Bode Plot

Frequency Response

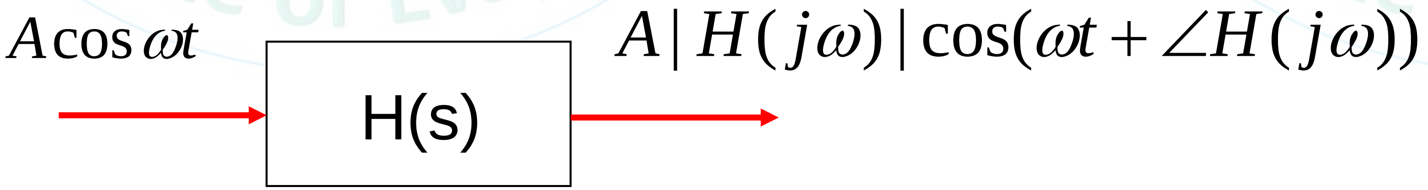
- Frequency Response: Given T.F. H(s)

$$\begin{cases} |H(j\omega)| & \text{v.s. } \omega \text{ is amp. resp.} \\ \angle H(j\omega) & \text{v.s. } \omega \text{ is phase resp.} \end{cases}$$

$$\text{most time, plot} \begin{cases} 20\log_{10}|H(j\omega)| & \text{v.s.} \quad \log_{10}\omega \\ \angle H(j\omega) \text{ in } ^\circ & \text{v.s.} \quad \log_{10}\omega \end{cases}$$

These are the Bode plot. In Matlab: bode

meaning:



Bode Plots

A better way to graphically display the frequency response!

Bode Magnitude Plot:

plots the magnitude of $G(j\omega)$ in decibels w.r.t. logarithmic frequency, i.e.,

$$\left|G(j\omega)\right|\Big|_{\text{dB}} = 20\log_{10}\left|G(j\omega)\right| \quad \text{vs} \quad \log_{10}\omega$$

Bode Phase Plot:

plots the phase angle of $G(j\omega)$ w.r.t. logarithmic frequency, i.e.,

$$\angle G(j\omega) \quad \text{vs} \quad \log_{10}\omega$$

Benefits:

- Display the dependence of magnitude of the frequency response on the input frequency better, especially for magnitude approaching zero
- Log axis converts the multiplications and divisions into additions and subtractions, which are easier to handle graphically
- Allow straight-line approximations for quick sketch

Example

$$\text{Example}: \quad H(s) = \frac{3}{s+3}$$

$$\text{If input} = 10\cos(3t), \omega = 3$$

$$|H(j\omega)| = \left|\frac{3}{3+j3}\right| = \frac{1}{\sqrt{2}}, \quad \angle H(j\omega) = \angle\frac{3}{3+j3} = -45^\circ$$

$$\therefore \text{output} = \frac{10}{\sqrt{2}}\cos(3t - 45^\circ)$$

Works only if H(s) is stable

Bode Plots

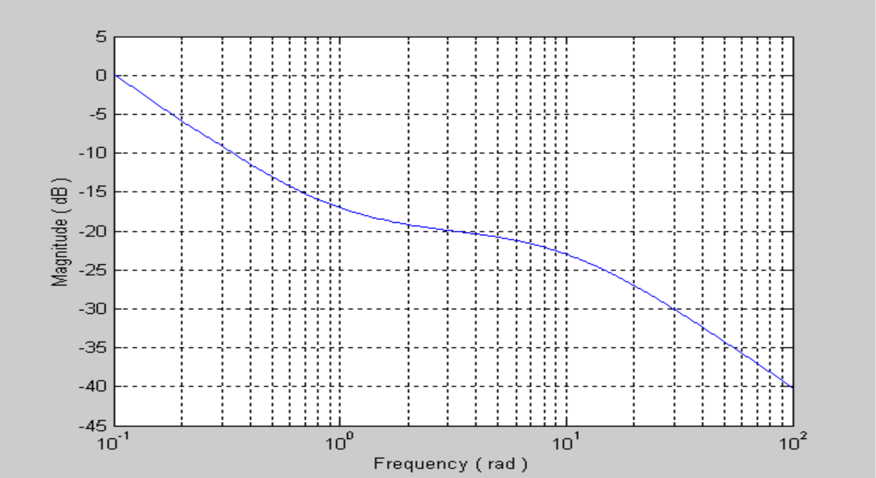
Ex: $G(s)=\dfrac{s+1}{s^2+10s} \Rightarrow G(j\omega)=\dfrac{(j\omega)+1}{(j\omega)^2+10(j\omega)}$

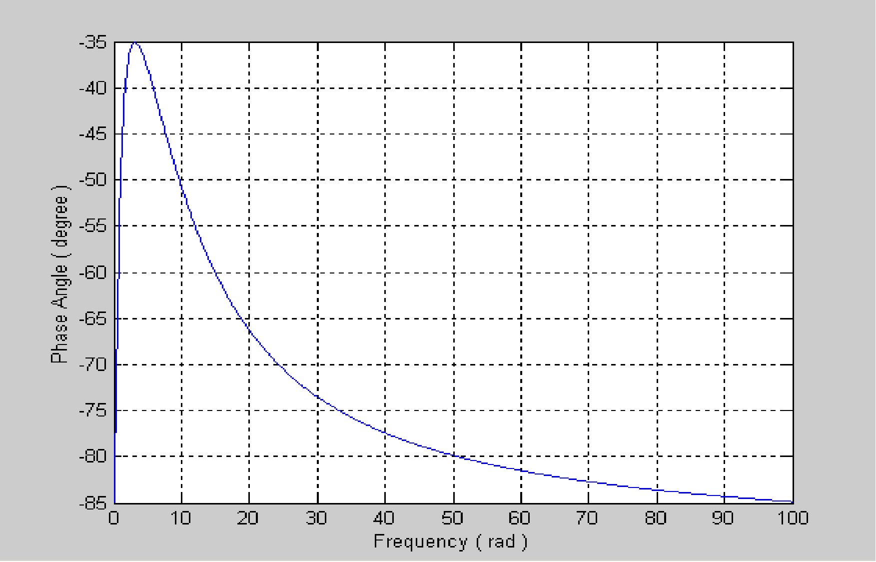
$$|G(j\omega)|=\frac{\sqrt{1+\omega^2}}{\omega\sqrt{100+\omega^2}}$$

$$\angle G(j\omega)=\angle(j\omega+1)-\angle\left(-\omega^2+10j\omega\right)$$

$$=\tan^{-1}(\omega)-\text{atan2}\left(10\omega,-\omega^2\right)$$

ω	$\lvert G(j\omega)\rvert$	$20\log_{10}\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0.1	1.0049	0.0428	-83.8623
0.2	0.5098	-5.8520	-79.8358
0.5	0.2233	-13.0211	-66.2974
1	0.1407	-17.0329	-50.7016
2	0.1096	-19.2012	-37.8750
5	0.0912	-20.7988	-37.8750
10	0.0711	-22.9671	-50.7106
20	0.0448	-26.9789	-66.2974
50	0.0196	-34.1480	-79.8358
100	0.0100	-40.0428	-84.8623





Bode Plots of LTI Systems

Transfer Function

$$G(s)=\frac{b_m s^m+b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}=\frac{b_m(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

Frequency Response

$$|G(j\omega)|=\left|\frac{b_m(j\omega-z_1)\cdots(j\omega-z_m)}{(j\omega-p_1)\cdots(j\omega-p_n)}\right|=|b_m|\cdot\left|\frac{1}{(j\omega-p_1)}\right|\cdots\left|\frac{1}{(j\omega-p_n)}\right|\cdot|(j\omega-z_1)|\cdots|(j\omega-z_m)|$$

Bode Magnitude Plot

$$20\log_{10}(|G(j\omega)|)=20\log_{10}(|b_m|)+20\log_{10}\left(\left|\frac{1}{(j\omega-p_1)}\right|\right)+\cdots+20\log_{10}\left(\left|\frac{1}{(j\omega-p_n)}\right|\right)$$
$$+20\log_{10}(|(j\omega-z_1)|)+\cdots+20\log_{10}(|(j\omega-z_1)|)$$

Bode Phase Plot

$$\angle G(j\omega)=\angle\frac{b_m(j\omega-z_1)\cdots(j\omega-z_m)}{(j\omega-p_1)\cdots(j\omega-p_n)}=\angle b_m+\angle(j\omega-z_1)+\angle(j\omega-z_2)+\cdots+\angle(j\omega-z_m)$$
$$-\angle(j\omega-p_1)-\angle(j\omega-p_2)\cdots-\angle(j\omega-p_n)$$

Example

Ex: Find the magnitude and the phase of the following transfer function:

$$G(s)=\frac{3s^3+12s^2+9s}{2s^3+22s^2+76s+80}=\frac{3s(s+3)(s+1)}{2(s+2)(s+4)(s+5)}$$

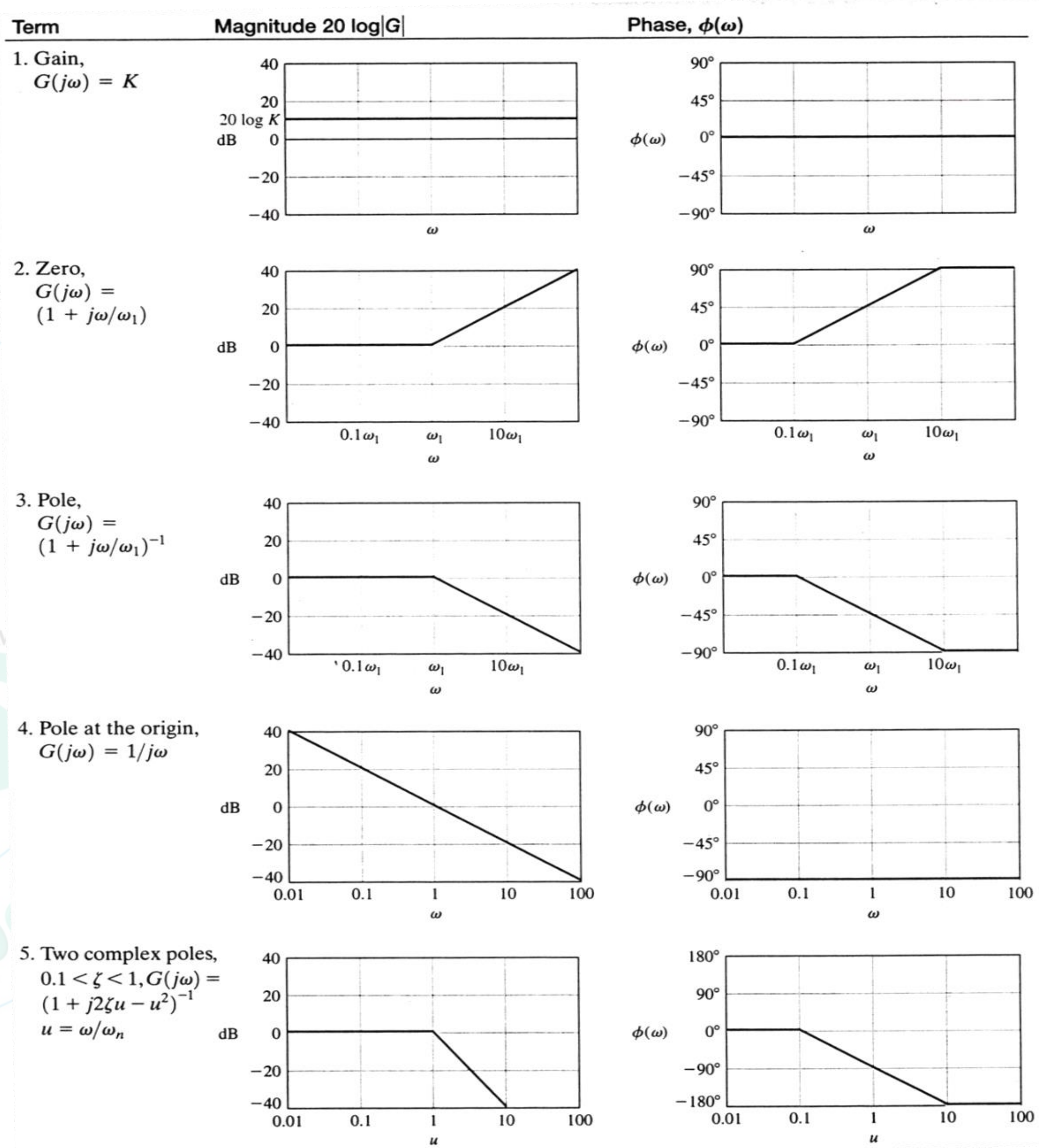
$$=\frac{\frac{3\times 3}{2\times 2\times 4\times 5}s\left(\frac{1}{3}s+1\right)(s+1)}{\left(\frac{1}{2}s+1\right)\left(\frac{1}{4}s+1\right)\left(\frac{1}{5}s+1\right)}=\frac{9}{80}\frac{s\left(\frac{1}{3}s+1\right)(s+1)}{\left(\frac{1}{2}s+1\right)\left(\frac{1}{4}s+1\right)\left(\frac{1}{5}s+1\right)}$$

$$|G(j\omega)|=\frac{\left|\frac{9}{80}\right|\,|j\omega|\,\left|\frac{1}{3}j\omega+1\right|}{\left|\frac{1}{2}j\omega+1\right|\,\left|\frac{1}{4}j\omega+1\right|\,\left|\frac{1}{5}j\omega+1\right|}$$

$$20\log_{10}\{|G(j\omega)|\}=20\log_{10}\left\{\frac{9}{80}\right\}+20\log_{10}\{|j\omega|\}+20\log_{10}\left\{\left|\frac{1}{3}j\omega+1\right|\right\}$$
$$-20\log_{10}\left\{\left|\frac{1}{2}j\omega+1\right|\right\}-20\log_{10}\left\{\left|\frac{1}{4}j\omega+1\right|\right\}-20\log_{10}\left\{\left|\frac{1}{5}j\omega+1\right|\right\}$$

$$\angle G(j\omega)=\angle\frac{9}{80}+\angle(j\omega)+\angle\left(\frac{1}{3}j\omega+1\right)+\angle(j\omega+1)$$
$$-\angle\left(\frac{1}{2}j\omega+1\right)-\angle\left(\frac{1}{4}j\omega+1\right)-\angle\left(\frac{1}{5}j\omega+1\right)$$

