paths, or loop and a path are said to be **non-touching** if they have no nodes in common.



Signal-Flow Graph Models

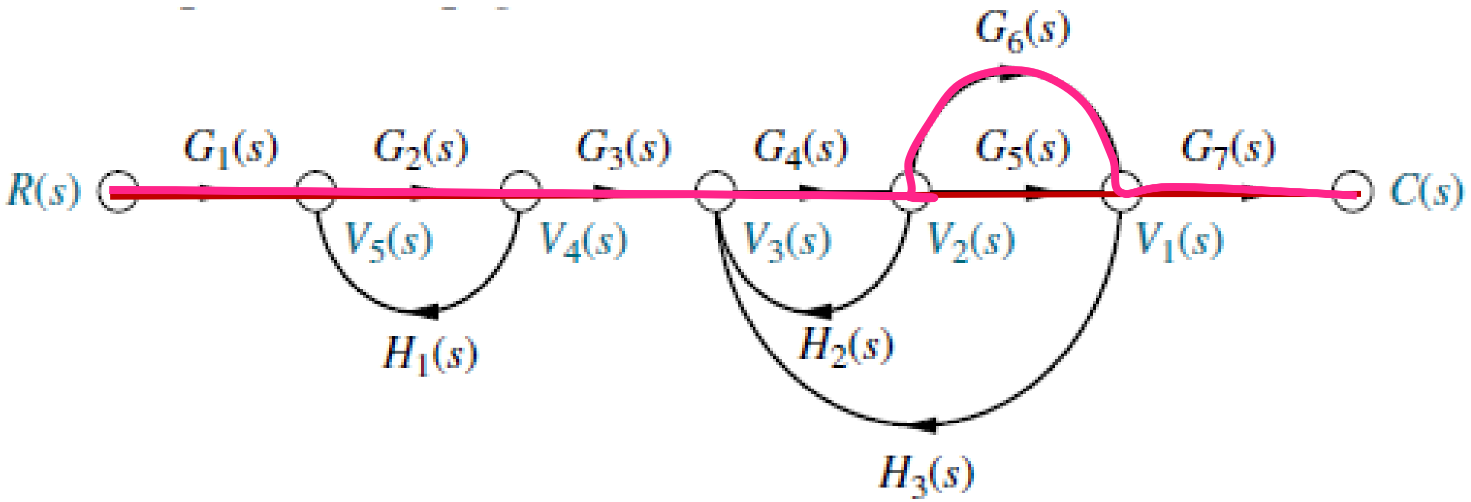
Consider the signal flow graph below and identify the following



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths (loops).
e) Determine the loop gains of the feedback loops.
f) Determine the path gains of the forward paths.
g) Non-touching loops

Signal-Flow Graph Models

Consider the signal flow graph below and identify the following



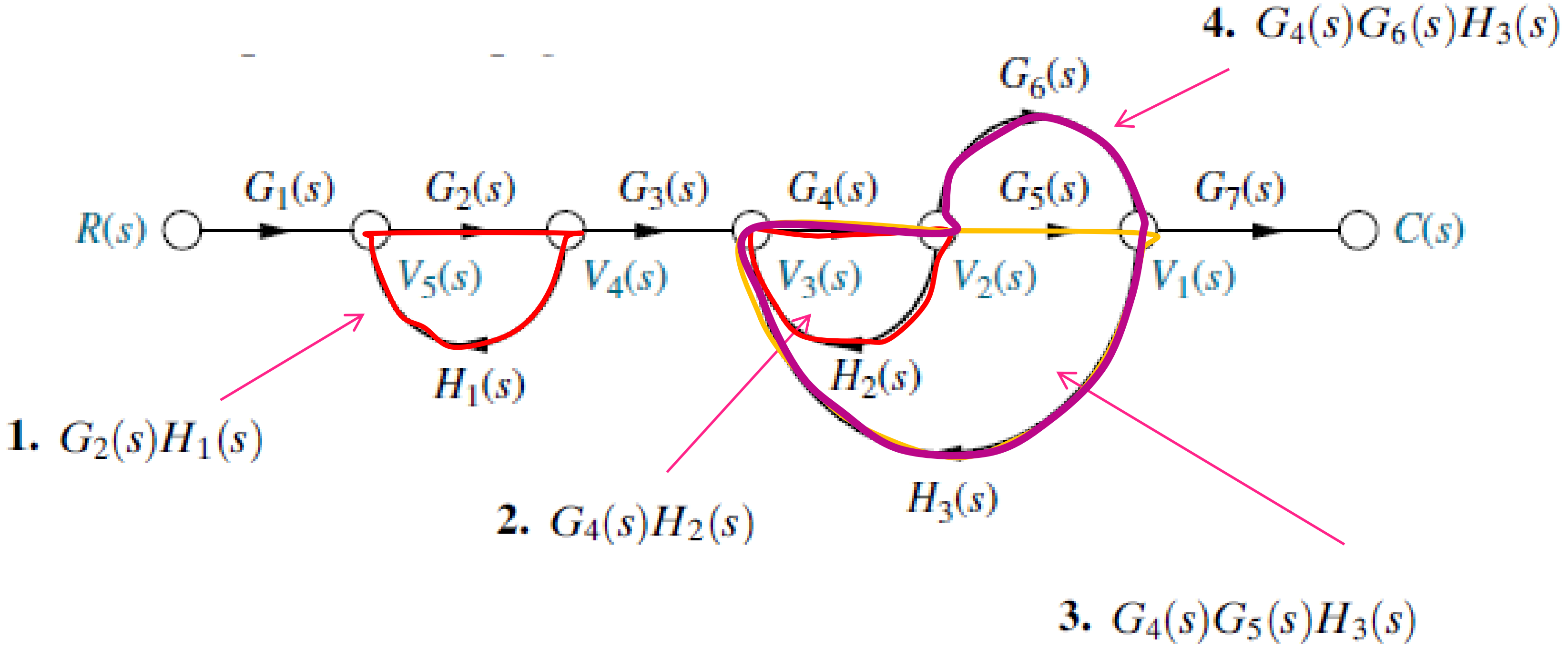
- There are two forward path gains;

1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$ 2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

Signal-Flow Graph Models

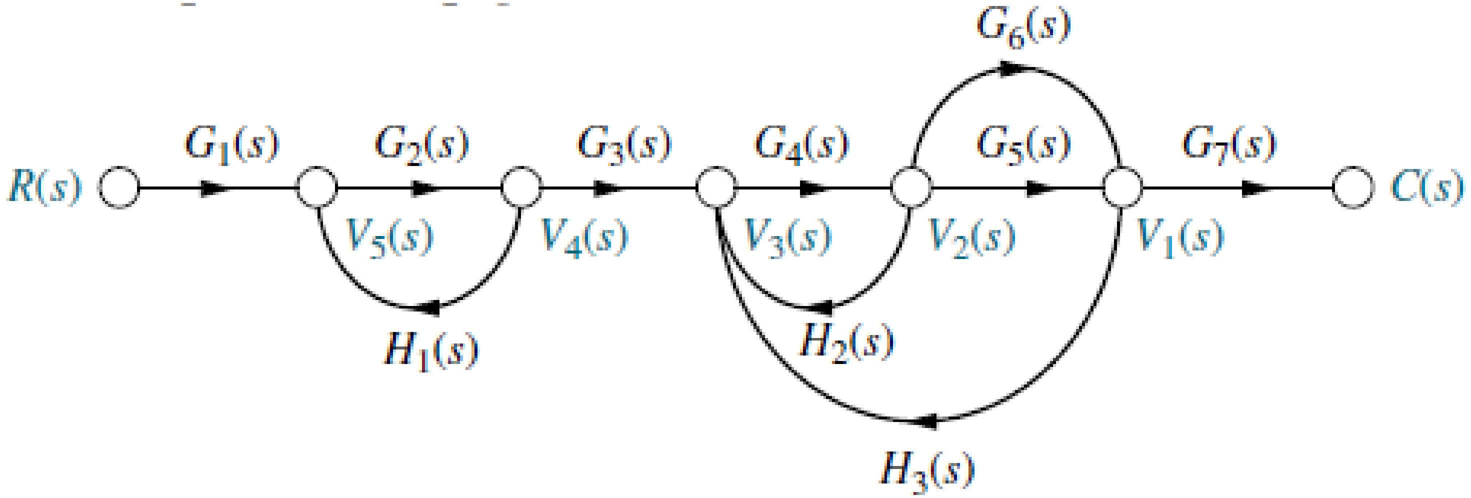
Consider the signal flow graph below and identify the following

- There are four loops



Signal-Flow Graph Models

Consider the signal flow graph below and identify the following

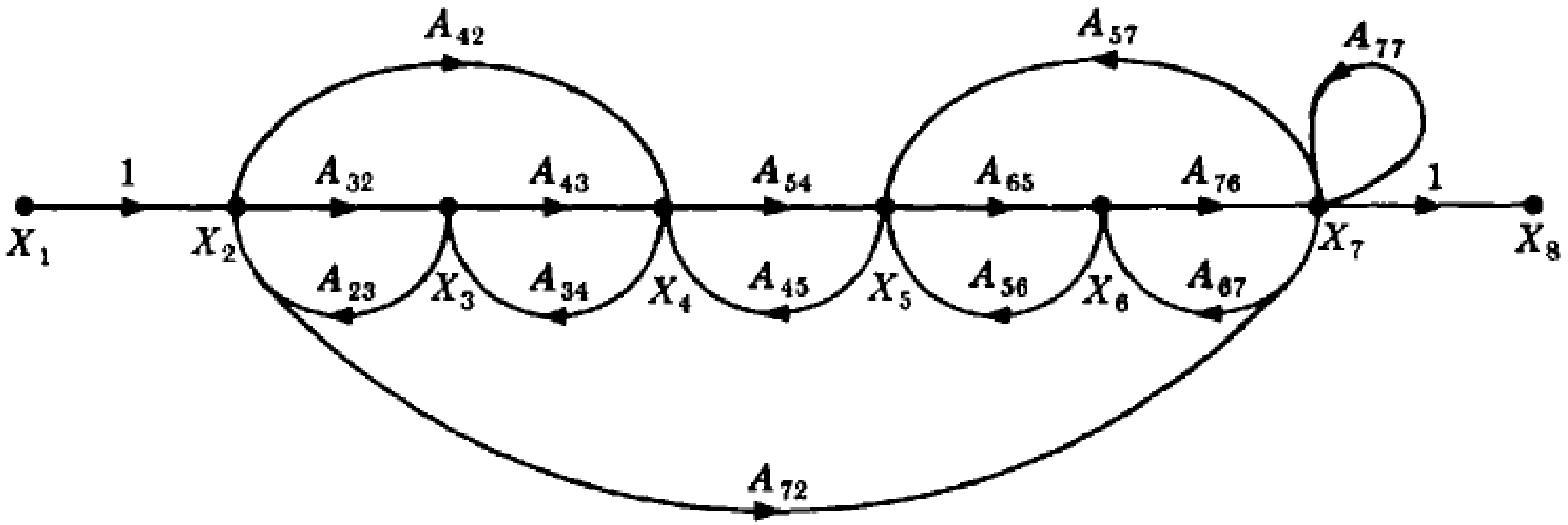


- Nontouching loop gains;

1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Signal-Flow Graph Models

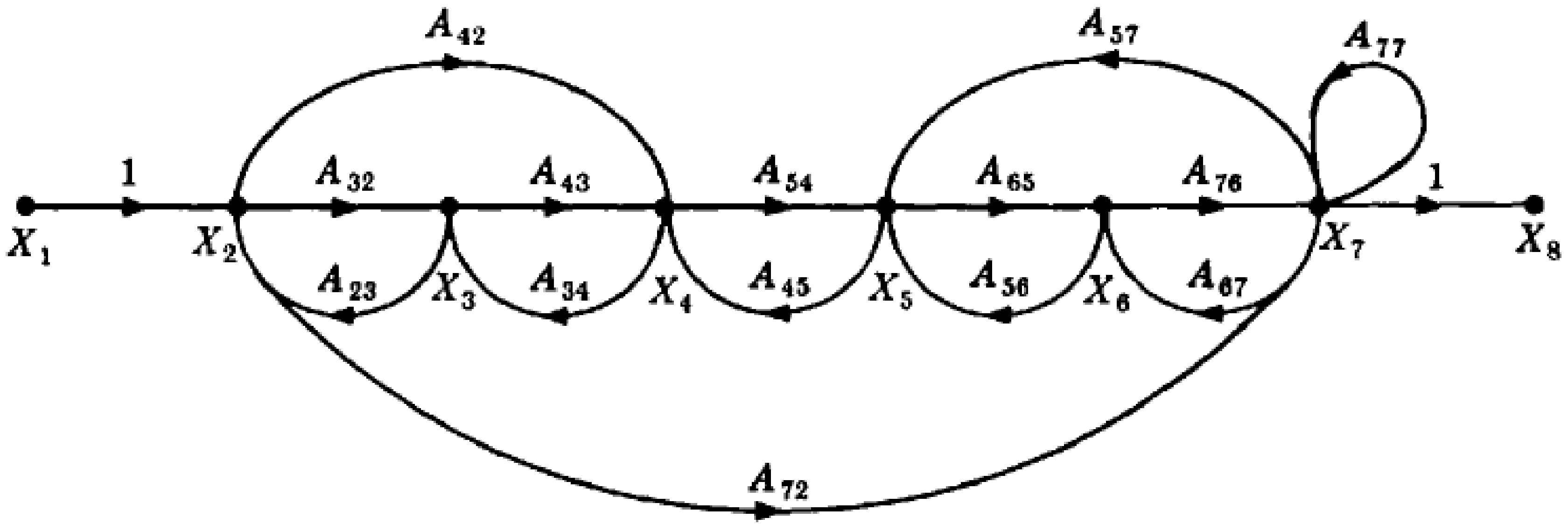
Consider the signal flow graph below and identify the following



a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths.
e) Self loop.
f) Determine the loop gains of the feedback loops.
g) Determine the path gains of the forward paths.

