MENG366

Bode Plot

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Frequency Response

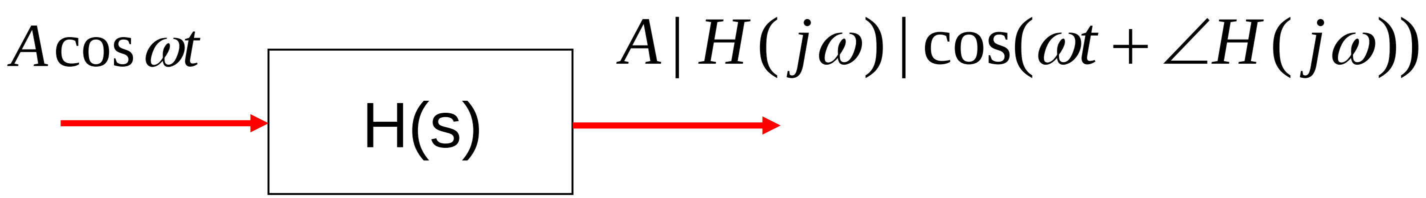
- Frequency Response: Given T.F. H(s)

$$\begin{cases} |H(j\omega)| & \text{v.s. } \omega \text{ is amp. resp.} \\ \angle H(j\omega) & \text{v.s. } \omega \text{ is phase resp.} \end{cases}$$

$$\text{most time, plot} \begin{cases} 20\log_{10}|H(j\omega)| & \text{v.s.} \quad \log_{10}\omega \\ \angle H(j\omega) \quad \text{in } ^\circ & \text{v.s.} \quad \log_{10}\omega \end{cases}$$

These are the Bode plot. In Matlab: bode

meaning:



Bode Plots

A better way to graphically display the frequency response!

Bode Magnitude Plot:

plots the magnitude of $G(j\omega)$ in decibels w.r.t. logarithmic frequency, i.e.,

$$\left|G(j\omega)\right|\Big|_{\text{dB}} = 20\log_{10}\left|G(j\omega)\right| \quad \text{vs} \quad \log_{10}\omega$$

Bode Phase Plot:

plots the phase angle of $G(j\omega)$ w.r.t. logarithmic frequency, i.e.,

$$\angle G(j\omega) \quad \text{vs} \quad \log_{10}\omega$$

Benefits:

- Display the dependence of magnitude of the frequency response on the input frequency better, especially for magnitude approaching zero
- Log axis converts the multiplications and divisions into additions and subtractions, which are easier to handle graphically
- Allow straight-line approximations for quick sketch

Example

$$\text{Example:} \quad H(s) = \frac{3}{s+3}$$

$$\text{If input} = 10\cos(3t), \omega = 3$$

$$|H(j\omega)| = \left|\frac{3}{3+j3}\right| = \frac{1}{\sqrt{2}}, \quad \angle H(j\omega) = \angle\frac{3}{3+j3} = -45^\circ$$

$$\therefore \text{output} = \frac{10}{\sqrt{2}}\cos(3t - 45^\circ)$$

Works only if H(s) is stable

Bode Plots

Ex:

$$G(s)=\frac{s+1}{s^2+10s}\Rightarrow G(j\omega)=\frac{(j\omega)+1}{(j\omega)^2+10(j\omega)}$$

ω	$\lvert G(j\omega)\rvert$	$20\log_{10}\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0.1			
0.2			
0.5			
1			
2			
5			
10			
20			
50			
100			

Bode Plots

Ex:

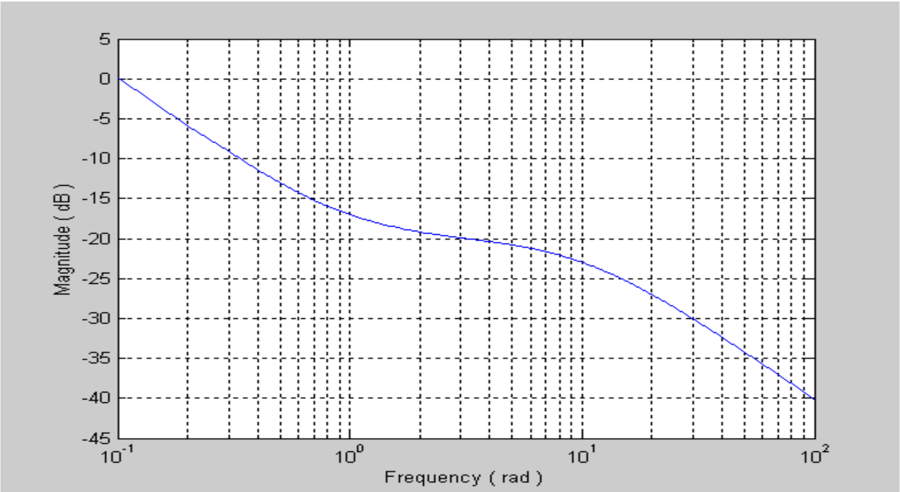
$$G(s)=\frac{s+1}{s^2+10s}\Rightarrow G(j\omega)=\frac{(j\omega)+1}{(j\omega)^2+10(j\omega)}$$

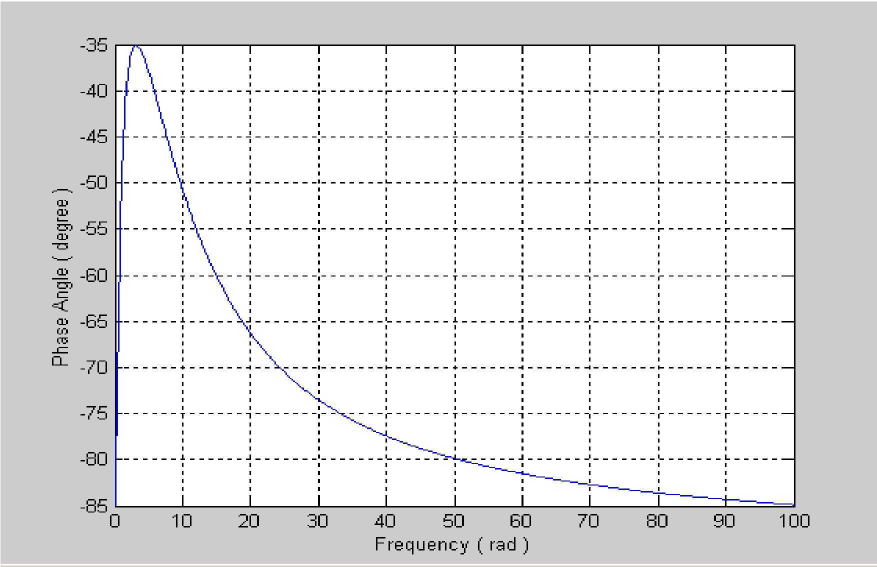
$$|G(j\omega)|=\frac{\sqrt{1+\omega^2}}{\omega\sqrt{100+\omega^2}}$$

$$\angle G(j\omega)=\angle(j\omega+1)-\angle\left(-\omega^2+10j\omega\right)$$

$$=\tan^{-1}(\omega)-\text{atan2}\left(10\omega,-\omega^2\right)$$

ω	$\lvert G(j\omega)\rvert$	$20\log_{10}\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0.1	1.0049	0.0428	-83.8623
0.2	0.5098	-5.8520	-79.8358
0.5	0.2233	-13.0211	-66.2974
1	0.1407	-17.0329	-50.7016
2	0.1096	-19.2012	-37.8750
5	0.0912	-20.7988	-37.8750
10	0.0711	-22.9671	-50.7106
20	0.0448	-26.9789	-66.2974
50	0.0196	-34.1480	-79.8358
100	0.0100	-40.0428	-84.8623





Bode Plots of LTI Systems

Transfer Function

$$G(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} = \frac{b_m (s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

Frequency Response

$$|G(j\omega)| = \left|\frac{b_m (j\omega - z_1)\cdots(j\omega - z_m)}{(j\omega - p_1)\cdots(j\omega - p_n)}\right| = |b_m| \cdot \left|\frac{1}{(j\omega - p_1)}\right| \cdots \left|\frac{1}{(j\omega - p_n)}\right| \cdot |(j\omega - z_1)| \cdots |(j\omega - z_m)|$$