Asymptotic curves for basic factors



Bode Plots of 1st Order Poles

Standard Form of Transfer Function:

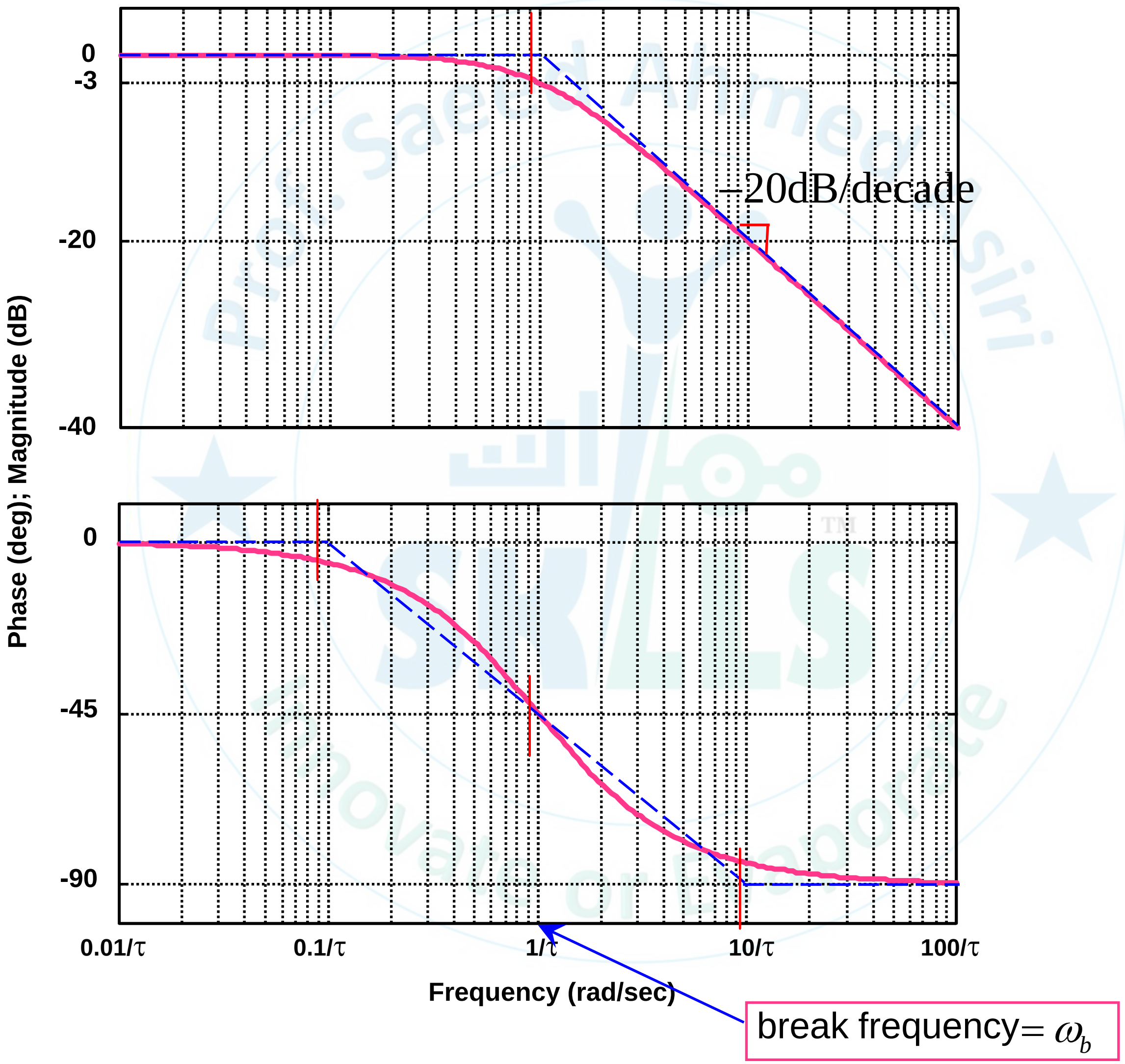
$$G_{p1}(s) = \frac{1}{\tau s + 1}, \quad \tau > 0$$

Frequency Response:

$$G_{p1}(j\omega) = \frac{1}{\tau j\omega + 1}, \quad \tau > 0$$

$$\begin{cases} |G_{p1}(j\omega)| = \dfrac{1}{\sqrt{\tau^2\omega^2 + 1}} \\ \angle G_{p1}(j\omega) = -\operatorname{atan2}(\tau\omega, 1) \\ \qquad = -\tan^{-1}(\tau\omega) \end{cases}$$

$$20\log_{10}|G_{p1}(j\omega)|$$
$$= -10\log_{10}(\tau^2\omega^2 + 1)$$
$$\approx \begin{cases} 0\text{dB}, & \tau\omega \ll 1 \text{ or } \omega \ll \dfrac{1}{\tau} = \omega_b \\ -3\text{dB}, & \tau\omega = 1 \text{ or } \omega = \dfrac{1}{\tau} = \omega_b \\ -20\log_{10}\left(\dfrac{\omega}{\omega_b}\right), & \tau\omega \gg 1 \text{ or } \omega \gg \dfrac{1}{\tau} = \omega_b \end{cases}$$



Q: By just looking at the Bode diagram, can you determine the time constant and the steady state gain of the system ?

Example

1st Order Real Poles

Transfer Function:

$$G(s) = \frac{50}{s+5}$$

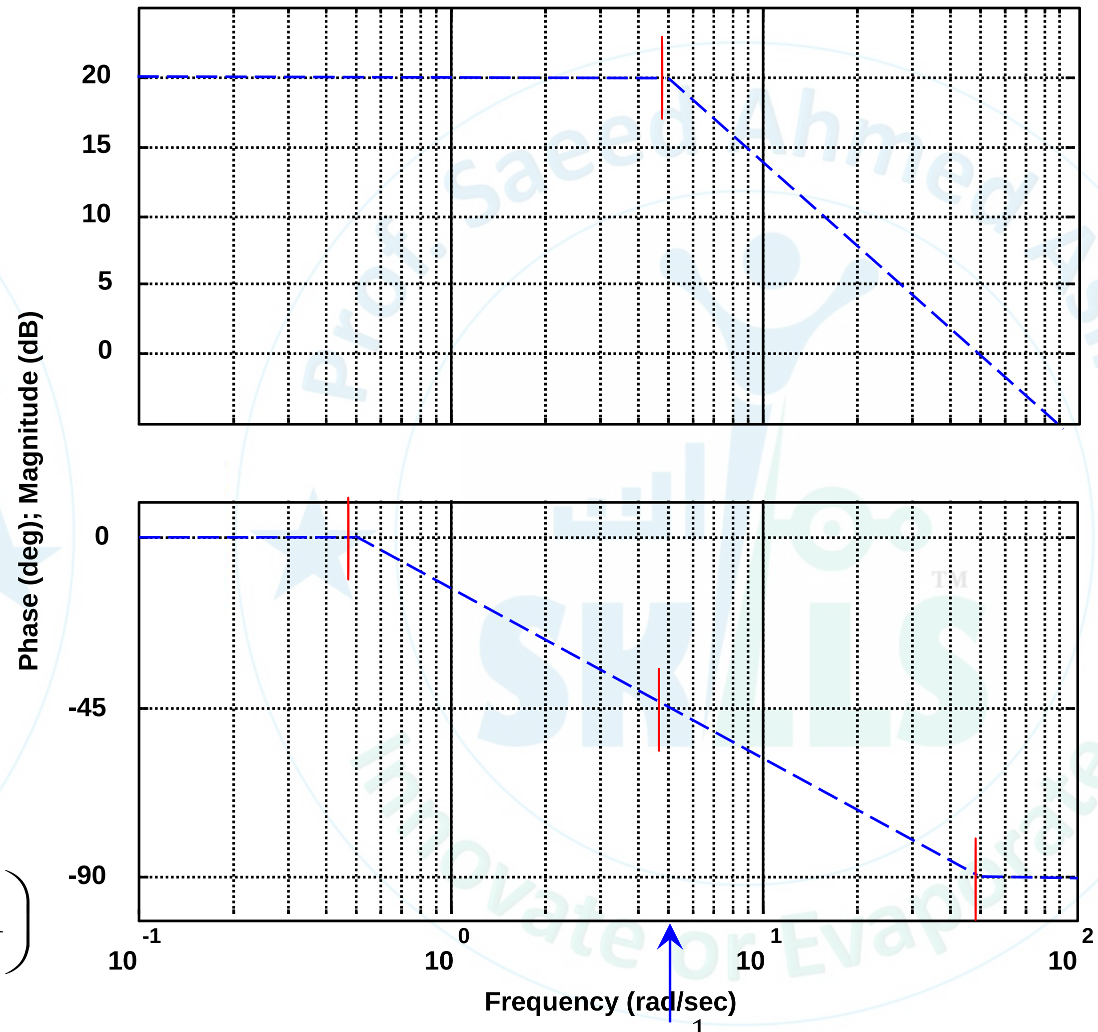
Plot the straight-line approximation of *G*(*s*)'s Bode diagram:

$$G(s) = \frac{50}{s+5} = 10 \times \frac{1}{\frac{1}{5}s+1}$$

τ

$$20\log_{10}|G(j\omega)| = 20 + 20\log_{10}\left|\frac{1}{\frac{1}{5}j\omega+1}\right|$$

$$\angle G(j\omega) = -\angle\left(\frac{1}{5}j\omega+1\right)$$



$$\omega_b = \frac{1}{\tau} = 5$$

Bode Plots of 1st Order Zeros

Standard Form of Transfer Function

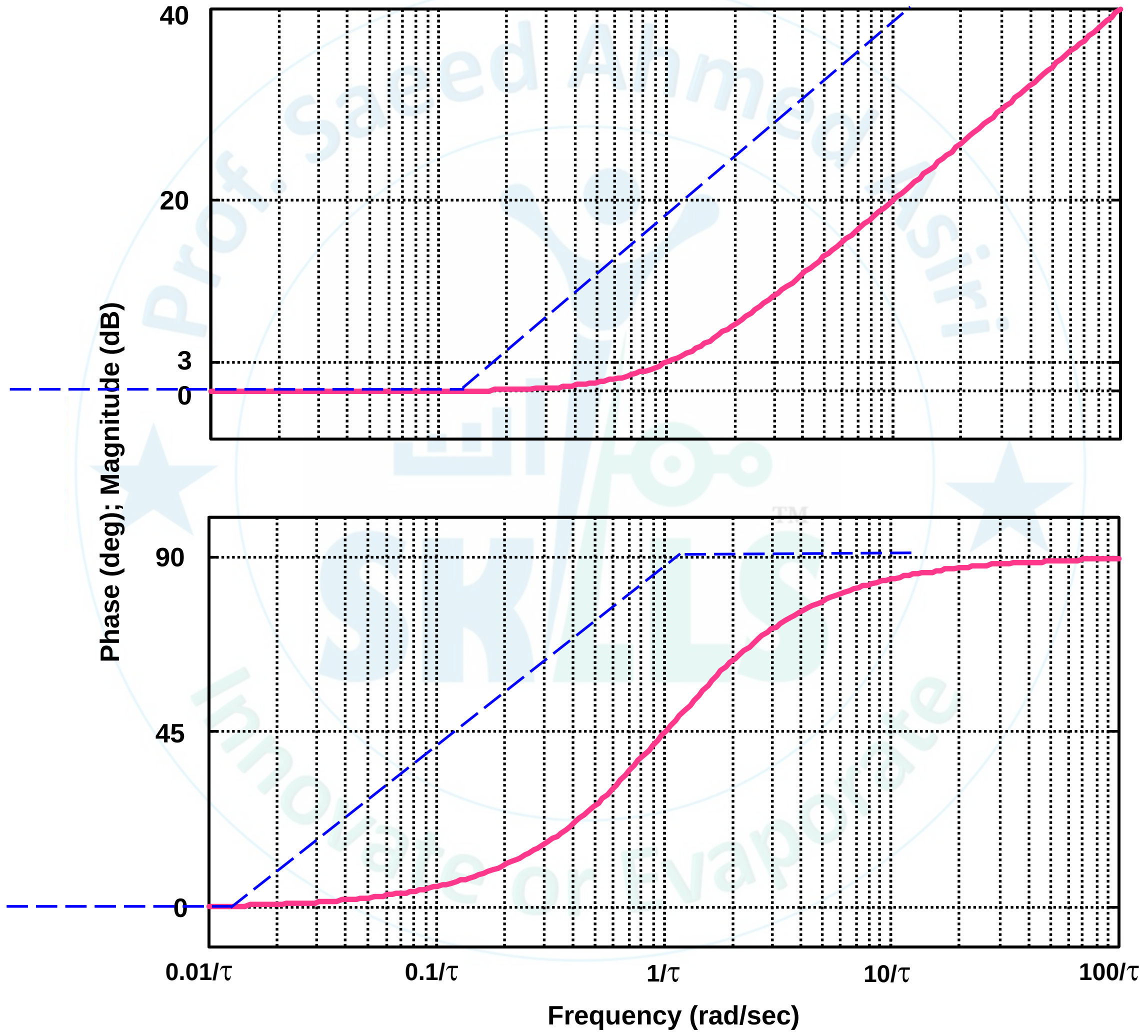
$$G_{z1}(s) = \tau s + 1 \quad , \qquad \tau > 0$$