Signal-Flow Graph Models

Input and output Nodes

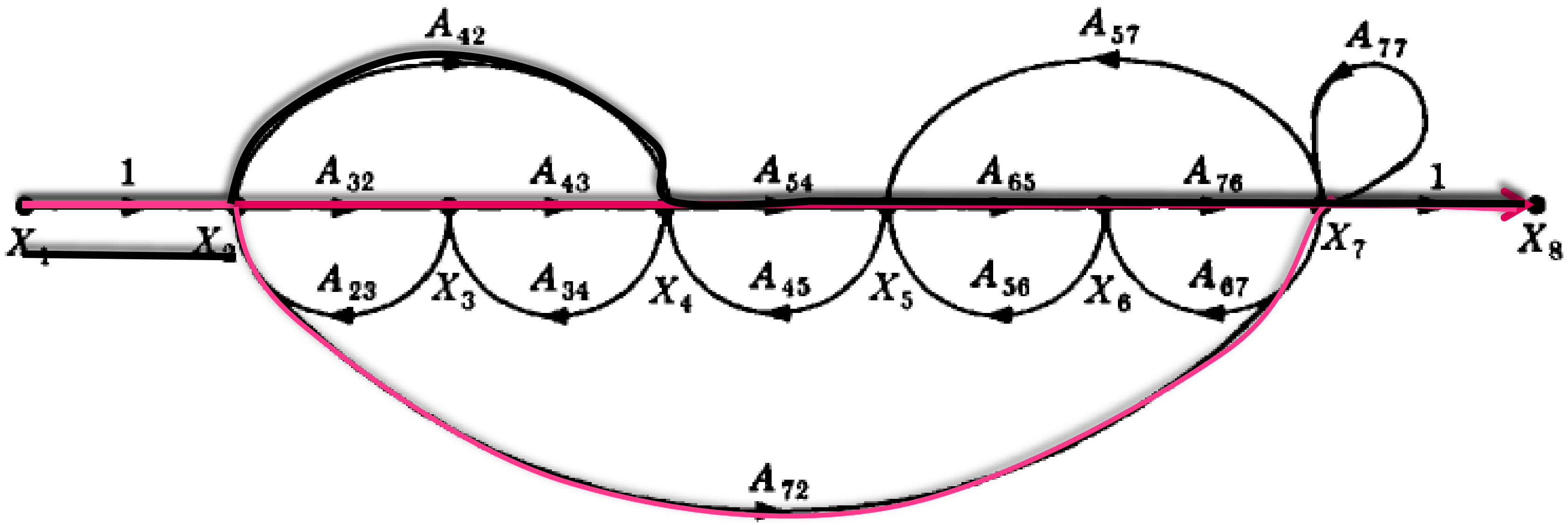


a) Input node X_1

b) Output node X_8

Signal-Flow Graph Models

(c) Forward Paths



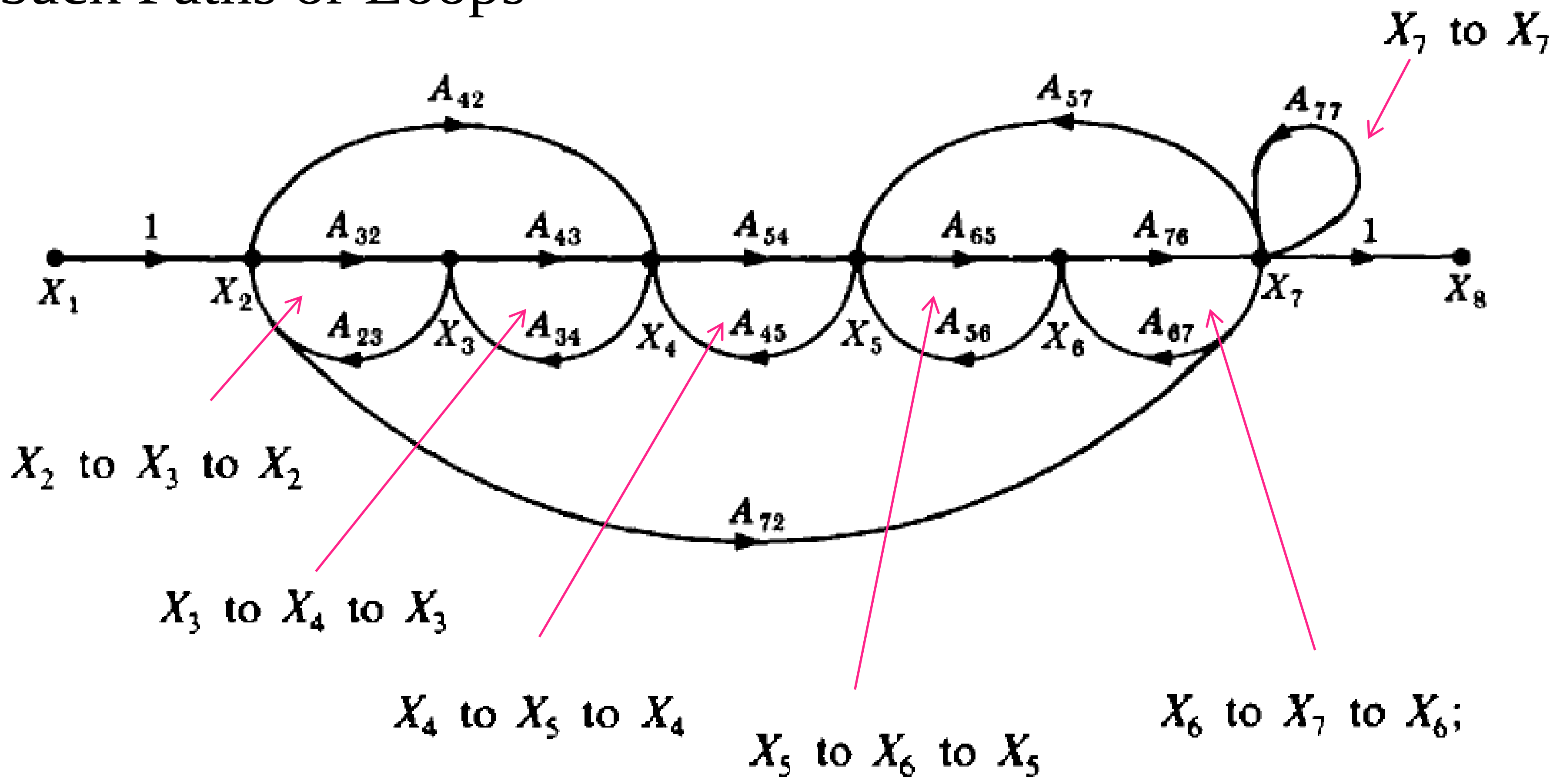
X_1 to X_2 to X_3 to X_4 to X_5 to X_6 to X_7 to X_8