Bode Magnitude Plot

$$20\log_{10}(|G(j\omega)|) = 20\log_{10}(|b_m|) + 20\log_{10}\left(\left|\frac{1}{(j\omega - p_1)}\right|\right) + \cdots + 20\log_{10}\left(\left|\frac{1}{(j\omega - p_n)}\right|\right)$$

$$+20\log_{10}(|(j\omega - z_1)|) + \cdots + 20\log_{10}(|(j\omega - z_1)|)$$

Bode Phase Plot

$$\angle G(j\omega) = \angle \frac{b_m (j\omega - z_1)\cdots(j\omega - z_m)}{(j\omega - p_1)\cdots(j\omega - p_n)} = \angle b_m + \angle(j\omega - z_1) + \angle(j\omega - z_2) + \cdots + \angle(j\omega - z_m)$$

$$-\angle(j\omega - p_1) - \angle(j\omega - p_2) \cdots - \angle(j\omega - p_n)$$

Example

Ex: Find the magnitude and the phase of the following transfer function:

$$G(s)=\frac{3s^3+12s^2+9s}{2s^3+22s^2+76s+80}=\frac{3s(s+3)(s+1)}{2(s+2)(s+4)(s+5)}$$

$$=\frac{\frac{3\times 3}{2\times 2\times 4\times 5}s\left(\frac{1}{3}s+1\right)(s+1)}{\left(\frac{1}{2}s+1\right)\left(\frac{1}{4}s+1\right)\left(\frac{1}{5}s+1\right)}=\frac{9}{80}\frac{s\left(\frac{1}{3}s+1\right)(s+1)}{\left(\frac{1}{2}s+1\right)\left(\frac{1}{4}s+1\right)\left(\frac{1}{5}s+1\right)}$$

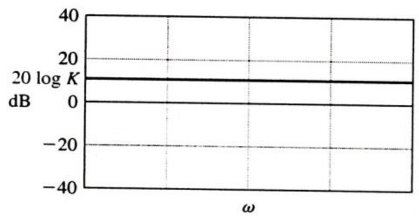
$$|G(j\omega)|=\frac{\left|\frac{9}{80}\right|\left|j\omega\right|\left|\frac{1}{3}j\omega+1\right|}{\left|\frac{1}{2}j\omega+1\right|\left|\frac{1}{4}j\omega+1\right|\left|\frac{1}{5}j\omega+1\right|}$$

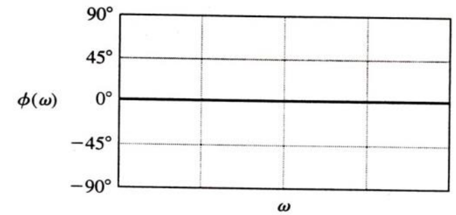
$$20\log_{10}\{|G(j\omega)|\}=20\log_{10}\left\{\frac{9}{80}\right\}+20\log_{10}\{|j\omega|\}+20\log_{10}\left\{\left|\frac{1}{3}j\omega+1\right|\right\}$$
$$-20\log_{10}\left\{\left|\frac{1}{2}j\omega+1\right|\right\}-20\log_{10}\left\{\left|\frac{1}{4}j\omega+1\right|\right\}-20\log_{10}\left\{\left|\frac{1}{5}j\omega+1\right|\right\}$$

$$\angle G(j\omega)=\angle\frac{9}{80}+\angle(j\omega)+\angle\left(\frac{1}{3}j\omega+1\right)+\angle(j\omega+1)$$
$$-\angle\left(\frac{1}{2}j\omega+1\right)-\angle\left(\frac{1}{4}j\omega+1\right)-\angle\left(\frac{1}{5}j\omega+1\right)$$

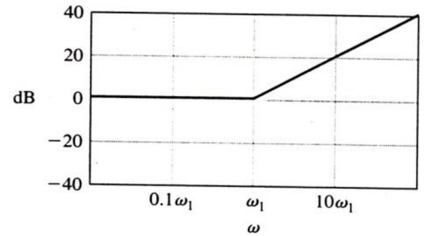
Term	Magnitude 20 log|G|	Phase, $\phi(\omega)$