Frequency Response

$$20\log_{10}\left|G_{p1}(j\omega)\right|$$

$$= 10\log_{10}\left(\tau^2\omega^2 + 1\right)$$

$$\approx \begin{cases} 0\text{dB}, & \tau\omega \ll 1 \text{ or } \omega \ll \dfrac{1}{\tau} = \omega_b \\ 3\text{dB}, & \tau\omega = 1 \text{ or } \omega = \dfrac{1}{\tau} = \omega_b \\ 20\log_{10}\left(\dfrac{\omega}{\omega_b}\right), & \tau\omega \gg 1 \text{ or } \omega \gg \dfrac{1}{\tau} = \omega_b \end{cases}$$



Example

1st Order Real Zeros

Transfer Function:

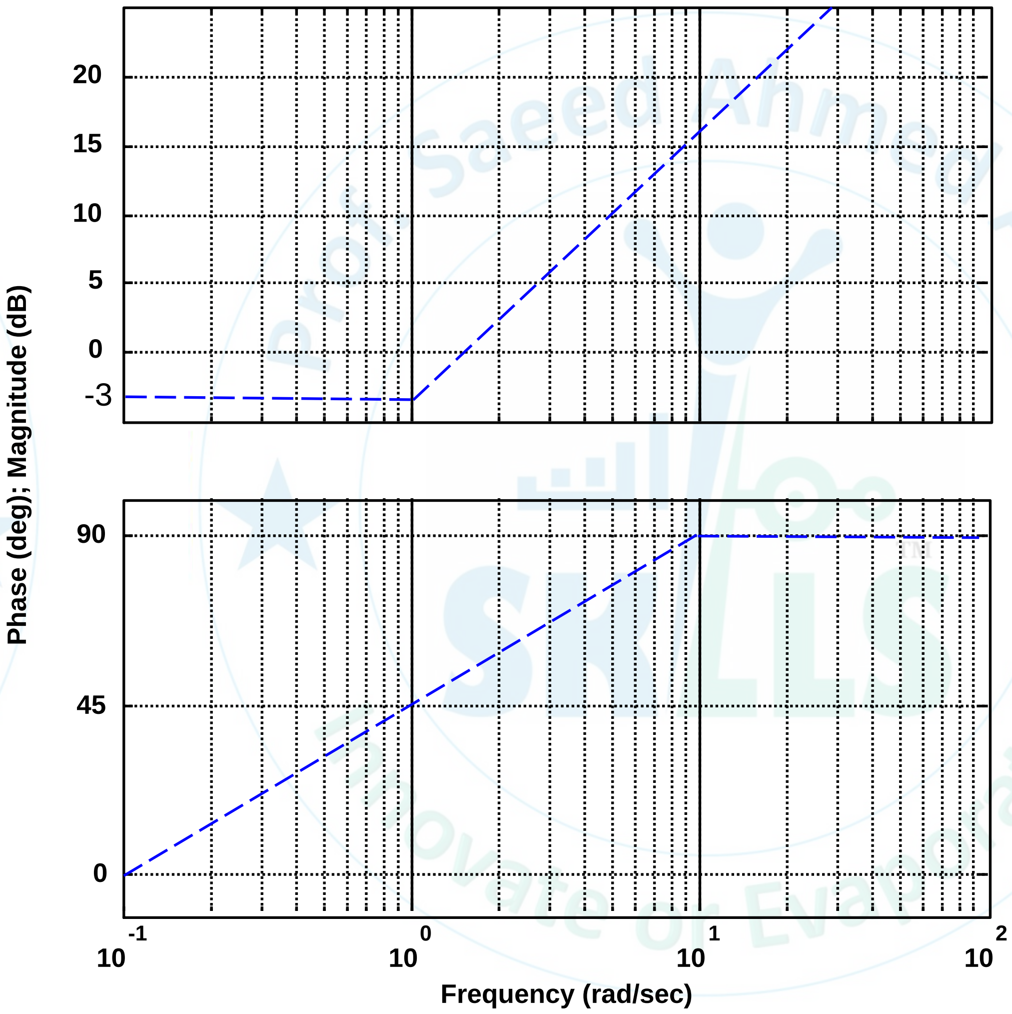
$$G(s) = 0.7s + 0.7$$

Plot the straight-line approximation of *G*(*s*)'s Bode diagram:

$$G(j\omega) = 0.7(j\omega + 1)$$

$$20\log_{10}|G(j\omega)| = 20\log_{10} 0.7 + 20\log_{10}|j\omega + 1|$$

$$\angle G(j\omega) = \angle(j\omega + 1)$$



Example

Lead Compensator

Transfer Function:

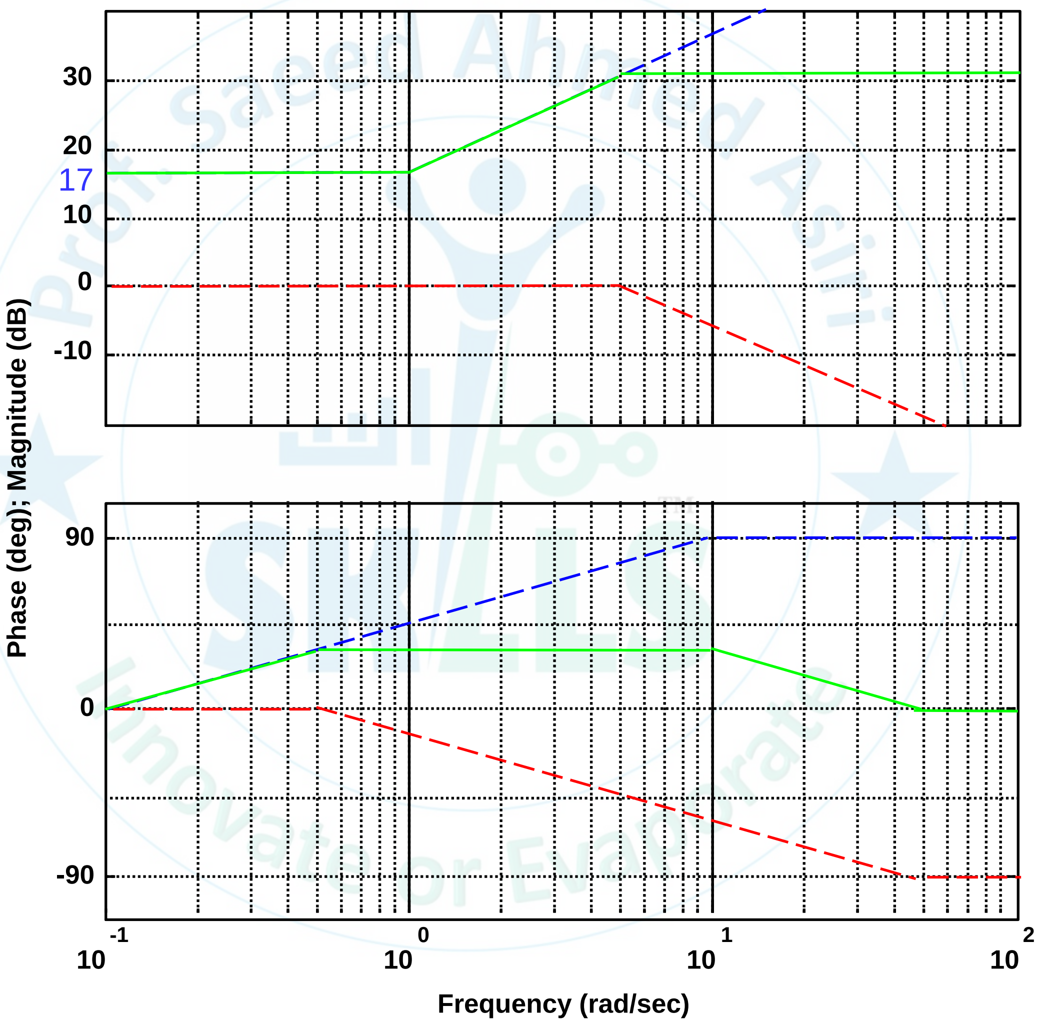
$$G(s)=\frac{35s+35}{s+5}$$

Plot the straight-line approximation of *G*(*s*)'s Bode diagram:

$$G(j\omega)=\frac{35}{5}(j\omega+1)\frac{1}{\frac{1}{5}j\omega+1}$$

$$20\log_{10}|G(j\omega)|=\underbrace{16.9}_{20\log_{10}7}+20\log_{10}|j\omega+1|+20\log_{10}\left|\frac{1}{\frac{1}{5}j\omega+1}\right|$$

$$\angle G(j\omega)=\angle(j\omega+1)+\angle\left(\frac{1}{\frac{1}{5}j\omega+1}\right)$$



1st Order Bode Plots Summary

- **1st Order Poles**

$$G_{p1}(s) = \frac{1}{\tau s + 1} \quad , \quad \tau > 0$$

  - Break Frequency

$$\omega_b = \frac{1}{\tau} \quad [\text{rad/s}]$$

  - Mag. Plot Approximation

    0 dB from DC to ω_b and a straight line with –20 dB/decade slope after ω_b

  - Phase Plot Approximation

    0 deg from DC to $\frac{1}{10}\omega_b$. Between $\frac{1}{10}\omega_b$ and $10\omega_b$, a straight line from 0 deg to –90 deg (passing –45 deg at ω_b). For frequency higher than $10\omega_b$, straight line on –90 deg.

- **1st Order Zeros**

$$G_{z1}(s) = \tau s + 1 \quad , \quad \tau > 0$$

  - Break Frequency

$$\omega_b = \frac{1}{\tau} \quad [\text{rad/s}]$$

  - Mag. Plot Approximation

    0 dB from DC to ω_b and a straight line with 20 dB/decade slope after ω_b

  - Phase Plot Approximation

    0 deg from DC to $\frac{1}{10}\omega_b$. Between $\frac{1}{10}\omega_b$ and $10\omega_b$, a straight line from 0 deg to 90 deg (passing 45 deg at ω_b). For frequency higher than $10\omega_b$, straight line on 90 deg.

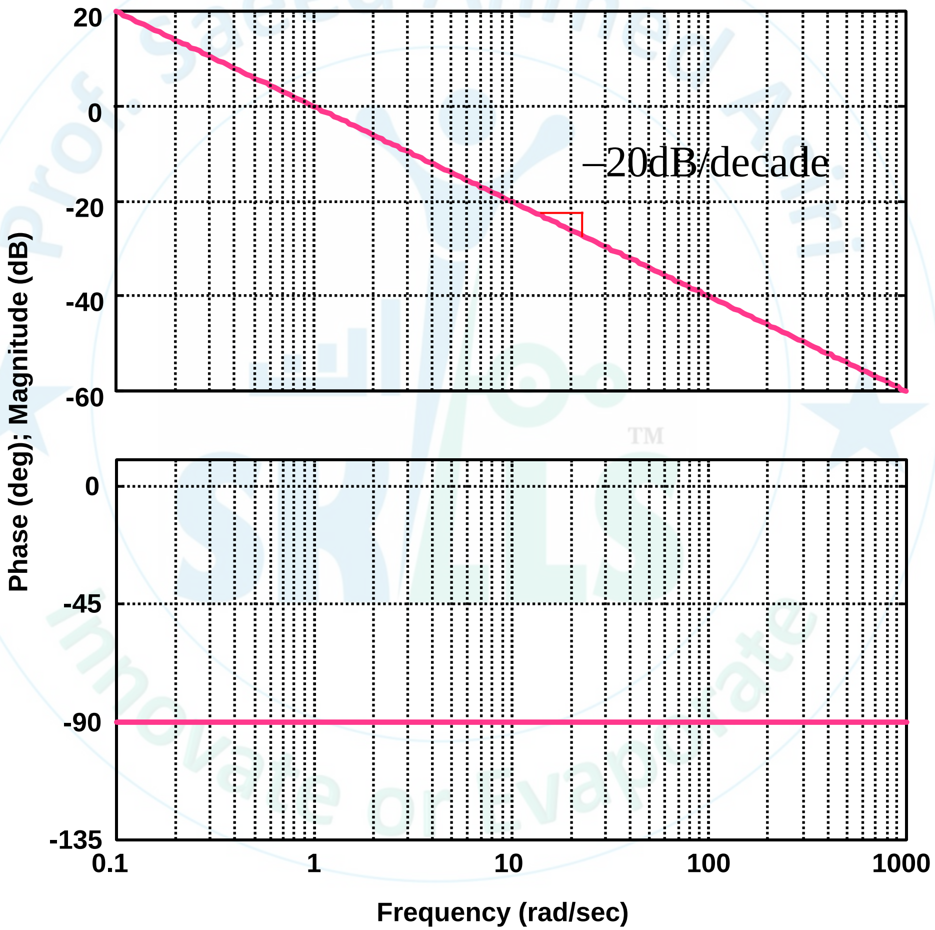
Note: By looking at Bode plots you should be able to determine the relative order of the system, its break frequency, and DC (steady-state) gain. This process should also be reversible, i.e., given a transfer function, be able to plot a straight line approximation of Bode plots.