X_1 to X_2 to X_7 to X_8

X_1 to X_2 to X_4 to X_5 to X_6 to X_7 to X_8

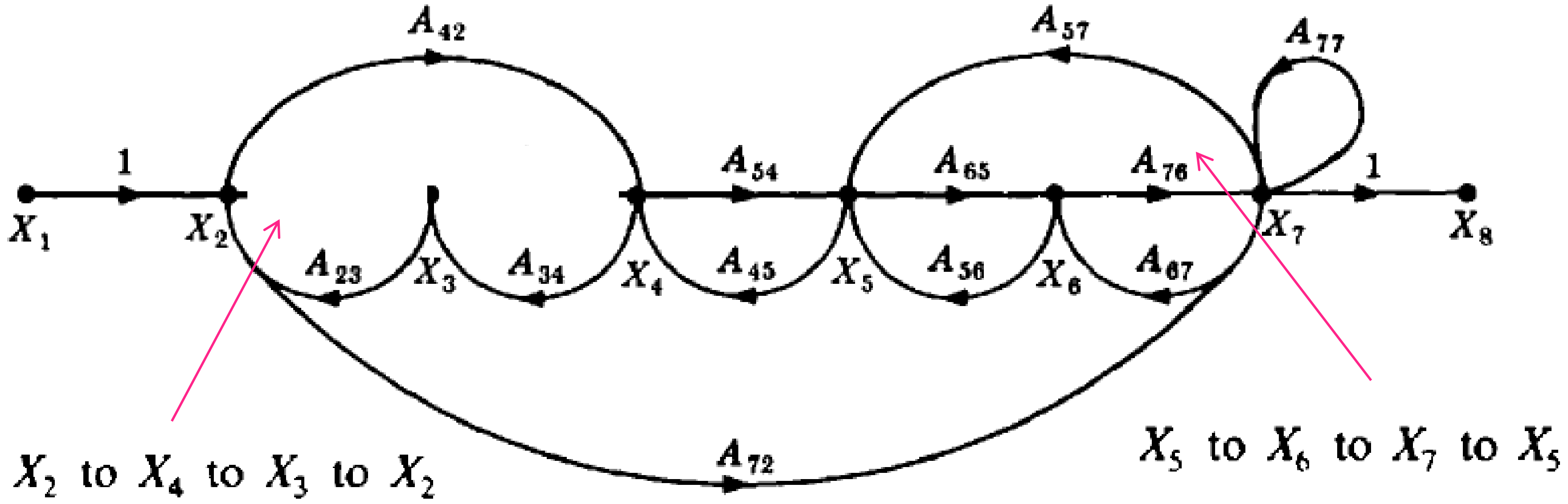
Signal-Flow Graph Models

(d) Feedback Paths or Loops



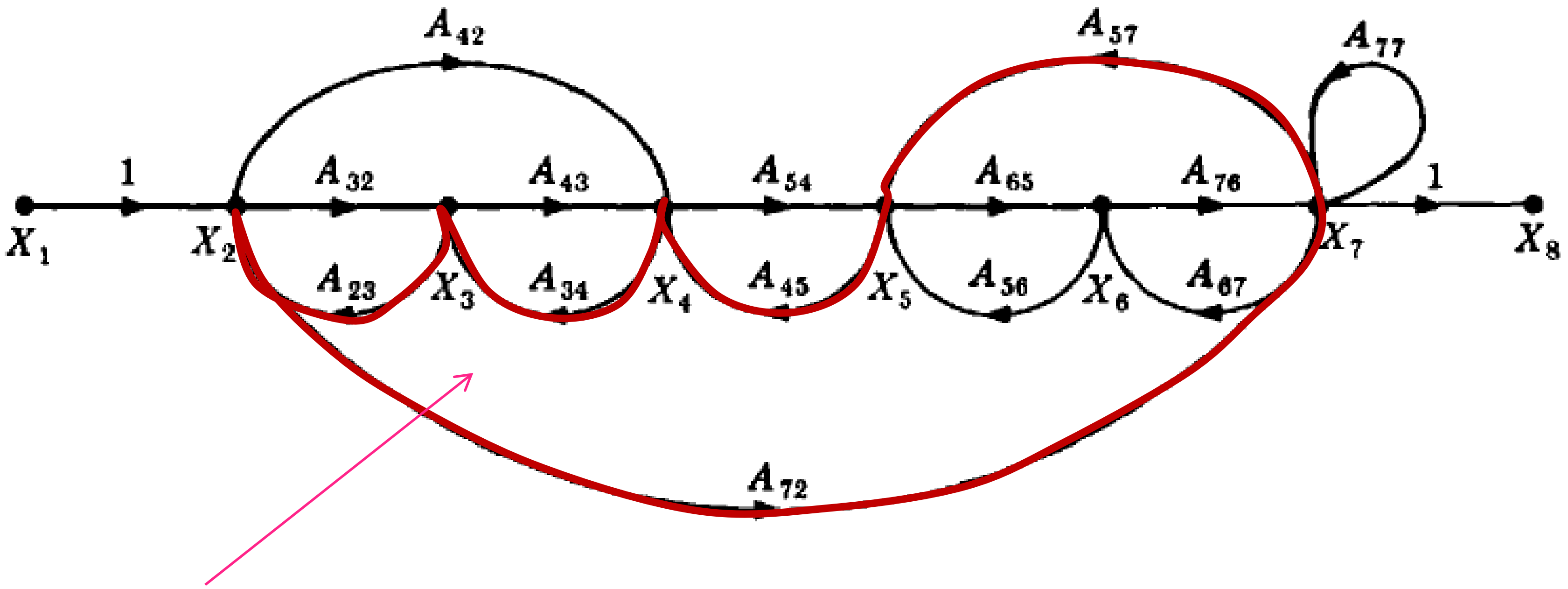
Signal-Flow Graph Models

(d) Feedback Paths or Loops



Signal-Flow Graph Models

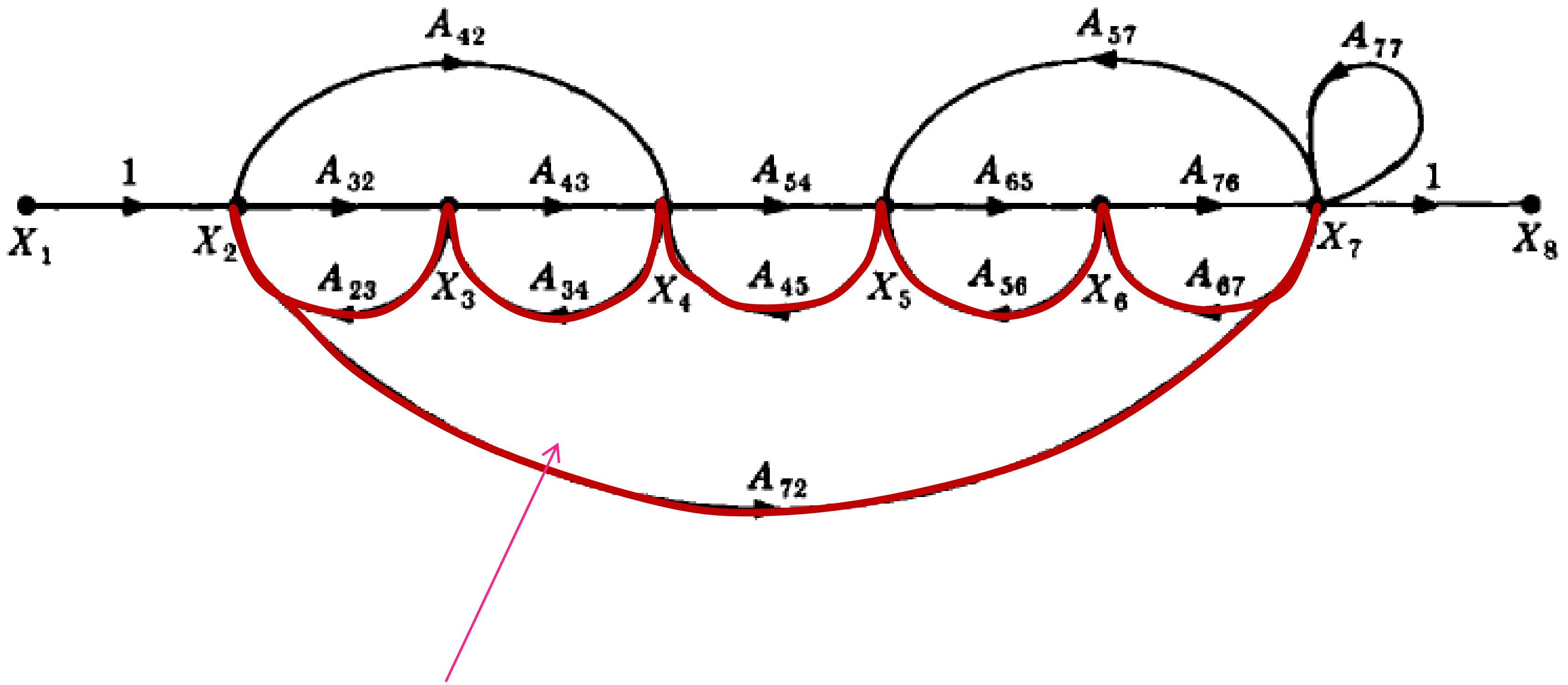
(d) Feedback Paths or Loops



X_2 to X_7 to X_5 to X_4 to X_3 to X_2

Signal-Flow Graph Models

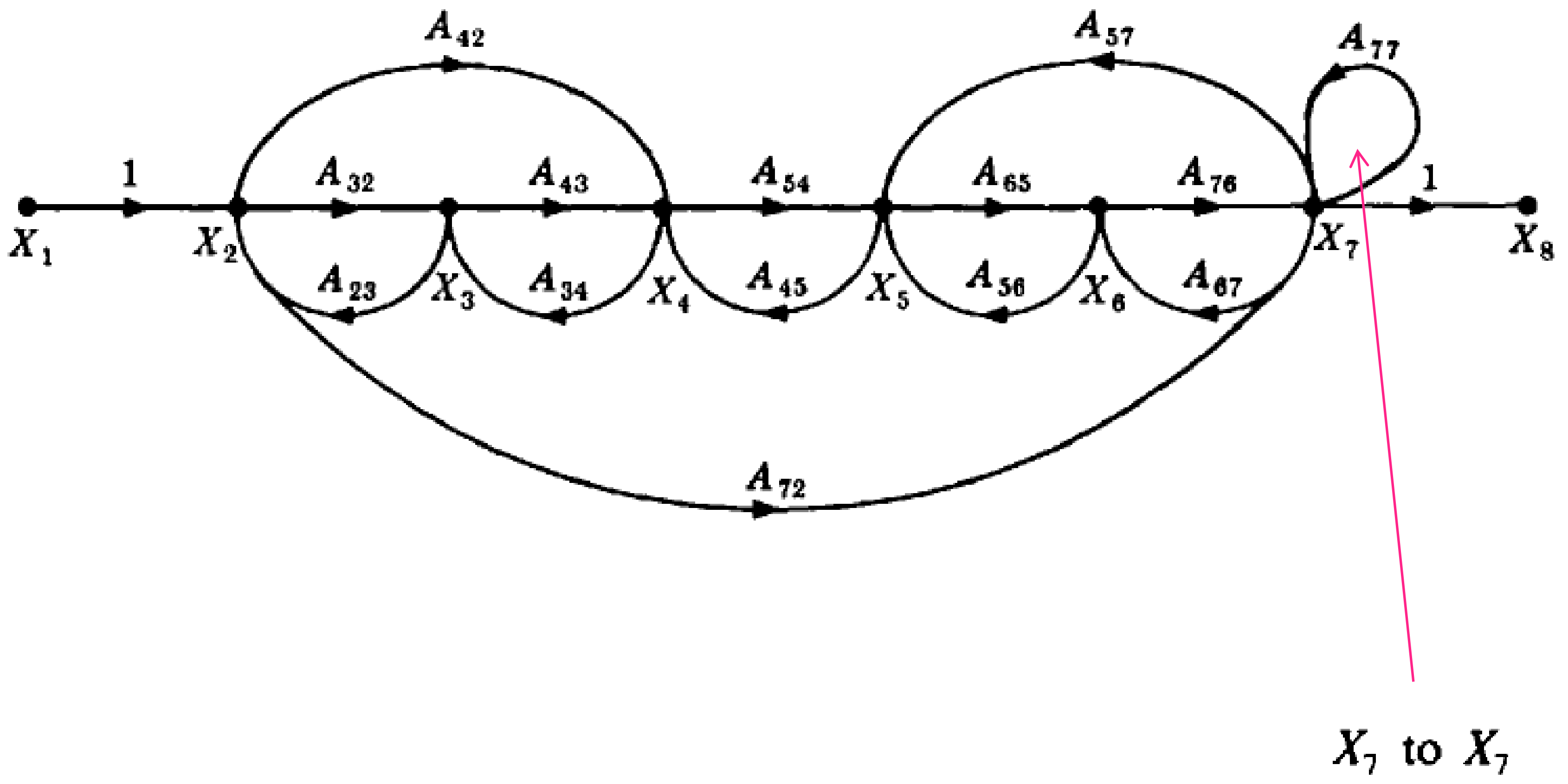
(d) Feedback Paths or Loops



X_2 to X_7 to X_6 to X_5 to X_4 to X_3 to X_2

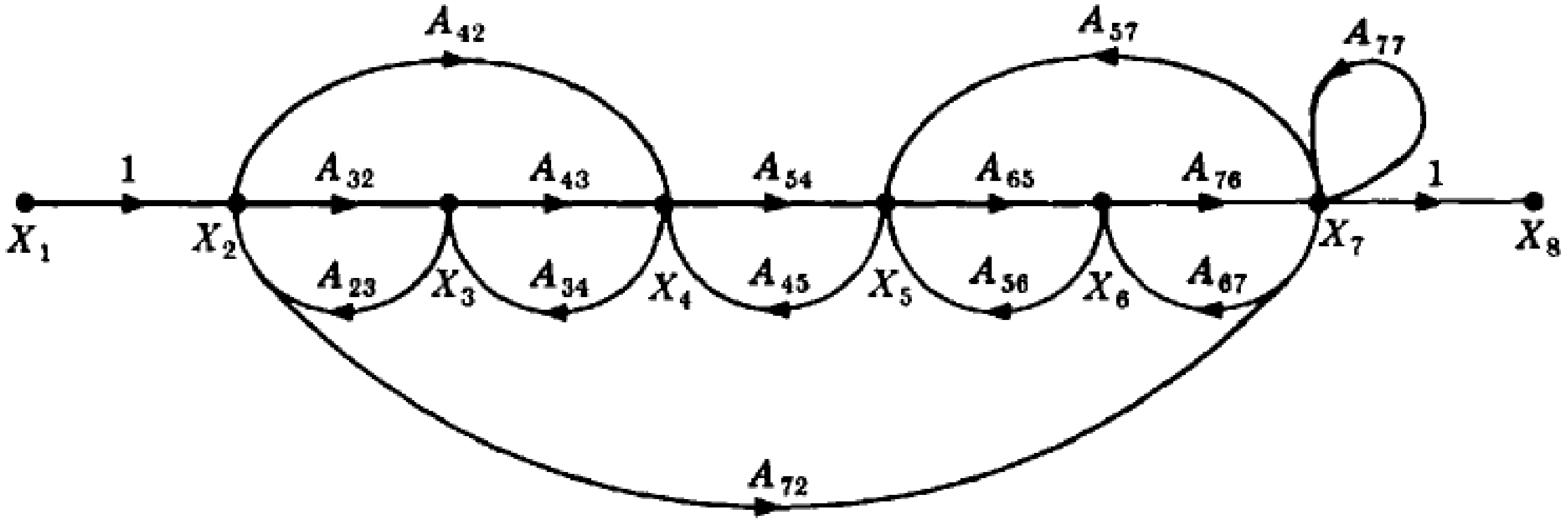
Signal-Flow Graph Models

(e) Self Loop(s)



Signal-Flow Graph Models

(f) Loop Gains of the Feedback Loops



$A_{32}A_{23}$ $A_{76}A_{67}$ $A_{72}A_{57}A_{45}A_{34}A_{23}$

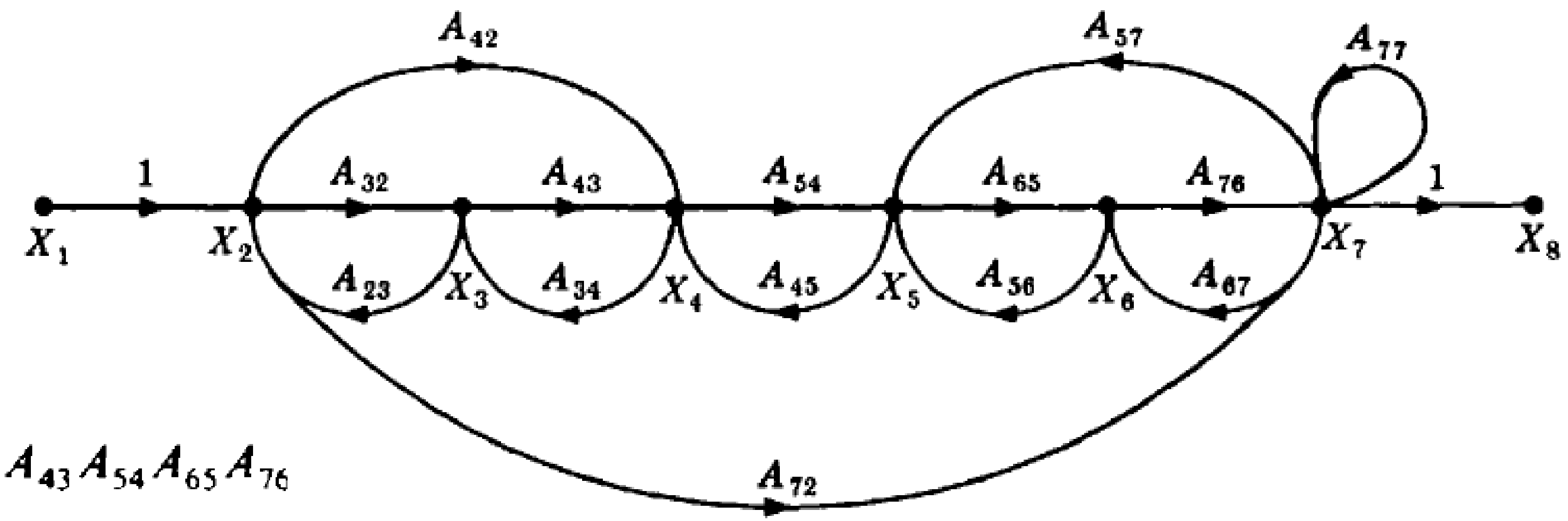
$A_{43}A_{34}$ $A_{65}A_{76}A_{57}$ $A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$

$A_{54}A_{45}$ A_{77}

$A_{65}A_{56}$ $A_{42}A_{34}A_{23}$

Signal-Flow Graph Models

(g) Path Gains of the Forward Paths



$A_{32}A_{43}A_{54}A_{65}A_{76}$

A_{72}

$A_{42}A_{54}A_{65}A_{76}$

Mason’s Rule (Mason, 1953)

- The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function.
- On the other hand, Mason’s rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.
- The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.

Mason's Rule

- The transfer function, ***C(s)/R(s),*** of a system represented by a signal-flow graph is;

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

Where

n = number of forward paths.
P_i = the i^{th} forward-path gain.
Δ = Determinant of the system
Δ_i = Determinant of the i^{th} forward path

- Δ is called the signal flow graph determinant or characteristic function. Since Δ=0 is the system characteristic equation.

Mason's Rule

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

Δ = 1- (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + … and so forth with sums of higher number of non-touching loop gains

Δ_i = value of Δ for the part of the block diagram that does not touch the i-th forward path (Δ_i = 1 if there are no non-touching loops to the i-th path.)