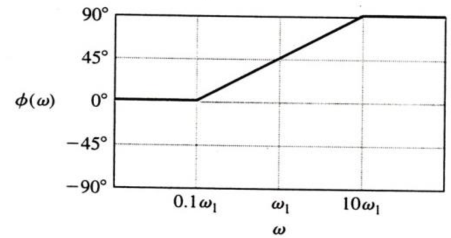
1. Gain, $G(j\omega) = K$



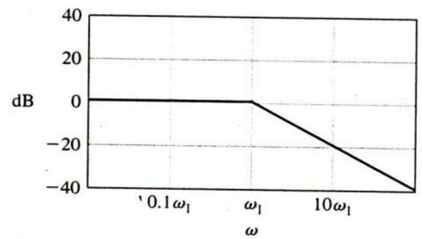


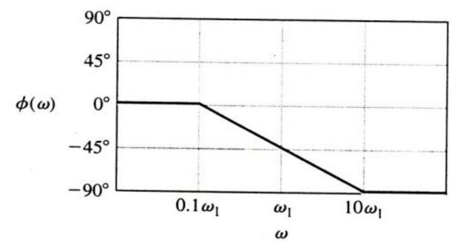
2. Zero, $G(j\omega) = (1 + j\omega/\omega_1)$



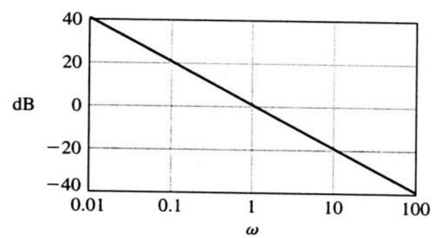


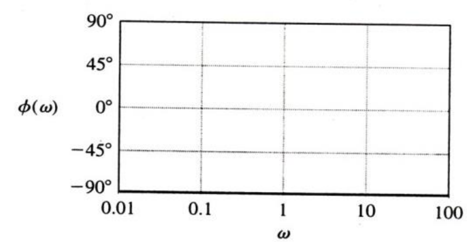
3. Pole, $G(j\omega) = (1 + j\omega/\omega_1)^{-1}$



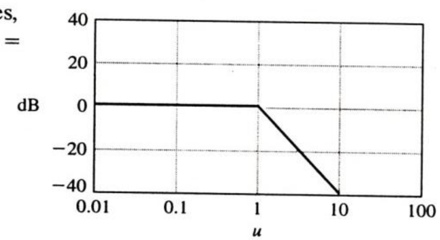


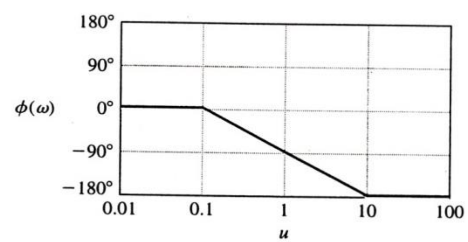
4. Pole at the origin, $G(j\omega) = 1/j\omega$





5. Two complex poles, $0.1 < \zeta < 1, G(j\omega) = (1 + j2\zeta u - u^2)^{-1}$, $u = \omega/\omega_n$





Asymptotic curves for basic factors

Bode Plots of 1st Order Poles

Standard Form of Transfer Function:

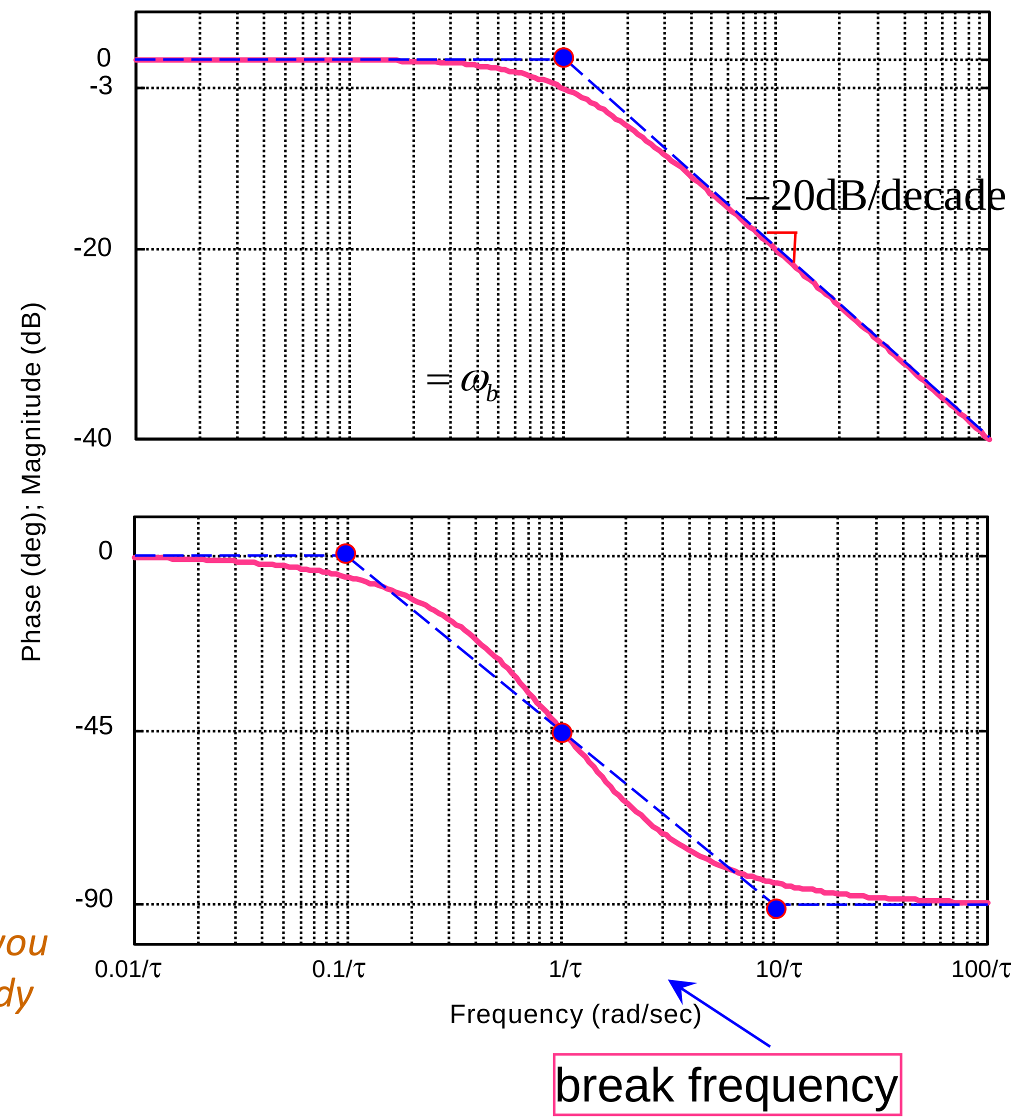
$$G_{p1}(s) = \frac{1}{\tau s + 1}, \qquad \tau > 0$$

Frequency Response:

$$G_{p1}(j\omega) = \frac{1}{\tau j\omega + 1}, \qquad \tau > 0$$

$$\begin{cases} |G_{p1}(j\omega)| = \dfrac{1}{\sqrt{\tau^2\omega^2 + 1}} \\ \angle G_{p1}(j\omega) = -\,\mathrm{atan2}(\tau\omega, 1) \\ \qquad\qquad = -\tan^{-1}(\tau\omega) \end{cases}$$

$$20\log_{10}|G_{p1}(j\omega)|$$
$$= -10\log_{10}(\tau^2\omega^2 + 1)$$
$$\approx \begin{cases} 0\text{dB}, & \tau\omega \ll 1 \text{ or } \omega \ll \dfrac{1}{\tau} = \omega_b \\ -3\text{dB}, & \tau\omega = 1 \text{ or } \omega = \dfrac{1}{\tau} = \omega_b \\ -20\log_{10}\left(\dfrac{\omega}{\omega_b}\right), & \tau\omega \gg 1 \text{ or } \omega \gg \dfrac{1}{\tau} = \omega_b \end{cases}$$



Q: By just looking at the Bode diagram, can you determine the time constant and the steady state gain of the system ?

Example

- 1st Order Real Poles

Transfer Function:

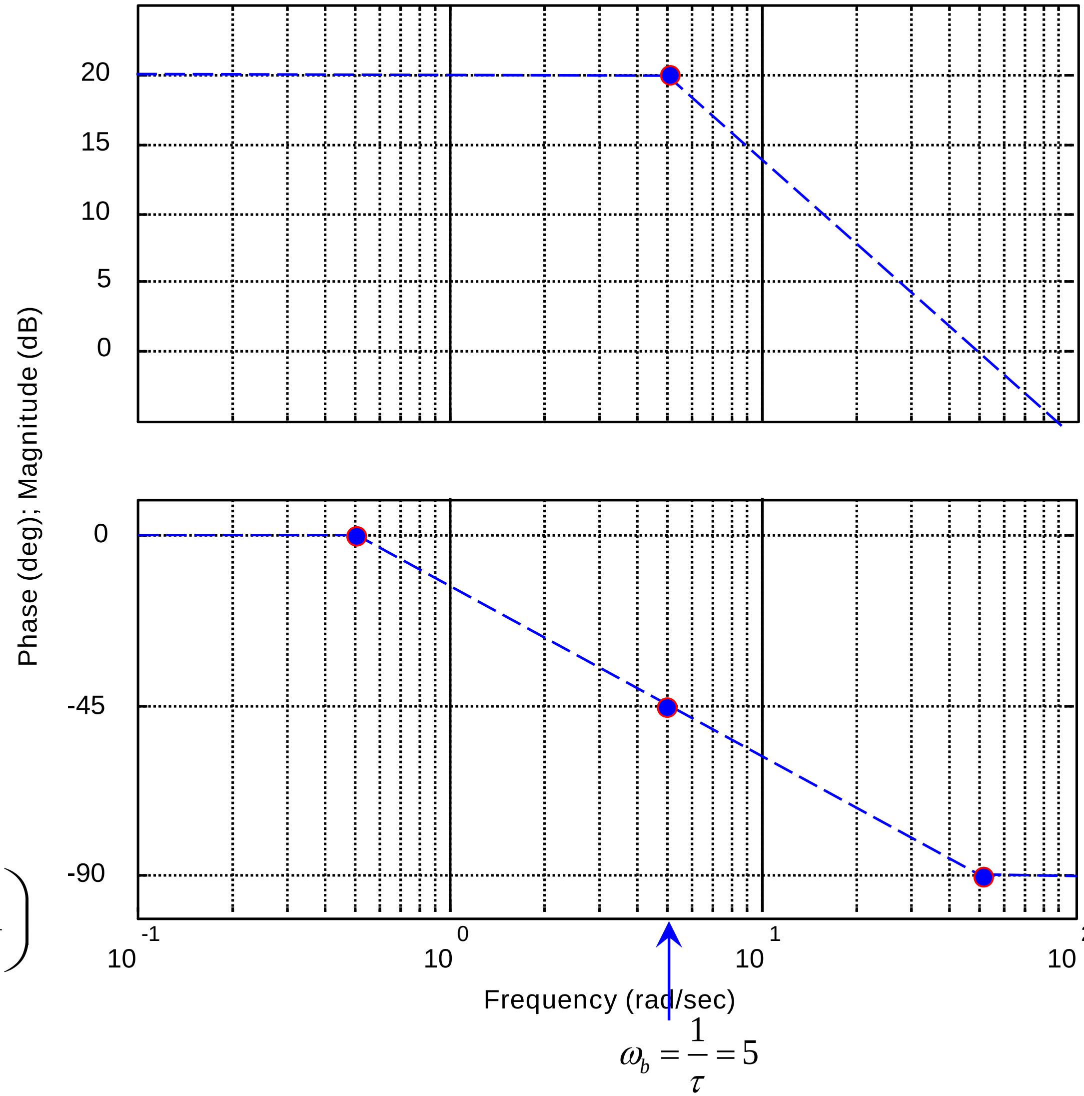
$$G(s) = \frac{50}{s+5}$$

Plot the straight line approximation of *G*(*s*)’s Bode diagram:

$$G(s) = \frac{50}{s+5} = 10 \times \frac{1}{\underbrace{\frac{1}{5}}_{\tau} s + 1}$$

$$20\log_{10}\left|G(j\omega)\right| = 20 + 20\log_{10}\left|\frac{1}{\frac{1}{5}j\omega + 1}\right|$$