Bode Plots of Integrators

Standard Form of Transfer Function

$$G_{p0}(s) = \frac{1}{s}$$

Frequency Response



$$20\log_{10}\left|G_{p0}(j\omega)\right| = 20\log_{10}\left|\frac{1}{j\omega}\right| = 20\log_{10}\left|\frac{1}{\omega}\right|$$

$$= 20\log_{10}\frac{1}{\omega} = -20\log_{10}\omega$$

Bode Plots of Differentiators

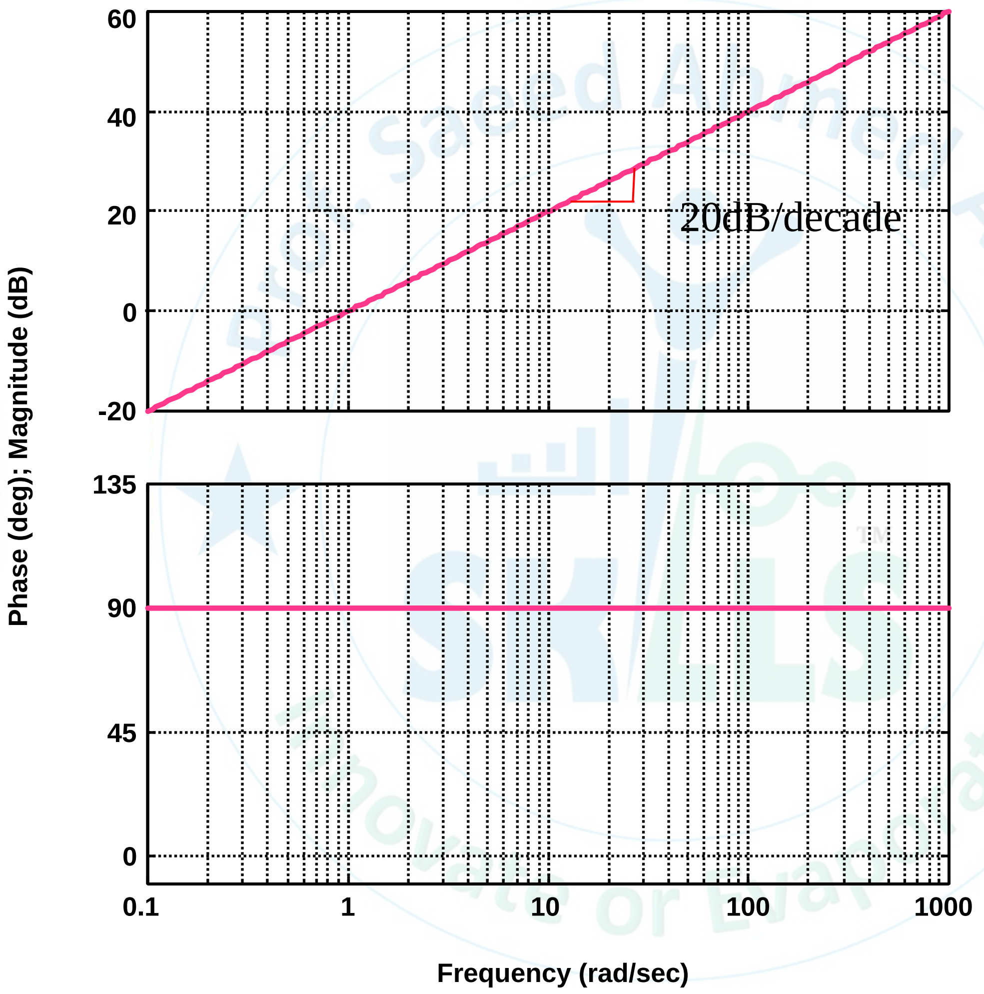
Standard Form of Transfer Function

$$G_{z0}(s) = s$$

Frequency Response

$$20\log_{10}\left|G_{z0}(j\omega)\right| = 20\log_{10}\left|j\omega\right|$$

$$= 20\log_{10}\omega$$



Bode Plots of Complex Poles

Standard Form of Transfer Function

$$G_{p2}(s)=\frac{{\omega_n}^2}{s^2+2\zeta\omega_n s+{\omega_n}^2}\ ,\quad 1\ge\zeta\ge 0$$

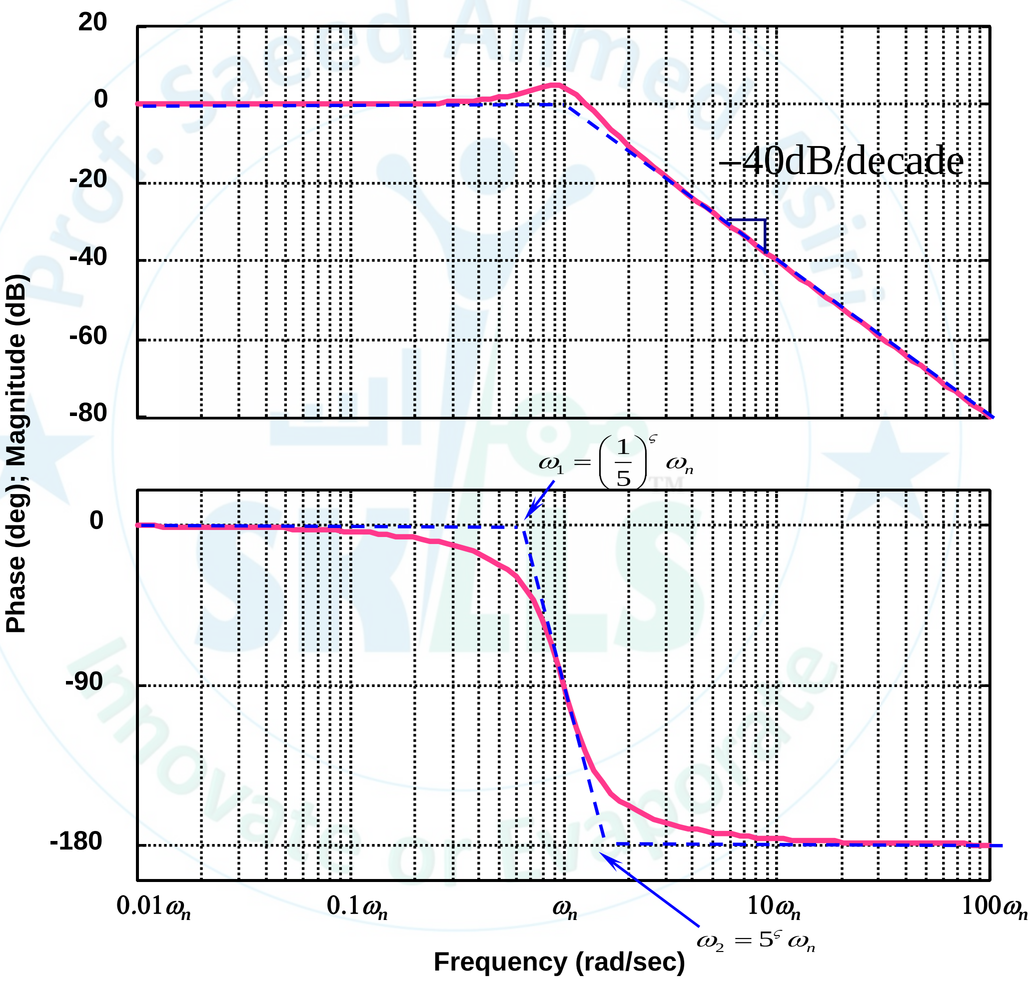
Frequency Response

$$G_{p2}(j\omega)=\frac{1}{\left(1-\frac{\omega^2}{{\omega_n}^2}\right)+j\frac{2\zeta\omega}{\omega_n}}$$

$$\left|G_{p2}(j\omega)\right|=\frac{1}{\sqrt{\left(1-\frac{\omega^2}{{\omega_n}^2}\right)^2+\frac{4\zeta^2\omega^2}{{\omega_n}^2}}}$$

$$\angle G_{p2}(j\omega)=-\angle\left(\left(1-\frac{\omega^2}{{\omega_n}^2}\right)+j\frac{2\zeta\omega}{\omega_n}\right)$$

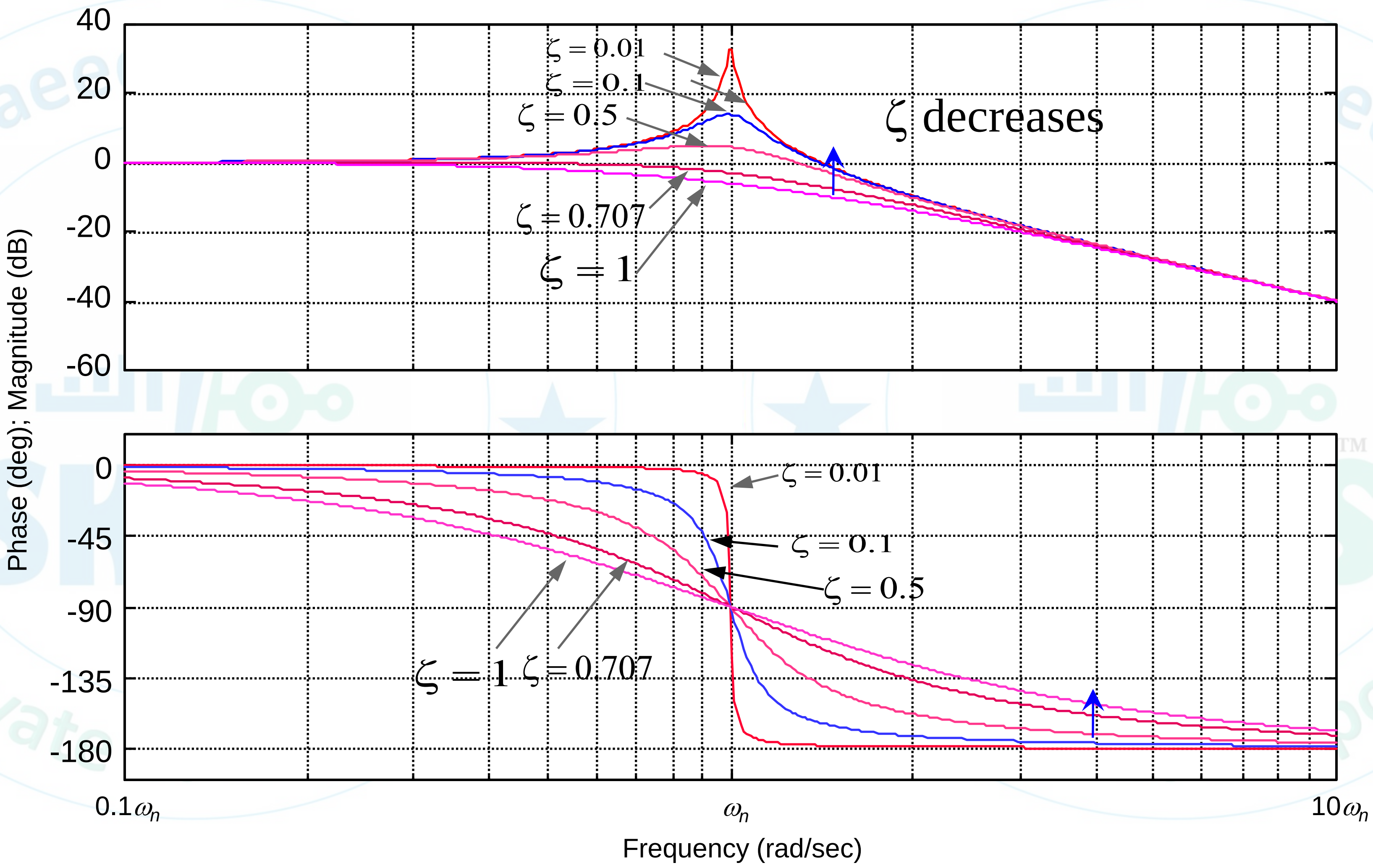
$$=-\,\mathrm{atan2}\left(\frac{2\zeta\omega}{\omega_n},\ \left(1-\frac{\omega^2}{{\omega_n}^2}\right)\right)$$



Peak (Resonant) Frequency and Magnitude for $\zeta\le\frac{1}{\sqrt{2}}=0.707$

$$\omega_r=\omega_n\sqrt{1-2\zeta^2}\quad \text{and}\quad \left|G_{p2}(j\omega_r)\right|=\frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

2nd Order System Frequency Response



2nd Order System Frequency Response

A Few Observations:

- Three *different* characteristic frequencies:
  - Natural Frequency (ω_n)
  - Damped Natural Frequency (ω_d): $\omega_d = \omega_n\sqrt{1-\zeta^2}$
  - Resonant (Peak) Frequency (ω_r): $\omega_r = \omega_n\sqrt{1-2\zeta^2}$

$$\omega_r \le \omega_d \le \omega_n$$

- When the damping ratio $\zeta > 0.707$, there is no peak in the Bode magnitude plot. DO NOT confuse this with the condition for over-damped and under-damped systems: when $\zeta < 1$ the system is under-damped (has overshoot) and when $\zeta > 1$ the system is over-damped (no overshoot).