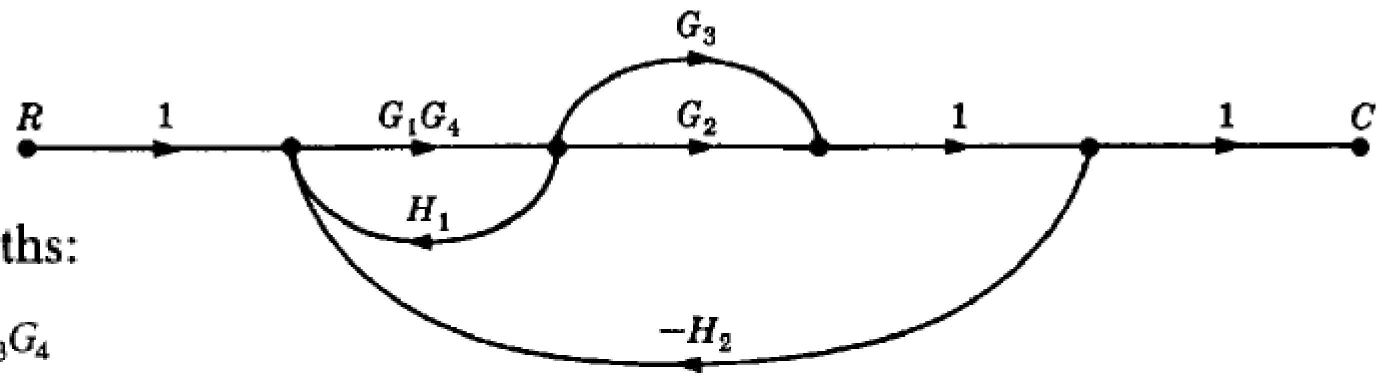
Signal-Flow Graph Models

Systematic approach

1. Calculate forward path gain P_i for each forward path *i*.
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time
5. etc
6. Calculate Δ from steps 2,3,4 and 5
7. Calculate Δ_i as portion of Δ not touching forward path *i*

Signal-Flow Graph Models

Example.1



There are two forward paths:

$P_1 = G_1G_2G_4 \qquad P_2 = G_1G_3G_4$

Therefore,

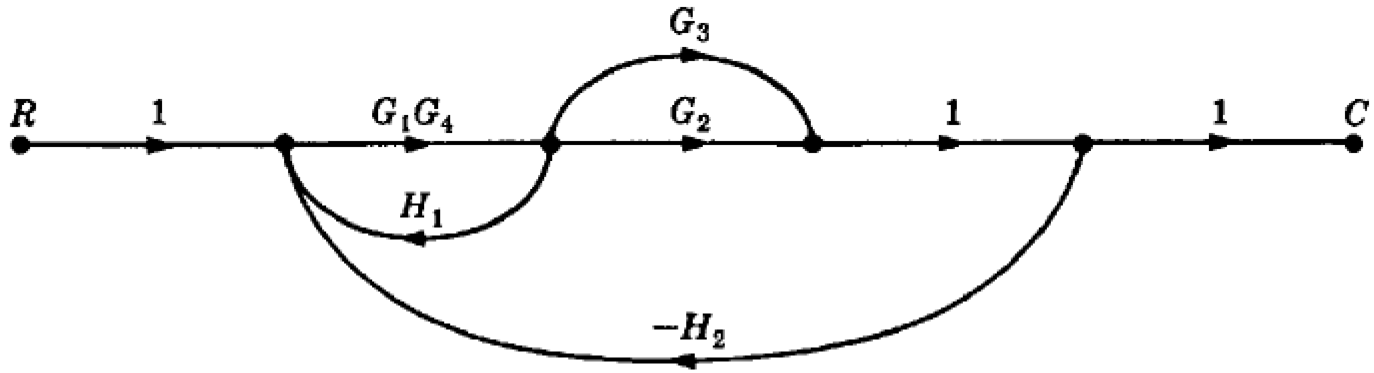
$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1G_4H_1, \quad L_2 = -G_1G_2G_4H_2, \quad L_3 = -G_1G_3G_4H_2$$

Signal-Flow Graph Models

Example.1



There are no non-touching loops, therefore

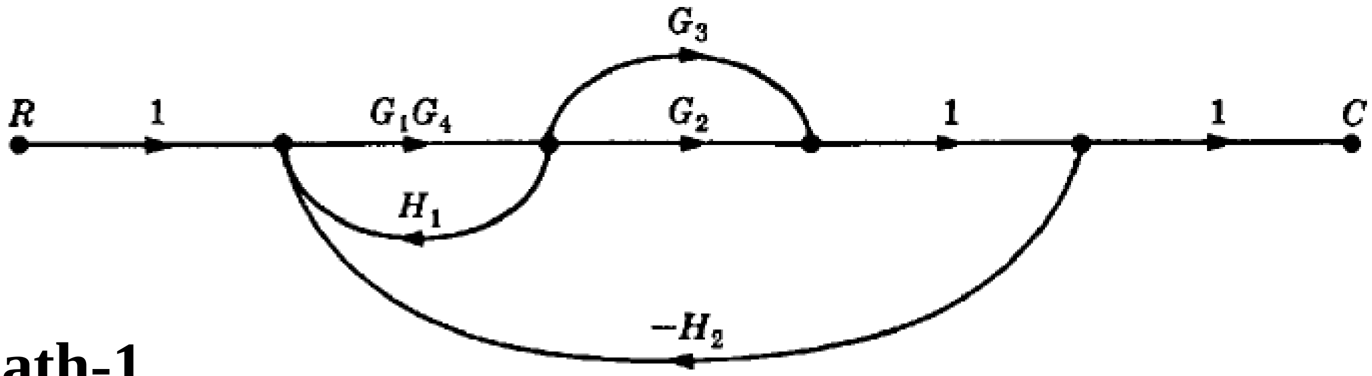
Δ = 1- (sum of all individual loop gains)

$$\Delta = 1 - \left(L_1 + L_2 + L_3\right)$$

$$\Delta = 1 - \left(G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2\right)$$

Signal-Flow Graph Models

Example.1



Eliminate forward path-1

Δ_1 = 1- (sum of all individual loop gains)+...

$\Delta_1 = 1$

Eliminate forward path-2

Δ_2 = 1- (sum of all individual loop gains)+...

$\Delta_2 = 1$

Signal-Flow Graph Models

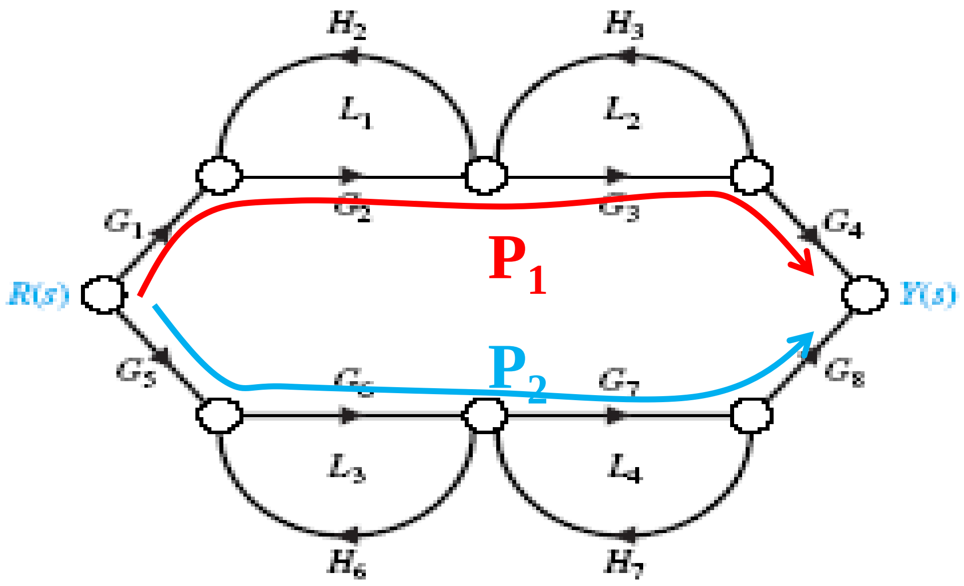
Example.1

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

$$= \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

Signal-Flow Graph Models

Example.2



1. **Calculate forward path gains for each forward path.**

2. **Calculate all loop gains.**

3. **Consider two non-touching loops.**

L_1L_3 L_1L_4

L_2L_4 L_2L_3

Signal-Flow Graph Models

Example.2

4. Consider three non-touching loops.

   None.

5. Calculate Δ from steps 2,3,4.

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$$

$$\Delta = 1 - (G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7) + (G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7)$$

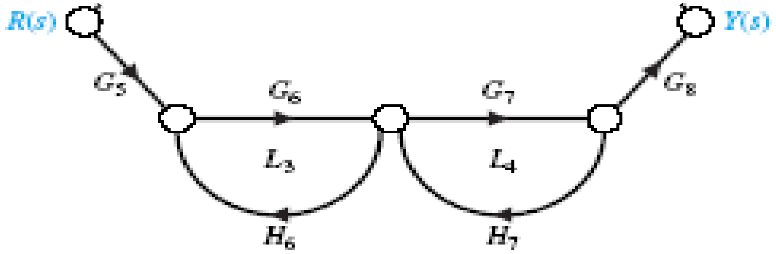
Signal-Flow Graph Models

Example.2

Eliminate forward path-1

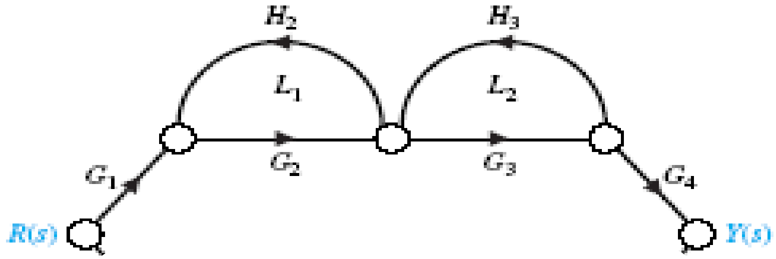
$$\Delta_1 = 1 - (L_3 + L_4)$$

$$\Delta_1 = 1 - (G_6 H_6 + G_7 H_7)$$



Eliminate forward path-2

$$\Delta_2 = 1 - (L_1 + L_2)$$



$$\Delta_2 = 1 - (G_2 H_2 + G_3 H_3)$$

Signal-Flow Graph Models

Example.2

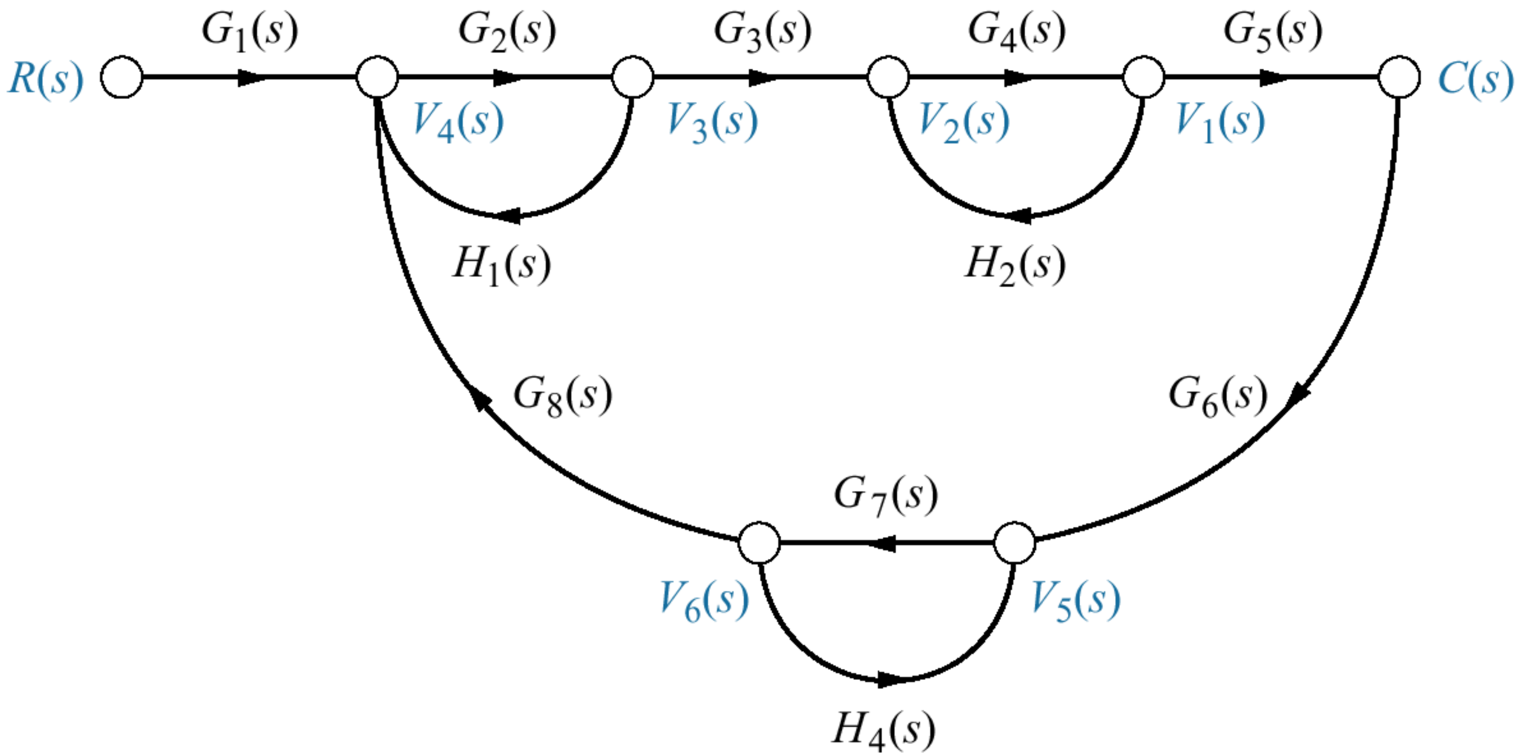
$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4\left[1-\left(G_6H_6 + G_7H_7\right)\right] + G_5G_6G_7G_8\left[1-\left(G_2H_2 + G_3H_3\right)\right]}{1-\left(G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7\right) + \left(G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7\right)}$$

Signal-Flow Graph Models

Example.3

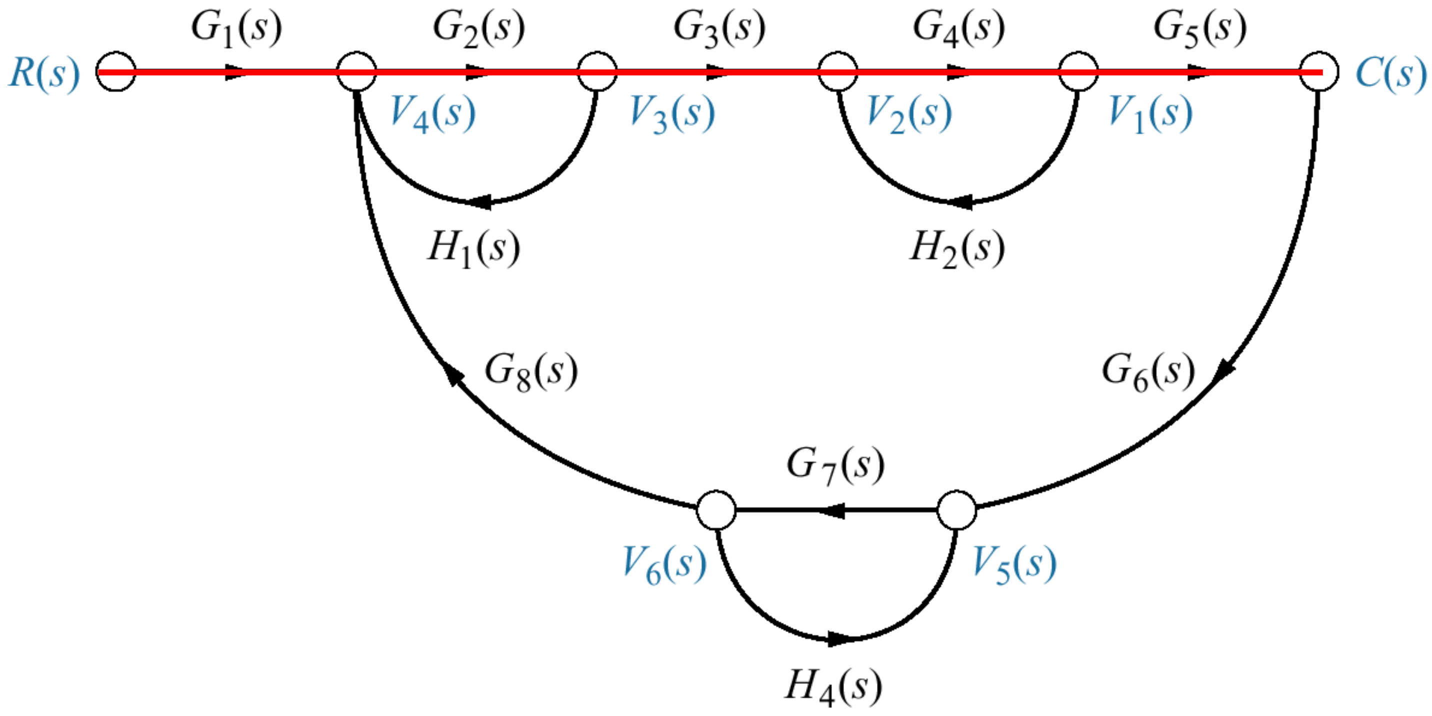
- Find the transfer function, C(s)/R(s), for the signal-flow graph in figure below.