Example

Second-Order System

Transfer Function:

$$G(s) = \frac{2500}{s^2 + 10s + 2500}$$

Plot the straight-line approximation of *G*(*s*)'s Bode diagram:

$$G_{p2}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad 1 \geq \zeta \geq 0$$

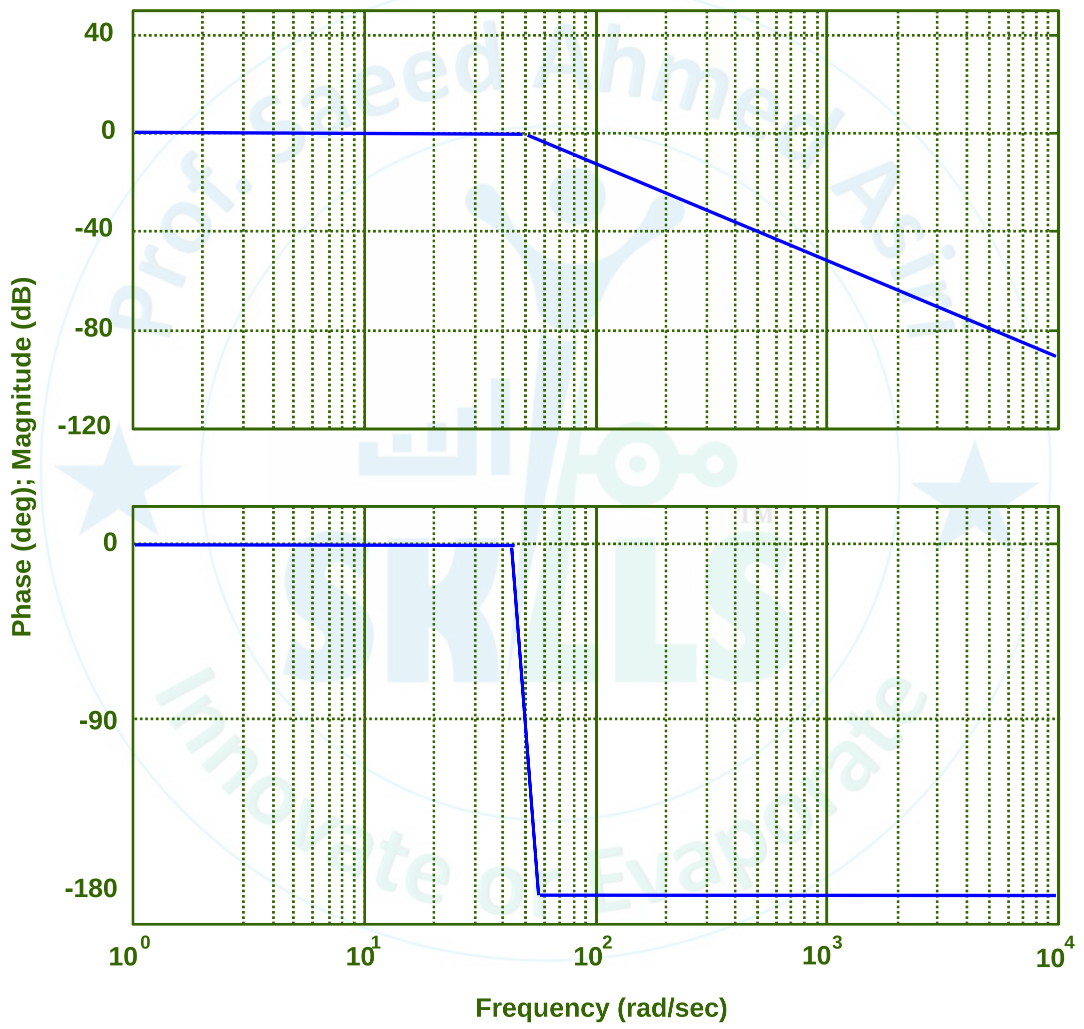
$$\omega_n^2 = 2500, \Rightarrow \omega_n = 50$$

$$2\varsigma\omega_n = 10, \Rightarrow \varsigma = \frac{10}{2\omega_n} = 0.1$$

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} \approx \omega_n$$

$$\omega_1 = \left(\frac{1}{5}\right)^{0.1}\omega_n = 42.6$$

$$\omega_2 = 5^{0.1}\omega_n = 58.7$$



Bode Plots of Complex Zeros

Standard Form of Transfer Function

$$G_{z2}(s)=\frac{s^2+2\zeta\omega_n s+\omega_n^2}{\omega_n^2},\quad 1\geq\zeta\geq 0$$

Frequency Response

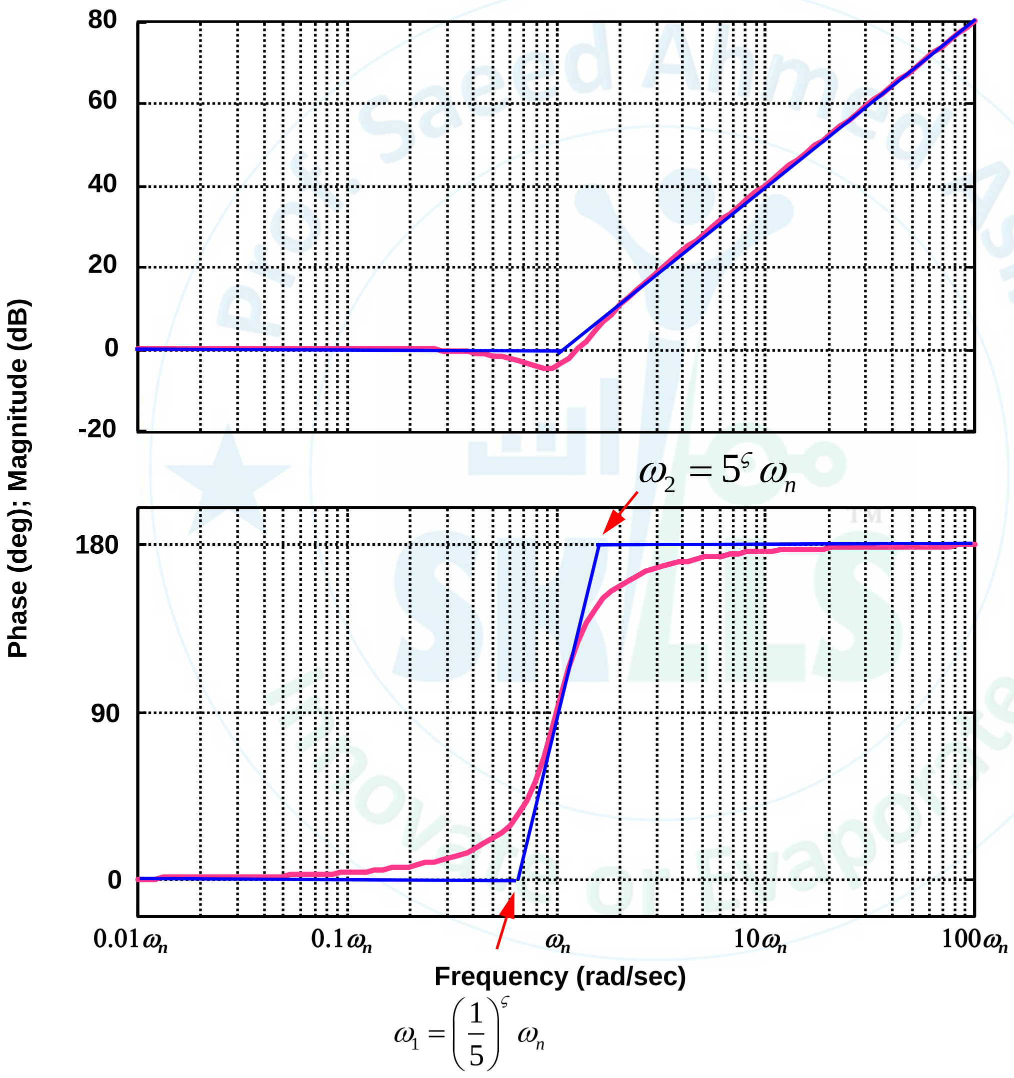
$$G_{z2}(j\omega)=\left(1-\frac{\omega^2}{\omega_n^2}\right)+j\frac{2\zeta\omega}{\omega_n}$$

$$|G_{z2}(j\omega)|=1/|G_{p2}(j\omega)|$$

$$=\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\frac{4\zeta^2\omega^2}{\omega_n^2}}$$

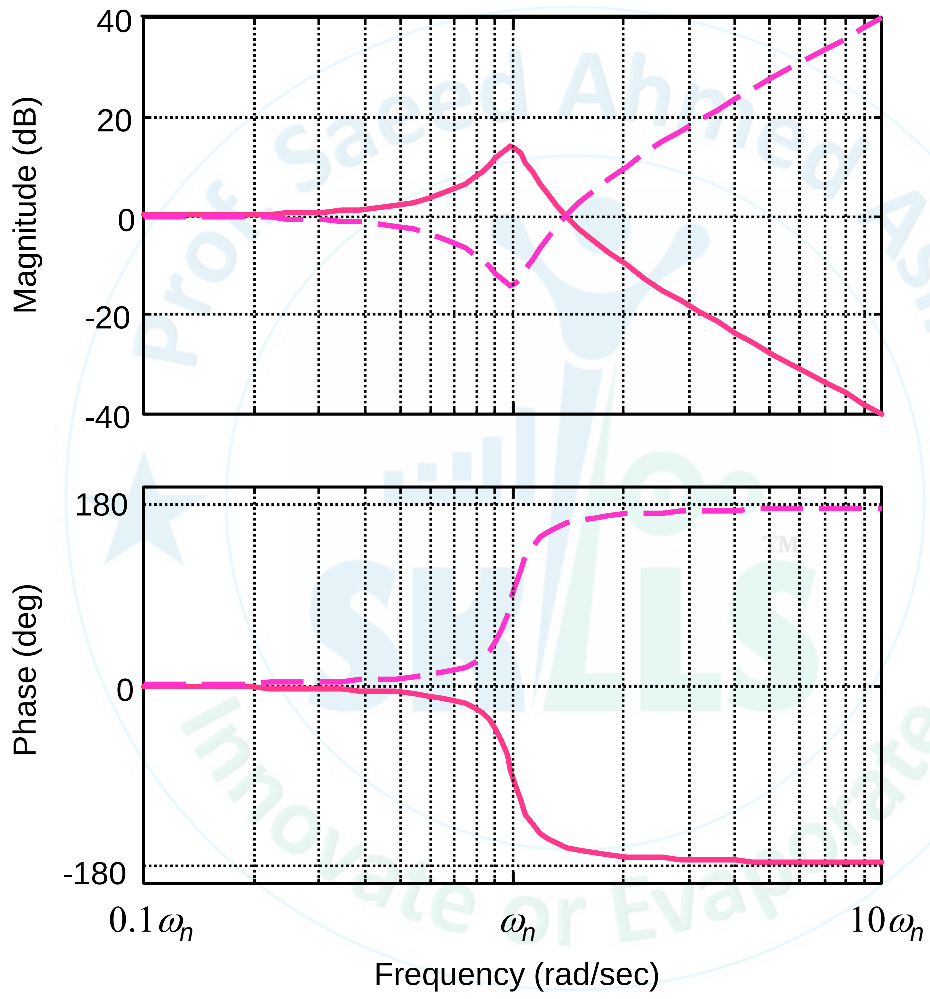
$$\angle G_{z2}(j\omega)=-\angle G_{p2}(j\omega)$$

$$=\text{atan2}\left(\frac{2\zeta\omega}{\omega_n},\quad 1-\frac{\omega^2}{\omega_n^2}\right)$$



Bode Plots of Poles and Zeros

Bode plots of zeros are the mirror images of the Bode plots of the identical poles w.r.t. the 0 dB line and the 0 deg line, respectively:



2nd Order Bode Diagram Summary

- **2nd Order Complex Poles**

$$G_{p2}(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2},\ 1\geq\zeta>0$$

  - Break Frequency

$$\omega_b=\omega_n \quad [\text{rad/s}]$$

  - Mag. Plot Approximation

    0 dB from DC to ω_n and a straight line with –40 dB/decade slope after ω_n. Peak value occurs at:

$$\omega_r=\omega_n\sqrt{1-2\zeta^2}$$

$$\Rightarrow\ \left|G_{p2}(j\omega_r)\right|_{MAX}=\frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

  - Phase Plot Approximation

    0 deg from DC to $(1/5)^{\zeta}\omega_n$. Between $(1/5)^{\zeta}\omega_n$ and $5^{\zeta}\omega_n$, a straight line from 0 deg to –180 deg (passing –90 deg at ω_n). For frequency higher than $5^{\zeta}\omega_n$, straight line on –180 deg.

- **2nd Order Complex Zeros**

$$G_{z2}(s)=\frac{s^2+2\zeta\omega_n s+\omega_n^2}{\omega_n^2},\ 1\geq\zeta>0$$

  - Break Frequency

$$\omega_b=\omega_n \quad [\text{rad/s}]$$

  - Mag. Plot Approximation

    0 dB from DC to ω_n and a straight line with 40 dB/decade slope after ω_n.

  - Phase Plot Approximation

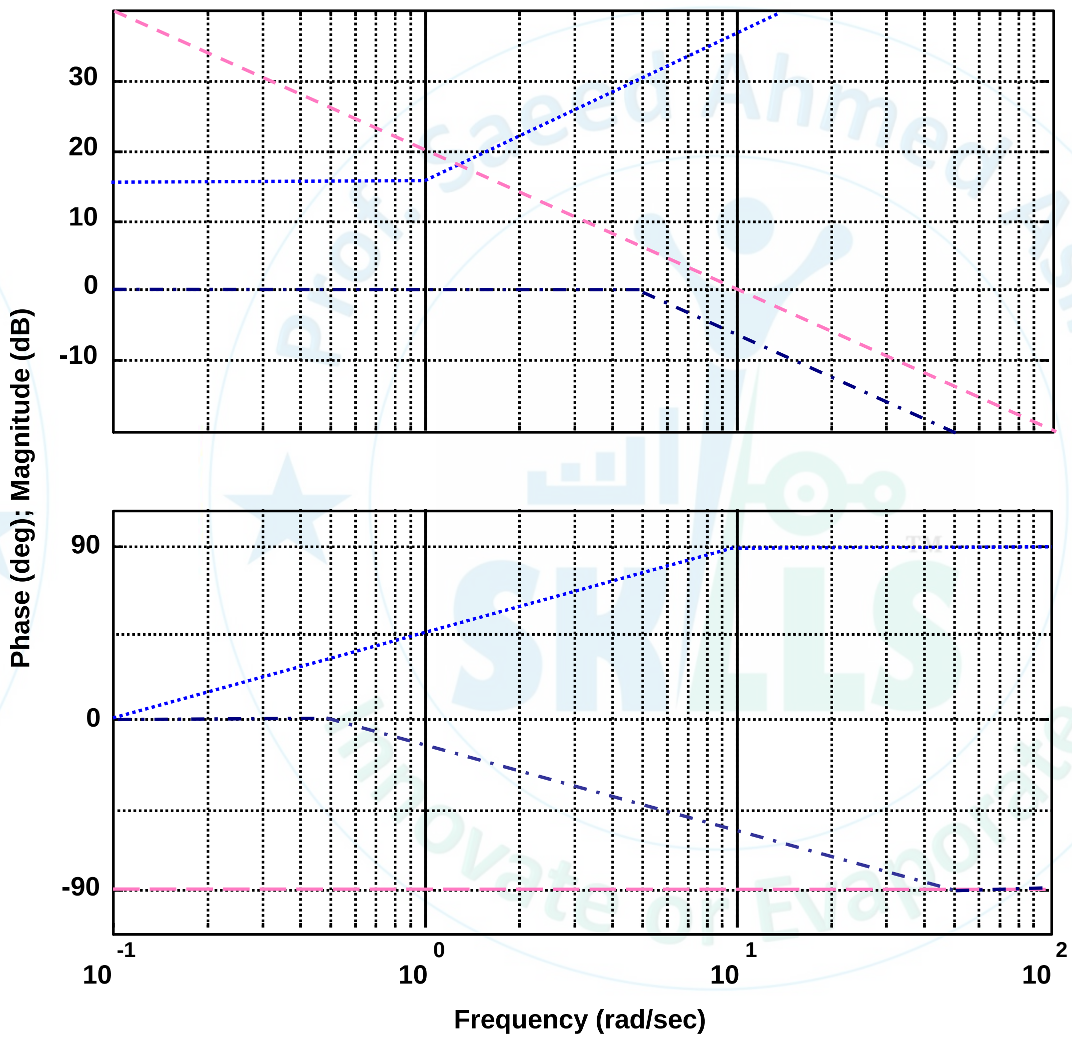
    0 deg from DC to $(1/5)^{\zeta}\omega_n$. Between $(1/5)^{\zeta}\omega_n$ and $5^{\zeta}\omega_n$, a straight line from 0 deg to 180 deg (passing 90 deg at ω_n). For frequency higher than $5^{\zeta}\omega_n$, straight line on 180 deg.

Example

Combination of Systems

Transfer Function: $$G(s) = \frac{35s + 35}{s(s+5)}$$

Plot the straight-line approximation of $G(s)$'s Bode plots:



SUM THEM

Example

Combination of Systems

Transfer Function:

$$G(s) = \frac{2500}{s(s^2 + 55s + 250)}$$

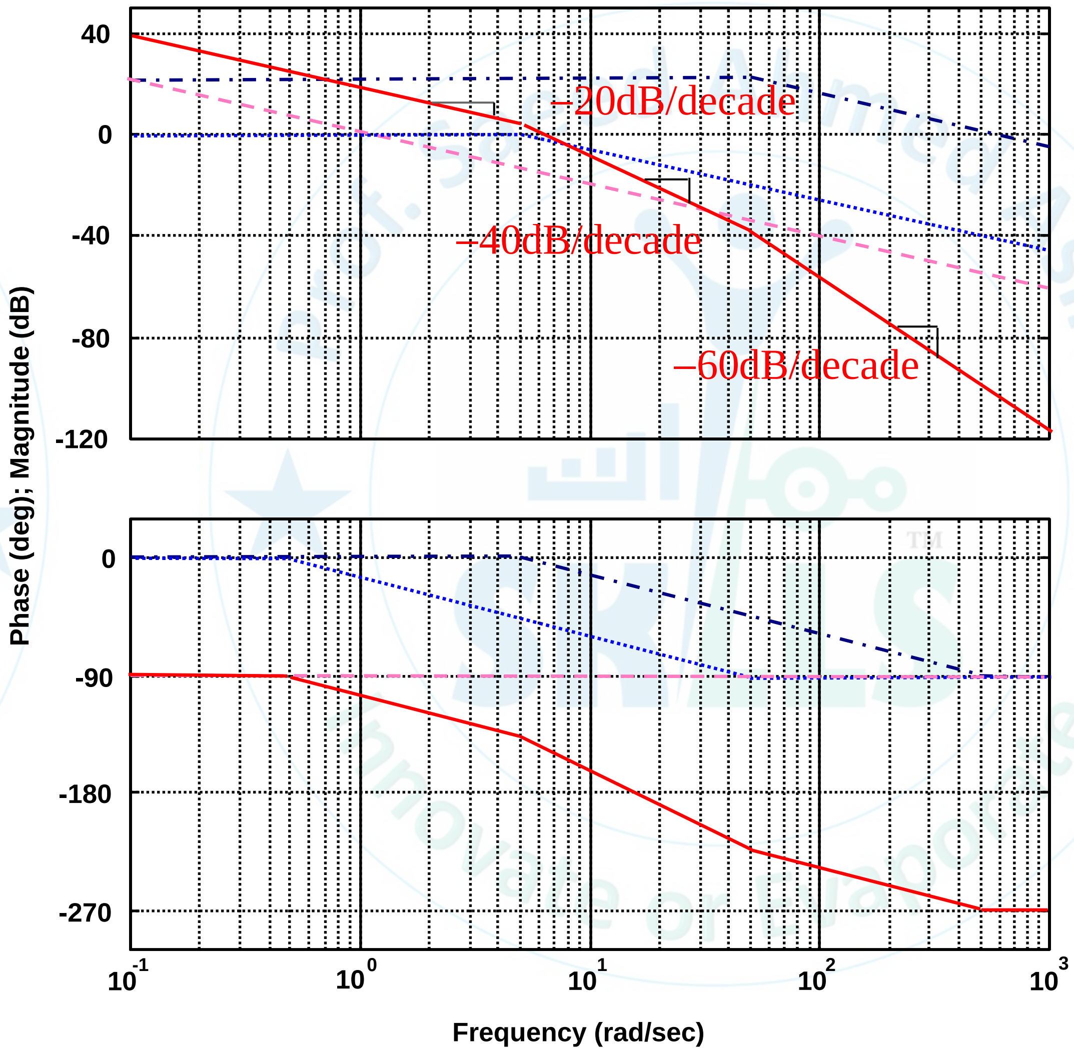
Plot the straight-line approximation of $G(s)$'s Bode plots:

$$G(s) = \frac{2500}{s(s+50)(s+5)} = \frac{2500}{50\times 5}\frac{1}{\frac{1}{50}s+1}\cdot\frac{1}{s}\cdot\frac{1}{\frac{1}{5}s+1}$$

$$G(j\omega) = 10\times\frac{1}{\frac{1}{50}j\omega+1}\cdot\frac{1}{j\omega}\cdot\frac{1}{\frac{1}{5}j\omega+1}$$

$$20\log_{10}|G(j\omega)| = \underbrace{20}_{20\log_{10}10} + 20\log_{10}\left|\frac{1}{\frac{1}{50}j\omega+1}\right|$$

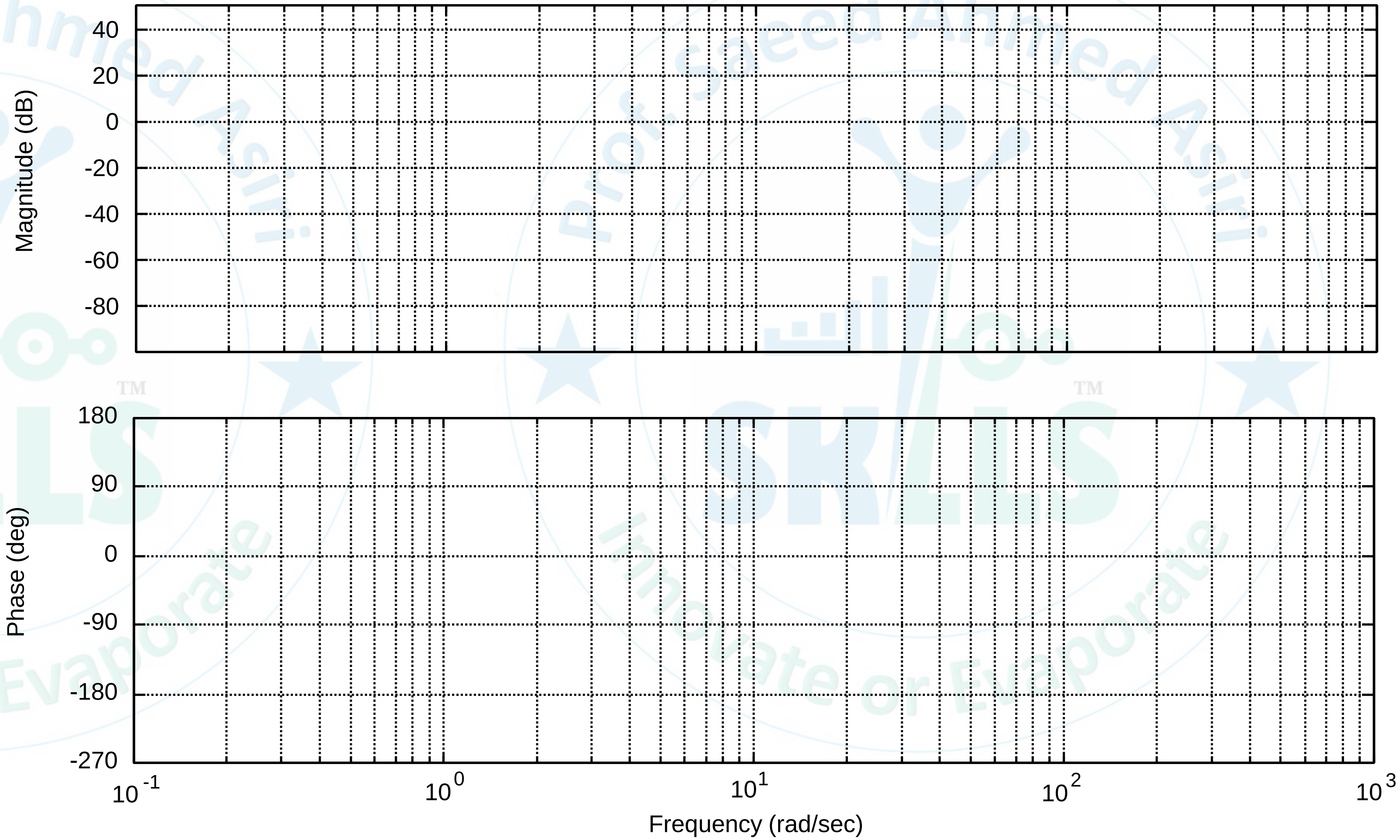
$$+20\log_{10}\left|\frac{1}{j\omega}\right| + 20\log_{10}\left|\frac{1}{\frac{1}{5}j\omega+1}\right|$$