Signal-Flow Graph Models

Example.3

- There is only one forward Path.

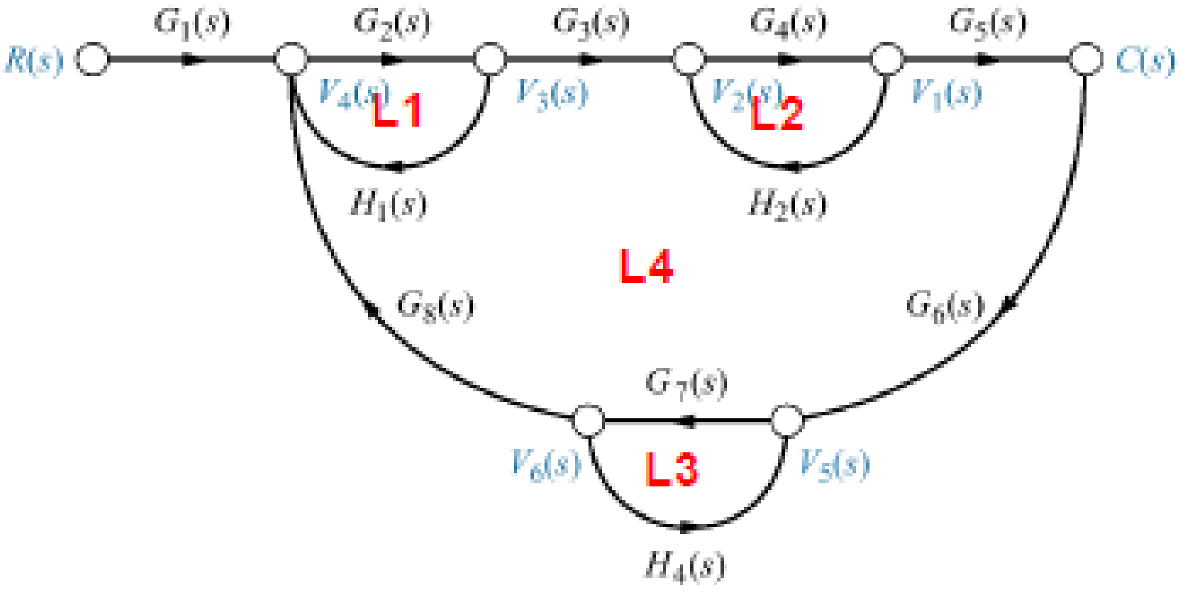


$$P_1 = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$$

Signal-Flow Graph Models

Example.3

- There are four feedback loops.



L1. $G_2(s)H_1(s)$

L2. $G_4(s)H_2(s)$

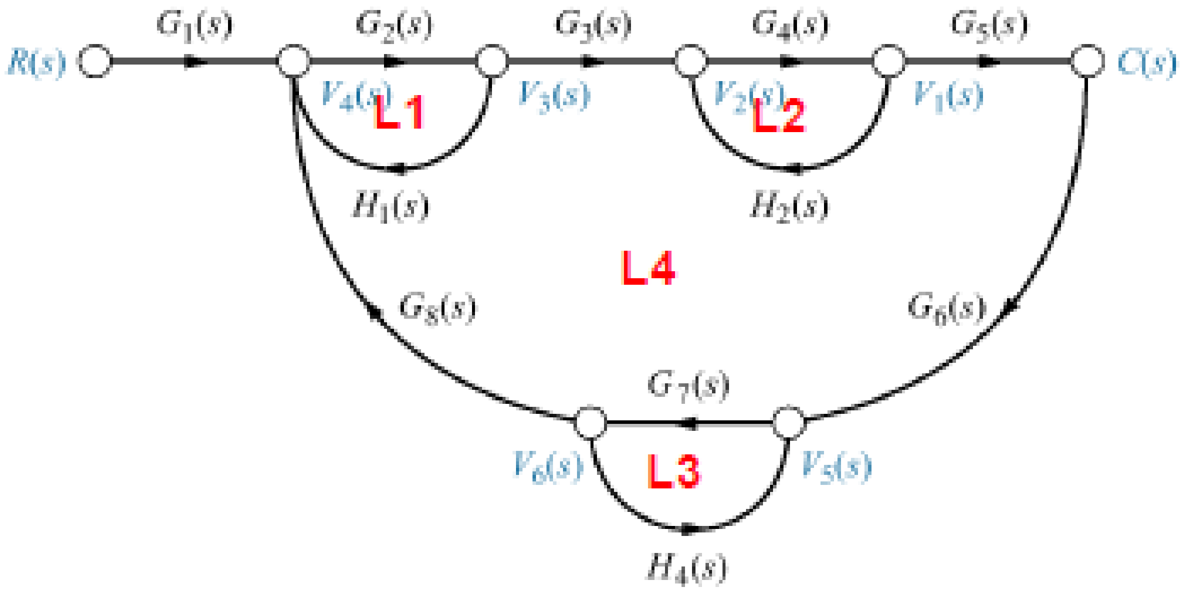
L3. $G_7(s)H_4(s)$

L4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$

Signal-Flow Graph Models

Example.3

Non-touching loops taken two at a time.



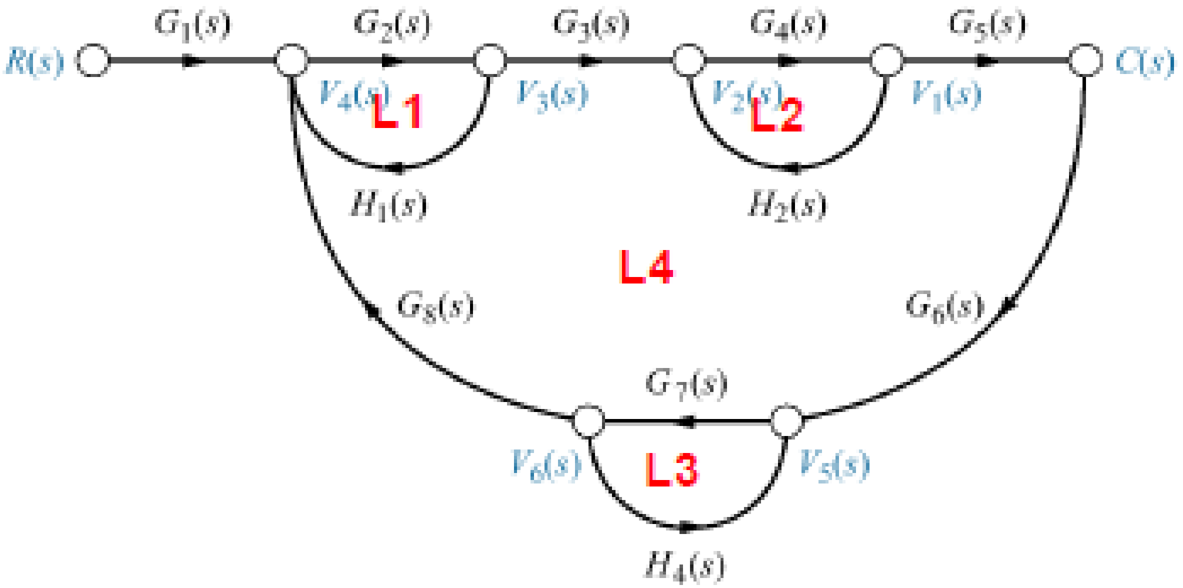
L1 and L2: $G_2(s)H_1(s)G_4(s)H_2(s)$ L2 and L3: $G_4(s)H_2(s)G_7(s)H_4(s)$

L1 and L3: $G_2(s)H_1(s)G_7(s)H_4(s)$

Signal-Flow Graph Models

Example.3

Non-touching loops taken three at a time.



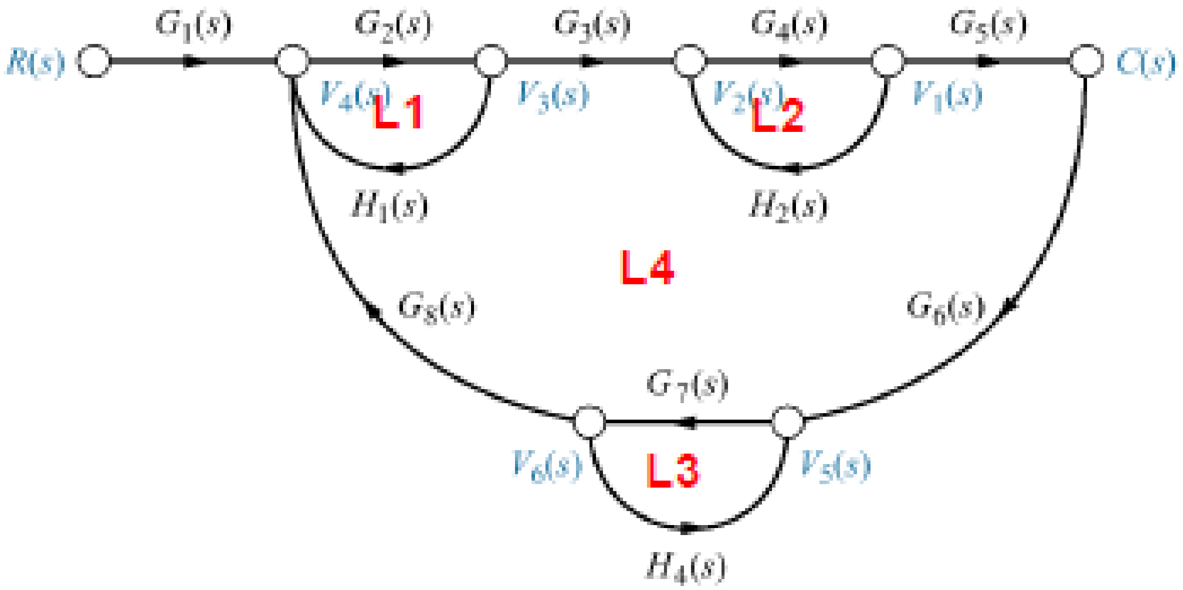
L1, L2, L3: $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$

Signal-Flow Graph Models

Example.3

$$\Delta = 1 - [G_2(s)H_1(s) + G_4(s)H_2(s)$$
$$+ G_7(s)H_4(s) + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)]$$
$$+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s)$$
$$+ G_4(s)H_2(s)G_7(s)H_4(s)]$$
$$- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]$$