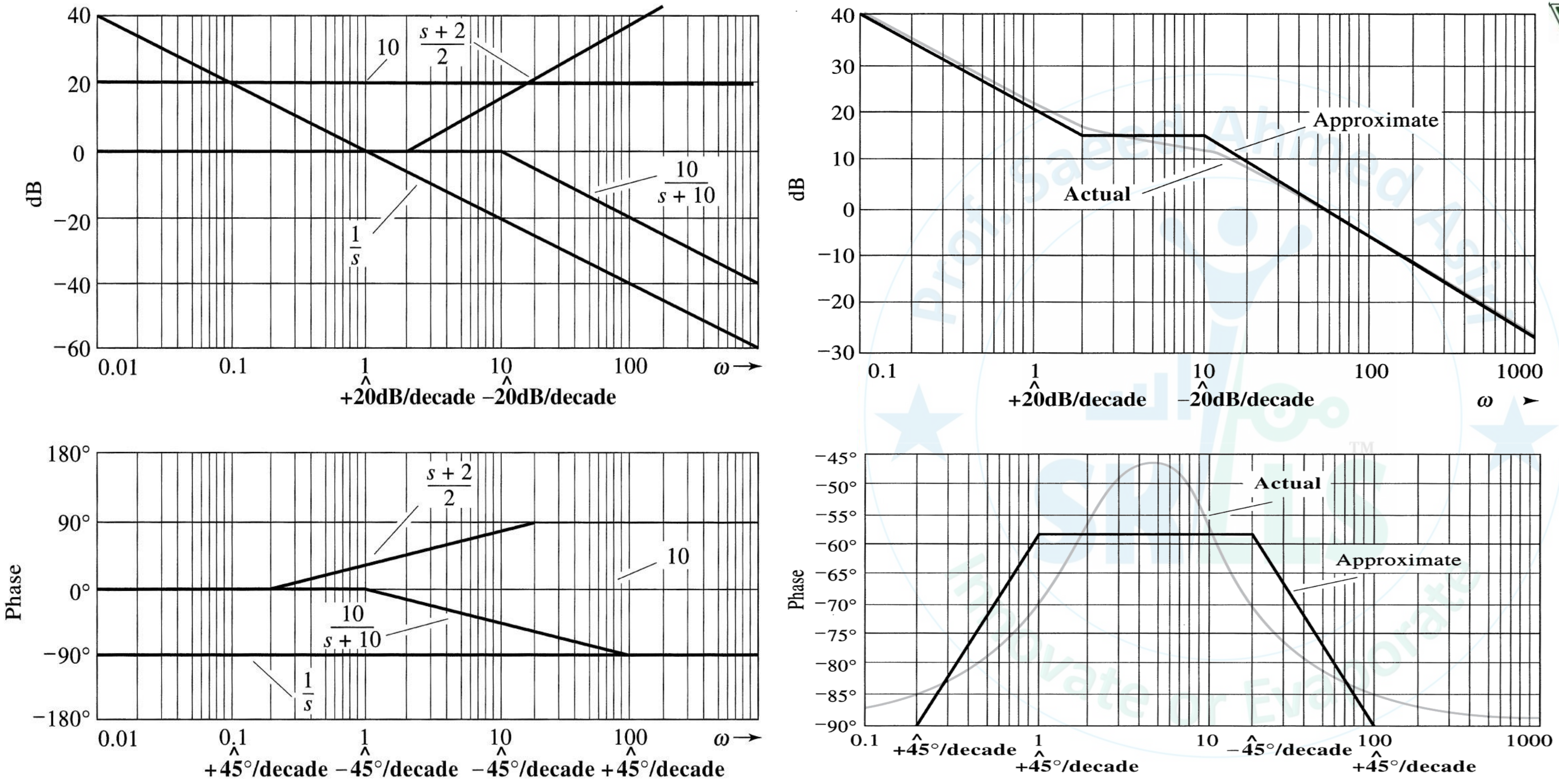


Example

$$T(s) = \frac{50(s+2)}{s(s+10)}$$

$$T(s) = 10\left[\frac{1}{s}\right]\left[\frac{s+2}{2}\right]\left[\frac{10}{s+10}\right]$$





40
20
0
−20
−40
−60
dB
10
s + 2
2
10
s + 10
1
s
0.01
0.1
1
10
100
ω
+20dB/decade −20dB/decade
180°
90°
0°
−90°
−180°
Phase
s + 2
2
10
10
s + 10
1
s
0.01
0.1
1
10
100
ω
+45°/decade −45°/decade −45°/decade +45°/decade
40
30
20
10
0
−10
−20
−30
dB
Approximate
Actual
0.1
1
10
100
1000
+20dB/decade −20dB/decade
ω
−45°
−50°
−55°
−60°
−65°
−70°
−75°
−80°
−85°
−90°
Phase
Actual
Approximate
0.1
1
10
100
1000
+45°/decade −45°/decade
+45°/decade +45°/decade

Example

Combination of Systems

Transfer Function: $$G(s)=\frac{2000(s^2+s+25)}{s(s+200)(s^2+10s+2500)}$$

Plot the straight-line approximation of *G*(*s*)'s Bode diagram:

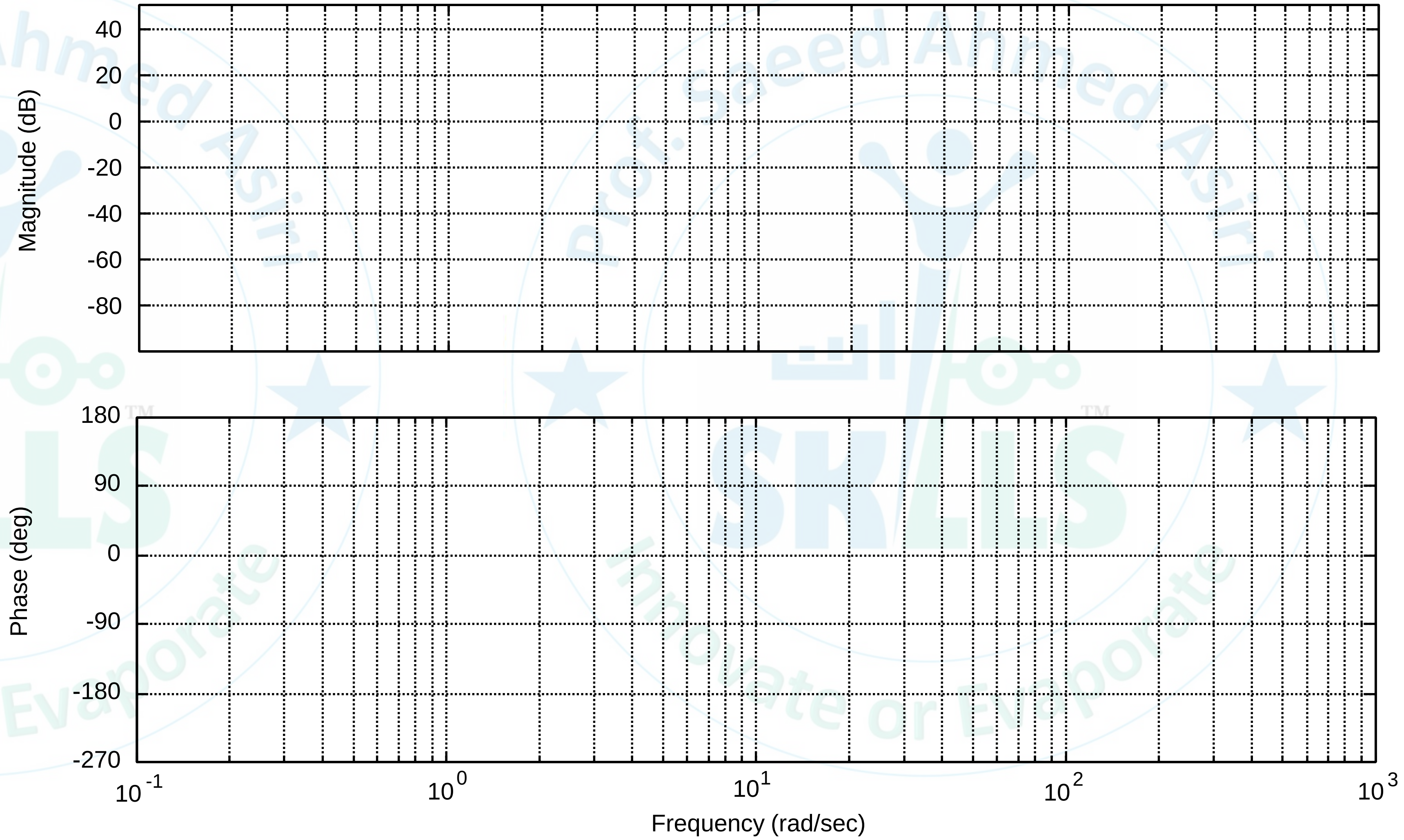
$$G(s)=\frac{2000\times 25}{200\times 2500}\left(\frac{s^2+s+25}{25}\right)\left(\frac{1}{s}\right)\left(\frac{1}{\frac{1}{200}s+1}\right)\left(\frac{2500}{s^2+10s+2500}\right)$$

$$20\log_{10}|G(j\omega)|=\underbrace{-20}_{20\log_{10}0.1}+20\log_{10}\left|\frac{(25-\omega^2)+j\omega}{25}\right|+20\log_{10}\left|\frac{1}{j\omega}\right|$$

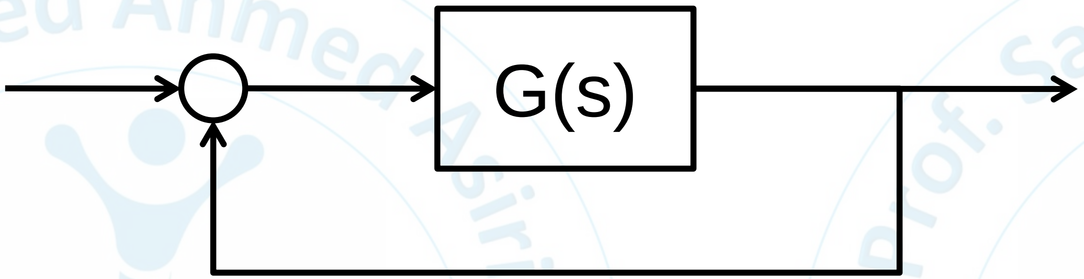
$$+20\log_{10}\left|\frac{1}{\frac{1}{200}j\omega+1}\right|+20\log_{10}\left|\frac{2500}{(2500-\omega^2)+10j\omega}\right|$$

Margins on Bode Plots

$$G(s) = \frac{2000(s^2 + s + 25)}{s(s+200)(s^2 + 10s + 2500)}$$



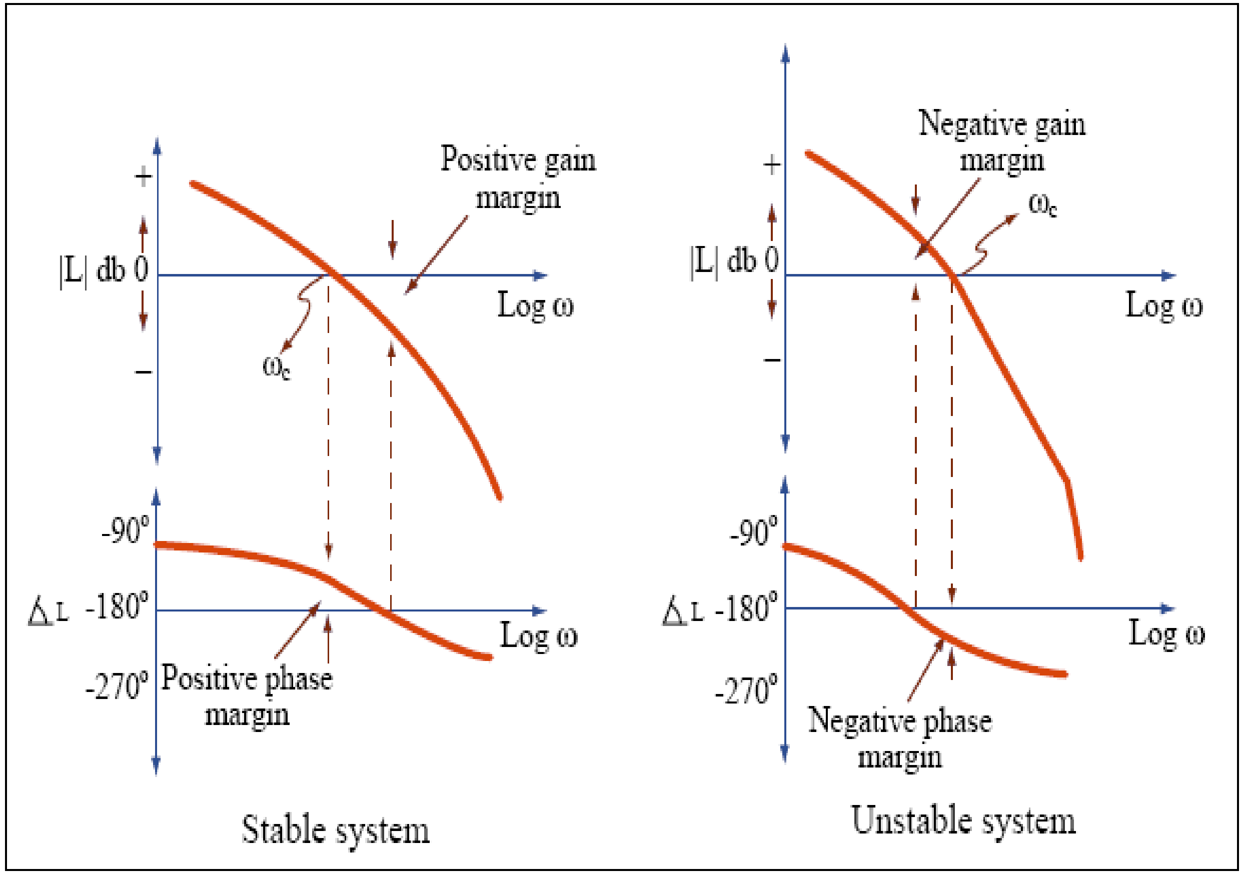
Margins on Bode Plots



In most cases, stability of this closed-loop can be determined from the Bode plot of G:

- Phase margin > 0
- Gain margin > 0

Margins on Bode Plots



Margins on Bode Plots

ω_{gc}: gain cross-over freq.

at ω_{gc}, $|G(j\omega)| = 1$ or $0\,\text{dB}$

PM : phase margin

$= 180° + \angle G(j\omega_{gc})$

ω_{pc}: phase cross-over freq.

$\angle G(j\omega_{pc}) = 180°$

GM : gain margin $= -20\log|G(j\omega_{pc})|\,\text{dB}$

$= 1/|G(j\omega_{pc})|$ in value

If $|G(j\omega)|$ never cross 0 dB line (always below 0 dB line), then $PM = \infty$.

If $\angle G(j\omega)$ never cross –180° line (always above –180°), then $GM = \infty$.

If $\angle G(j\omega)$ cross –180° several times, then there are several GM's.

If $|G(j\omega)|$ cross 0 dB several times, then there are several PM's.

Margins on Bode Plots

Example:

$$G(s)=\frac{100(s+1)}{(s+2)(s+5)}$$

$$=10\frac{s+1}{\left(\frac{1}{2}s+1\right)\left(\frac{1}{5}s+1\right)}$$

Bode plot on next page.

1. $|G(j\omega)|$ cross 0 dB line near $\omega=100$

   $\therefore \omega_{gc}\approx 100$

   $PM\approx$ ________

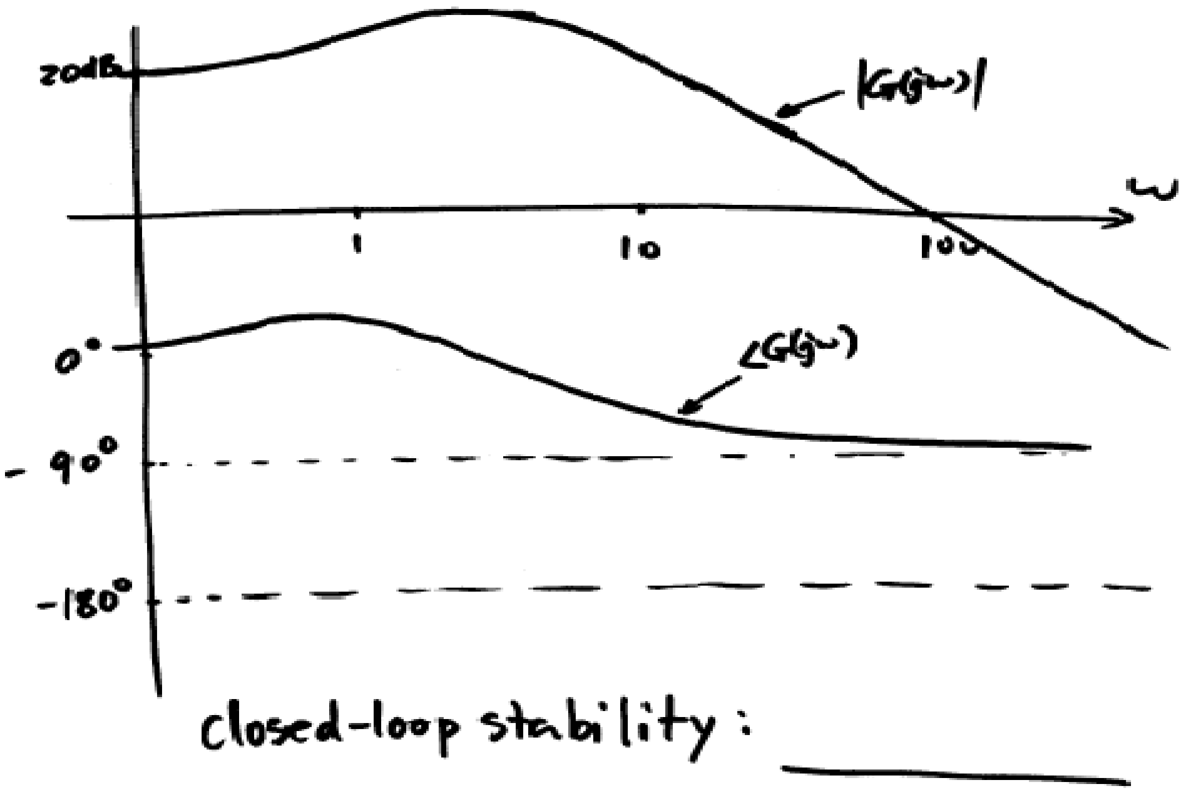
2. $\angle G(j\omega)$ cross $-180°$ at $\omega_{pc}=$ ________

   $\therefore GM=$ ________

Margins on Bode Plots

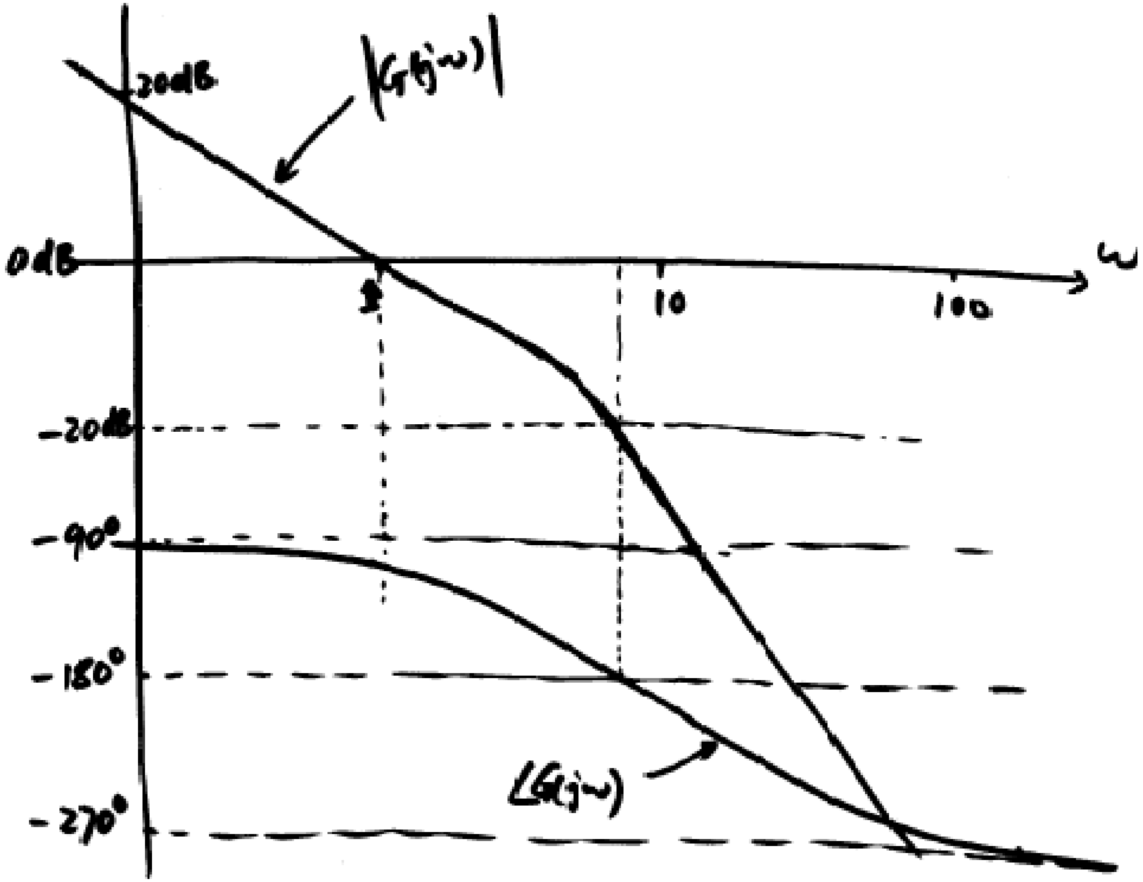
Example:

$$G(s)=\frac{25}{s\left(s^2+4s+25\right)}=\frac{1}{s\left(\frac{1}{25}s^2+\frac{4}{25}s+1\right)}$$

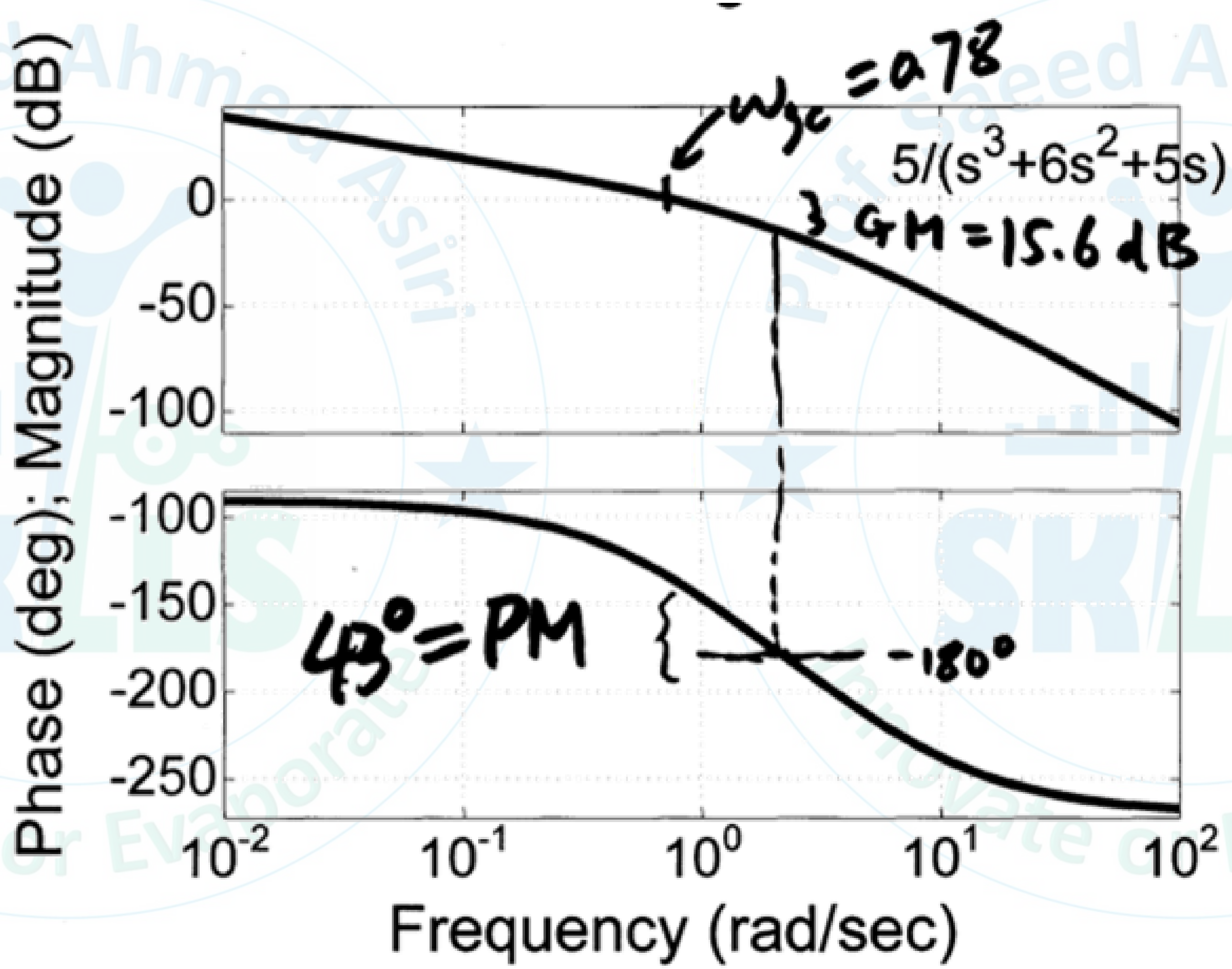


Margins on Bode Plots

$$G(s) = \frac{40}{s(s+2)} = 20\frac{1}{s\left(\frac{1}{2}s+1\right)}$$



Margins on Bode Plots



Margins on Bode Plots