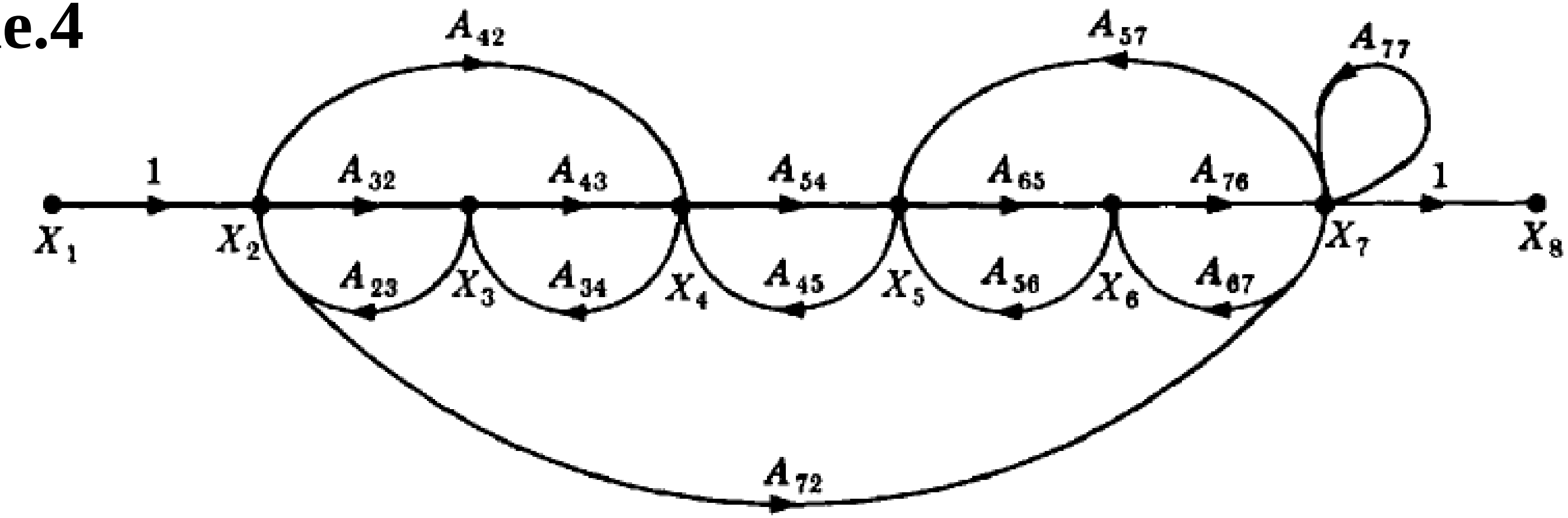
Eliminate forward path-



$$\Delta_1 = 1 - G_7(s)H_4(s)$$

Signal-Flow Graph Models

Example.4



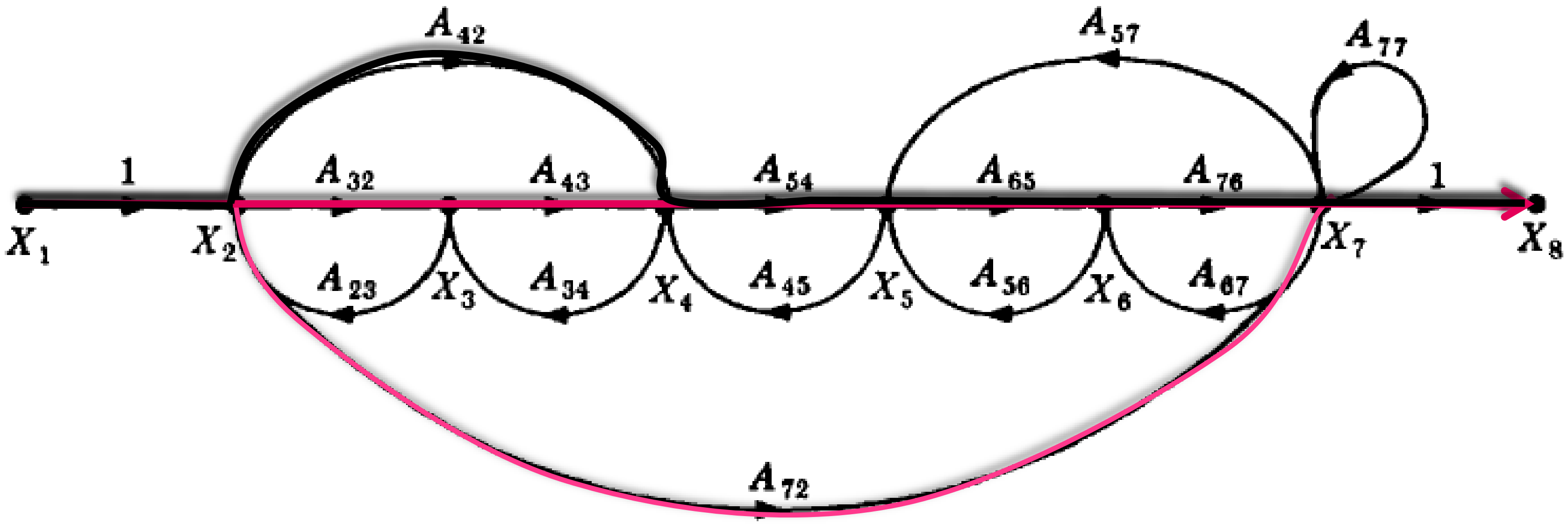
There are three forward paths, therefore n=3.

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{3} P_i \Delta_i}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

Signal-Flow Graph Models

Example.4

$$P_3 = A_{42}A_{54}A_{65}A_{76}$$

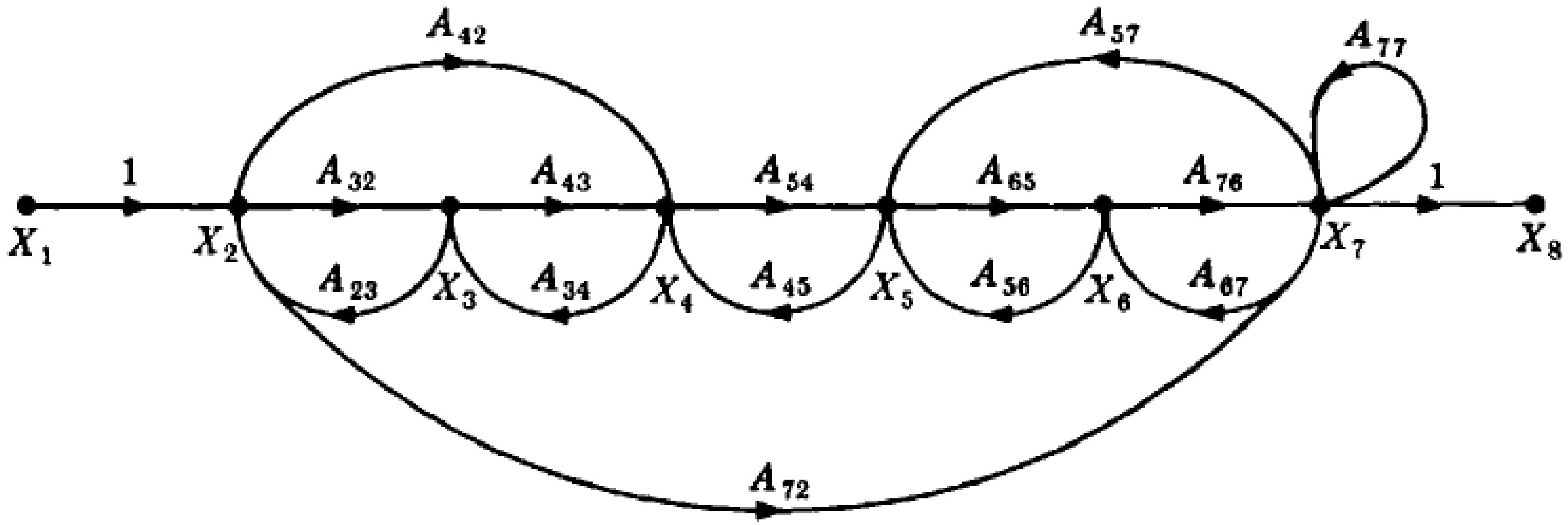


$$P_1 = A_{32}A_{43}A_{54}A_{65}A_{76}$$

$$P_2 = A_{72}$$

Signal-Flow Graph Models

Example.4



$$L_1 = A_{32}A_{23}$$

$$L_2 = A_{43}A_{34}$$

$$L_3 = A_{54}A_{45}$$

$$L_4 = A_{65}A_{56}$$

$$L_5 = A_{76}A_{67}$$

$$L_6 = A_{77}$$

$$L_7 = A_{42}A_{34}A_{23}$$

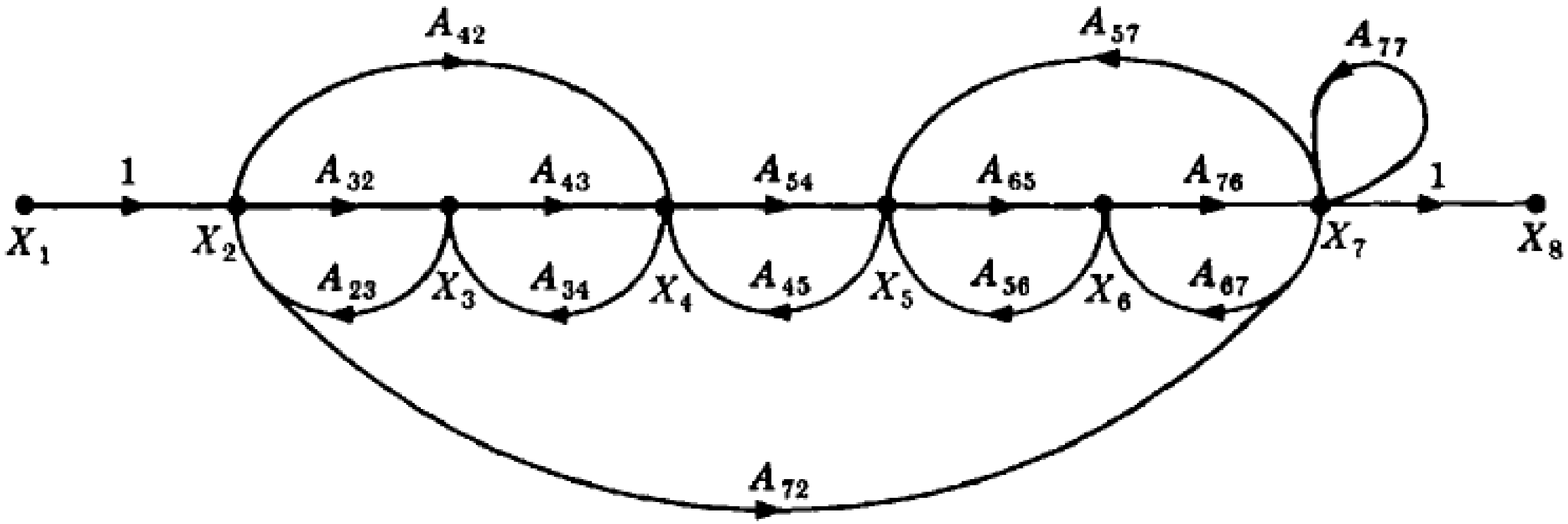
$$L_8 = A_{65}A_{76}A_{67}$$

$$L_9 = A_{72}A_{57}A_{45}A_{34}A_{23}$$

$$L_{10} = A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$$

Signal-Flow Graph Models

Example.4



L_1L_3 L_2L_4 L_3L_5 L_4L_6 L_5L_7 L_7L_8

L_1L_4 L_2L_5 L_3L_6 L_4L_7

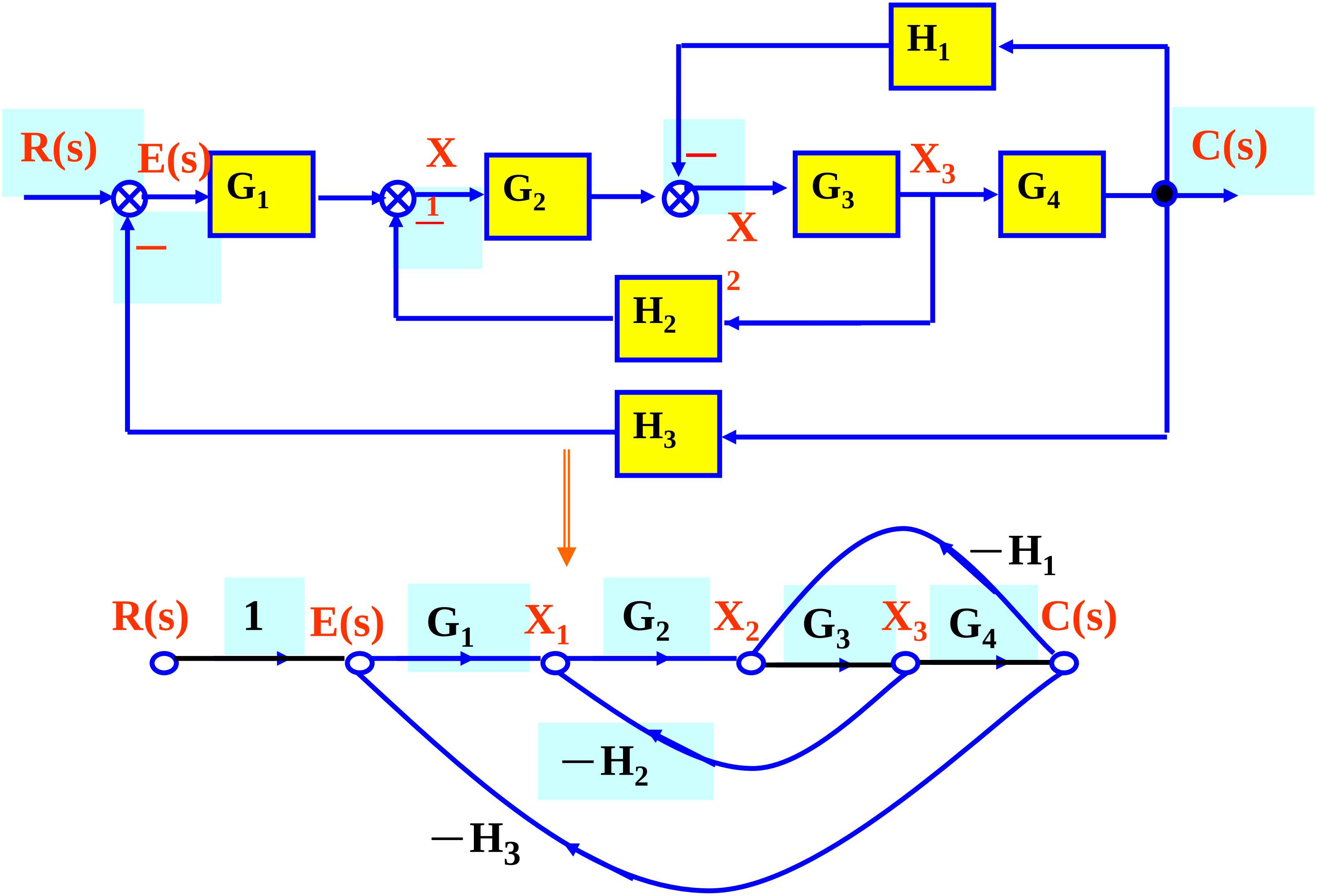
L_1L_5 L_2L_6

L_1L_6 L_2L_8

L_1L_8

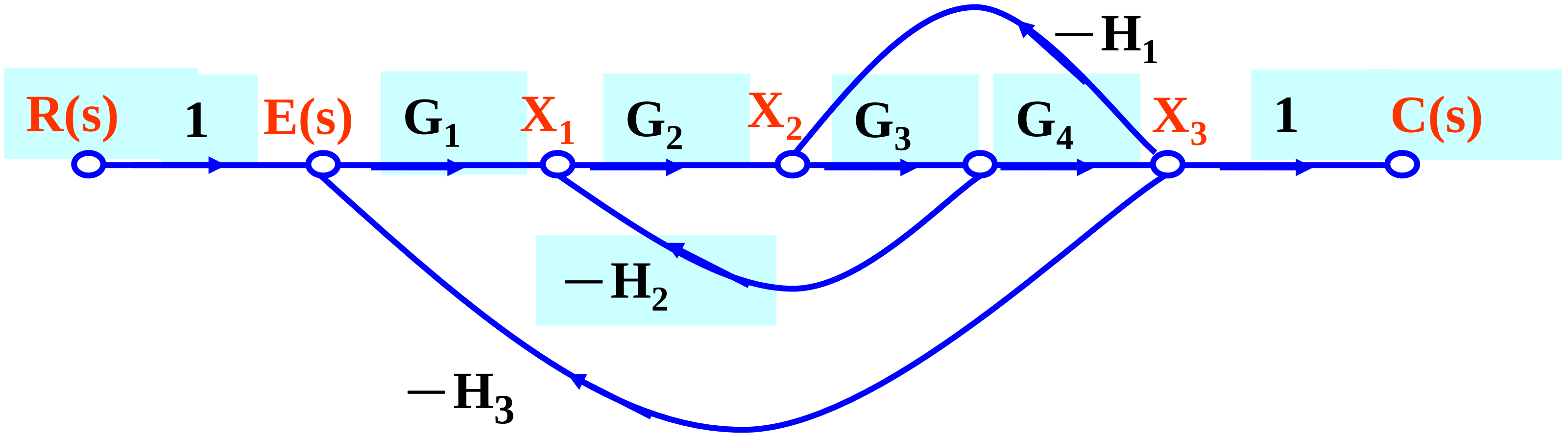
Block Diagram to Signal-Flow Graph

Example. 5:



Block Diagram to Signal-Flow Graph

Example. 5:



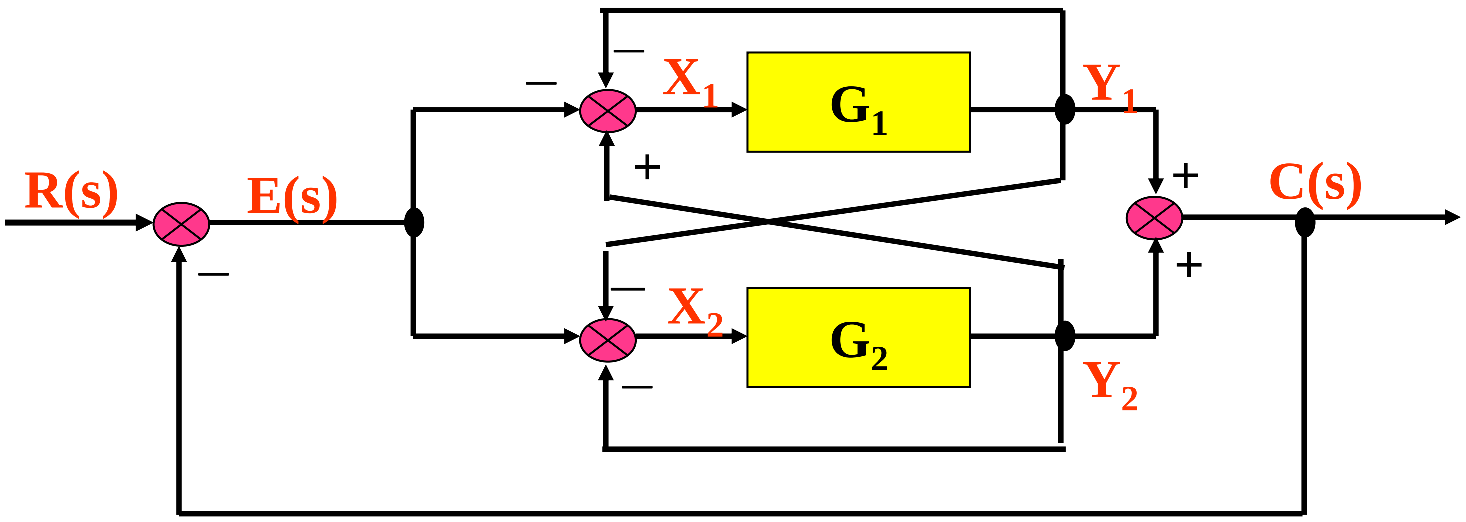
$$\Delta = 1 + (G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1)$$

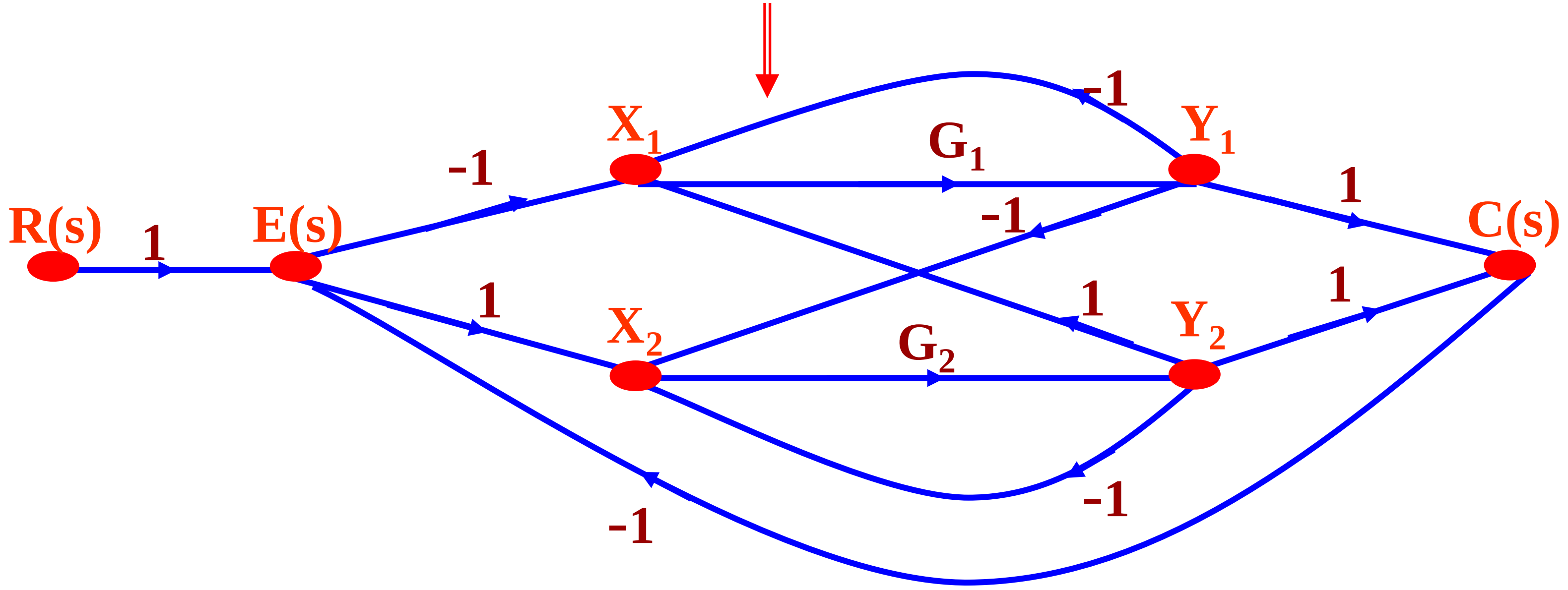
$$P_1 = G_1G_2G_3G_4; \qquad \Delta_1 = 1$$

$$G = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1}$$

Block Diagram to Signal-Flow Graph

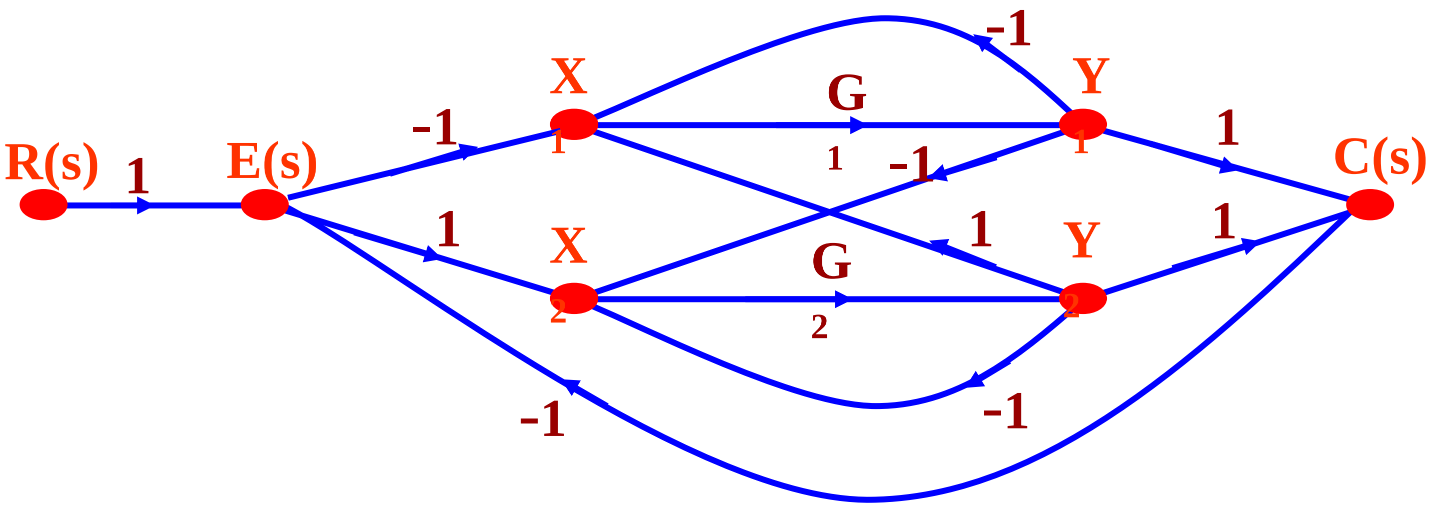
Example. 6:





Block Diagram to Signal-Flow Graph

Example. 6:



7 loops:

$[G_1\cdot(-1)]$; $[G_2\cdot(-1)]$; $[G_1\cdot(-1)\cdot G_2\cdot 1]$; $[(-1)\cdot G_1\cdot 1\cdot(-1)]$;

$[(-1)\cdot G_1\cdot(-1)\cdot G_2\cdot 1\cdot(-1)]$; $[1\cdot G_2\cdot 1\cdot(-1)]$; $[1\cdot G_2\cdot 1\cdot G_1\cdot 1\cdot(-1)]$.

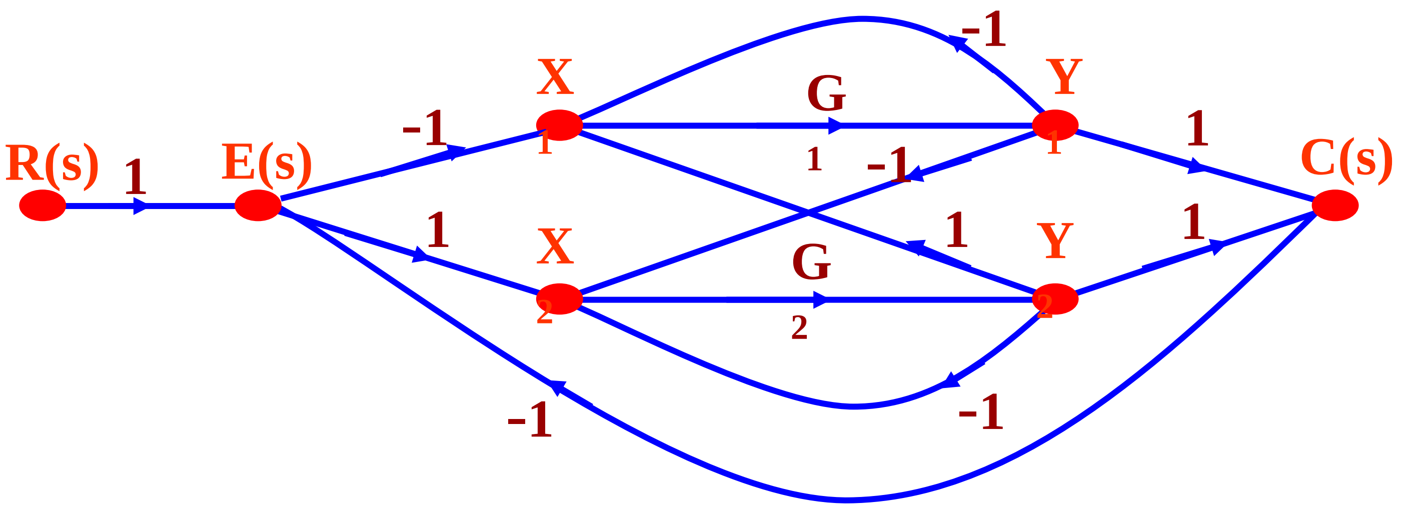
3 ‘2 non-touching loops’:

$[G_1\cdot(-1)]\cdot[G_2\cdot(-1)]$; $[(-1)\cdot G_1\cdot 1\cdot(-1)]\cdot[G_2\cdot(-1)]$;

$[1\cdot G_2\cdot 1\cdot(-1)]\cdot[G_1\cdot(-1)]$.

Block Diagram to Signal-Flow Graph

Example. 6:



Then: $$\Delta = 1 + 2G_2 + 4G_1G_2$$

4 forward paths:

$$p_1 = (-1)\cdot G_1 \cdot 1 \qquad \Delta_1 = 1 + G_2$$

$$p_2 = (-1)\cdot G_1 \cdot (-1)\cdot G_2 \cdot 1 \qquad \Delta_2 = 1$$

$$p_3 = 1\cdot G_2 \cdot 1 \qquad \Delta_3 = 1 + G_1$$

$$p_4 = 1\cdot G_2 \cdot 1\cdot G_1 \cdot 1 \qquad \Delta_4 = 1$$

Block Diagram to Signal-Flow Graph

Example. 6:

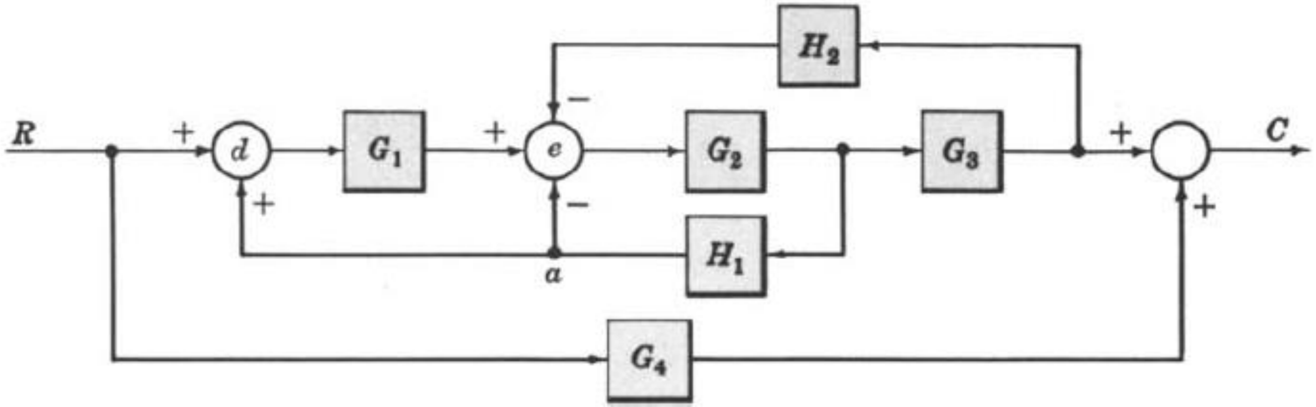
We have

$$\frac{C(s)}{R(s)} = \frac{\sum p_k \Delta_k}{\Delta}$$

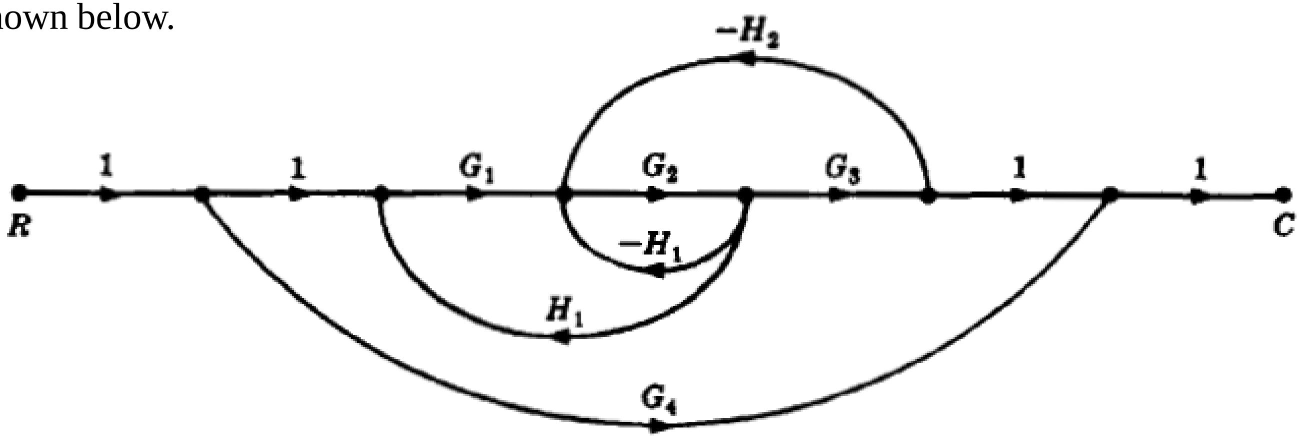
$$= \frac{G_2 - G_1 + 2G_1G_2}{1 + 2G_2 + 4G_1G_2}$$

Block Diagram to Signal-Flow Graph

Example. 7:



- The signal flow graph of the above block diagram is shown below.



- There are two forward paths. The path gains are

$$P_1 = G_1G_2G_3 \text{ and } P_2 = G_4$$

- The three feedback loop gains are

$$P_{11} = -G_2H_1,\ P_{21} = G_1G_2H_1,\ P_{31} = -G_2G_3H_2.$$

- No loops are non-touching, hence

$$\Delta = 1 - (P_{11} + P_{21} + P_{31})$$

- Because the loops touch the nodes of P1, hence

$$\Delta_1 = 1$$

- Since no loops touch the nodes of P2, therefore

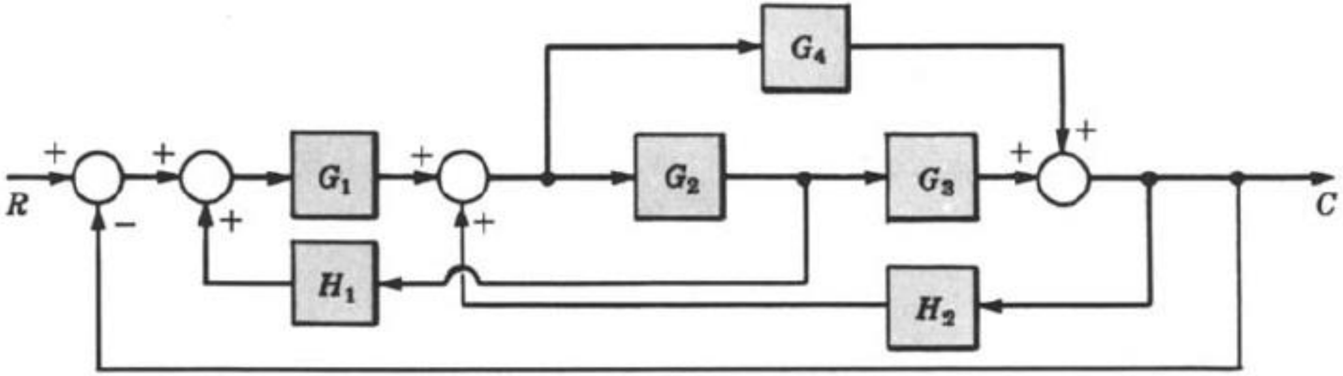
$$\Delta_2 = \Delta.$$

- Hence the control ratio T = C/R is

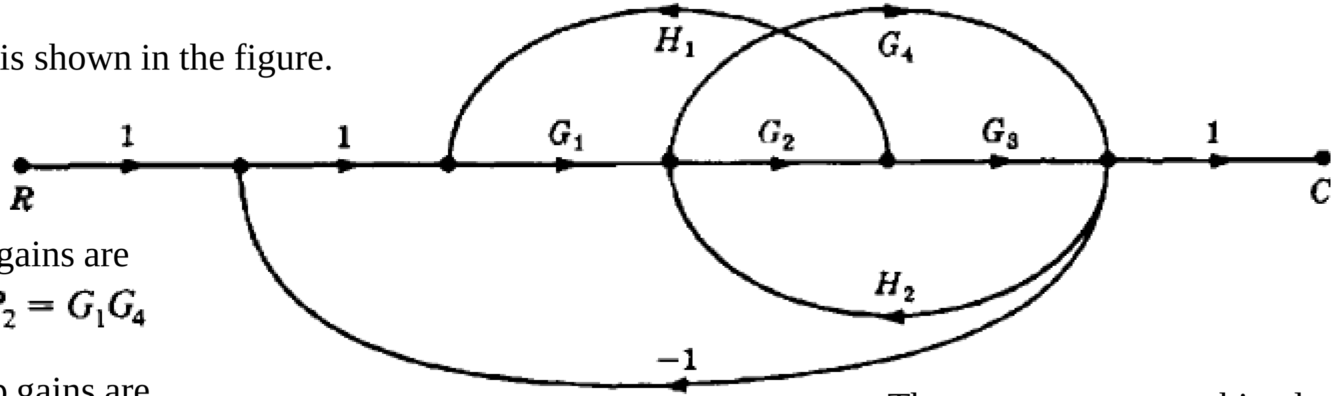
$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_4 + G_2G_4H_1 - G_1G_2G_4H_1 + G_2G_3G_4H_2}{1 + G_2H_1 - G_1G_2H_1 + G_2G_3H_2}$$

Block Diagram to Signal-Flow Graph

Example. 7:



- The signal flow graph is shown in the figure.



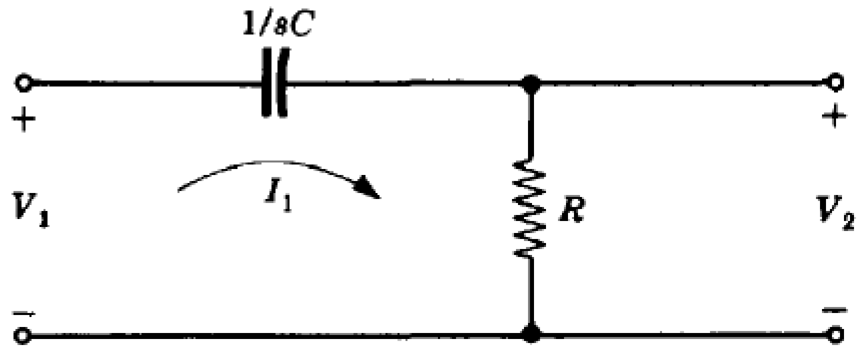
- The two forward path gains are
  $P_1 = G_1G_2G_3$ and $P_2 = G_1G_4$
- The five feedback loop gains are
  $P_{11} = G_1G_2H_1,\ P_{21} = G_2G_3H_2,\ P_{31} = -G_1G_2G_3,$
  $P_{41} = G_4H_2,$ and $P_{51} = -G_1G_4.$
- All feedback loops touches the two forward paths, hence $\Delta_1 = \Delta_2 = 1$
- There are no non-touching loops, hence

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51})$$
$$= 1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4$$

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4}$$

Electrical System to Signal-Flow Graph

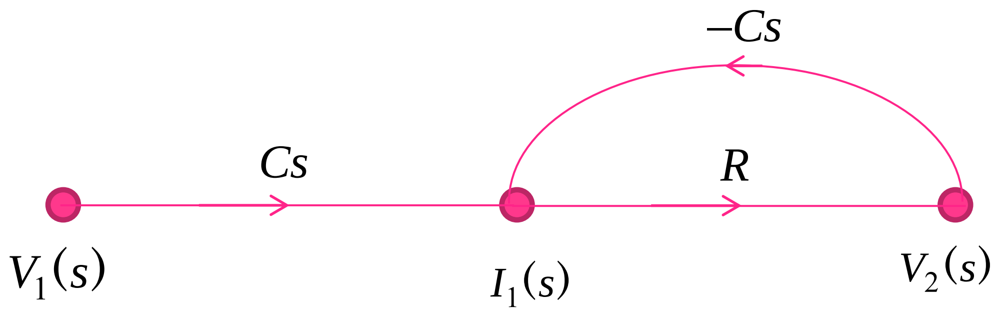
Example. 8:



$$V_1(s) = \frac{1}{Cs} I_1(s) + I_1(s)R$$

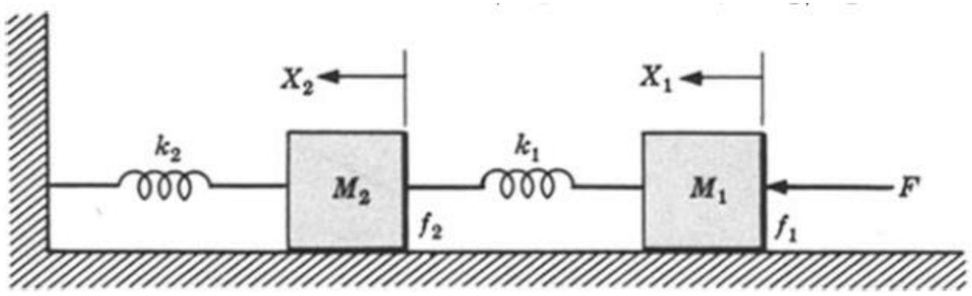
$$V_2(s) = I_1(s)R$$

$$\Rightarrow \quad CsV_1(s) - CsV_2(s) = I_1(s)$$



Electrical System to Signal-Flow Graph

Example. 9:



$$F = M_1 s^2 X_1 + k_1(X_1 - X_2)$$

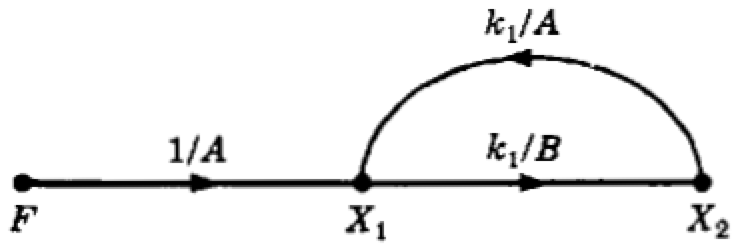
$$0 = M_2 s^2 X_2 + k_1(X_2 - X_1) + k_2 X_2$$

(i) $$F + k_1 X_2 = \left(M_1 s^2 + f_1 s + k_1\right) X_1$$

(ii) $$k_1 X_1 = \left(M_2 s^2 + f_2 s + k_1 + k_2\right) X_2$$

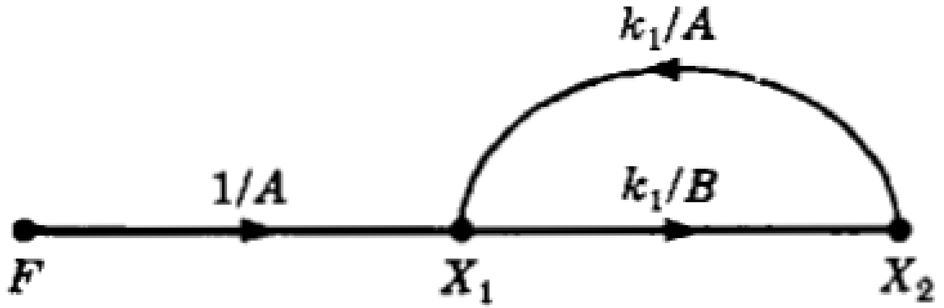
(iii) $$\left(\frac{1}{A}\right) F + \left(\frac{k_1}{A}\right) X_2 = X_1$$

(iv) $$\left(\frac{k_1}{B}\right) X_1 = X_2$$



Electrical System to Signal-Flow Graph

Example. 9:



The forward path gain is $P_1 = k_1/AB$. The feedback loop gain is $P_{11} = k_1^2/AB$. then $\Delta = 1 - P_{11} = (AB - k_1^2)/AB$ and $\Delta_1 = 1$. Finally,

$$\frac{X_2}{F} = \frac{P_1\Delta_1}{\Delta} = \frac{k_1}{AB - k_1^2} = \frac{k_1}{(M_1s^2 + f_1s + k_1)(M_2s^2 + f_2s + k_1 + k_2) - k_1^2}$$