$$\angle G(j\omega) = -\angle\left(\frac{1}{5}j\omega + 1\right)$$



Bode Plots of 1st Order Zeros

Standard Form of Transfer Function

$$G_{z1}(s) = \tau s + 1 \quad , \quad \tau > 0$$

Frequency Response

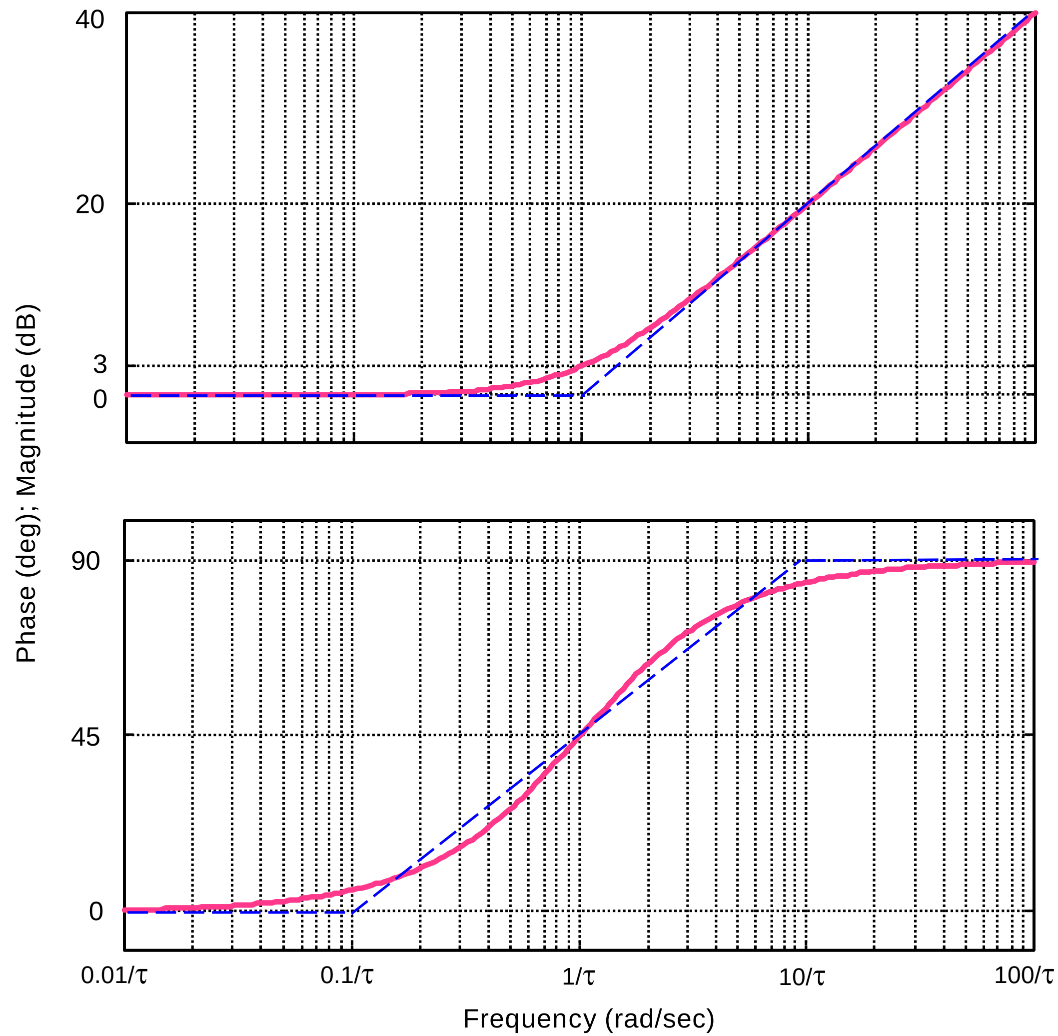
$$G_{z1}(j\omega) = \tau j\omega + 1 \quad , \quad \tau > 0$$

$$\left|G_{z1}(j\omega)\right| = \sqrt{\tau^2\omega^2 + 1}$$

$$\angle G_{z1}(j\omega) = \text{atan2}(\tau\omega, 1) = \tan^{-1}(\tau\omega)$$

$$20\log_{10}\left|G_{p1}(j\omega)\right| = 10\log_{10}\left(\tau^2\omega^2 + 1\right)$$

$$\approx \begin{cases} 0\text{dB}, & \tau\omega \ll 1 \text{ or } \omega \ll \frac{1}{\tau} = \omega_b \\ 3\text{dB}, & \tau\omega = 1 \text{ or } \omega = \frac{1}{\tau} = \omega_b \\ 20\log_{10}\left(\frac{\omega}{\omega_b}\right), & \tau\omega \gg 1 \text{ or } \omega \gg \frac{1}{\tau} = \omega_b \end{cases}$$



Example

- 1st Order Real Zeros

Transfer Function:

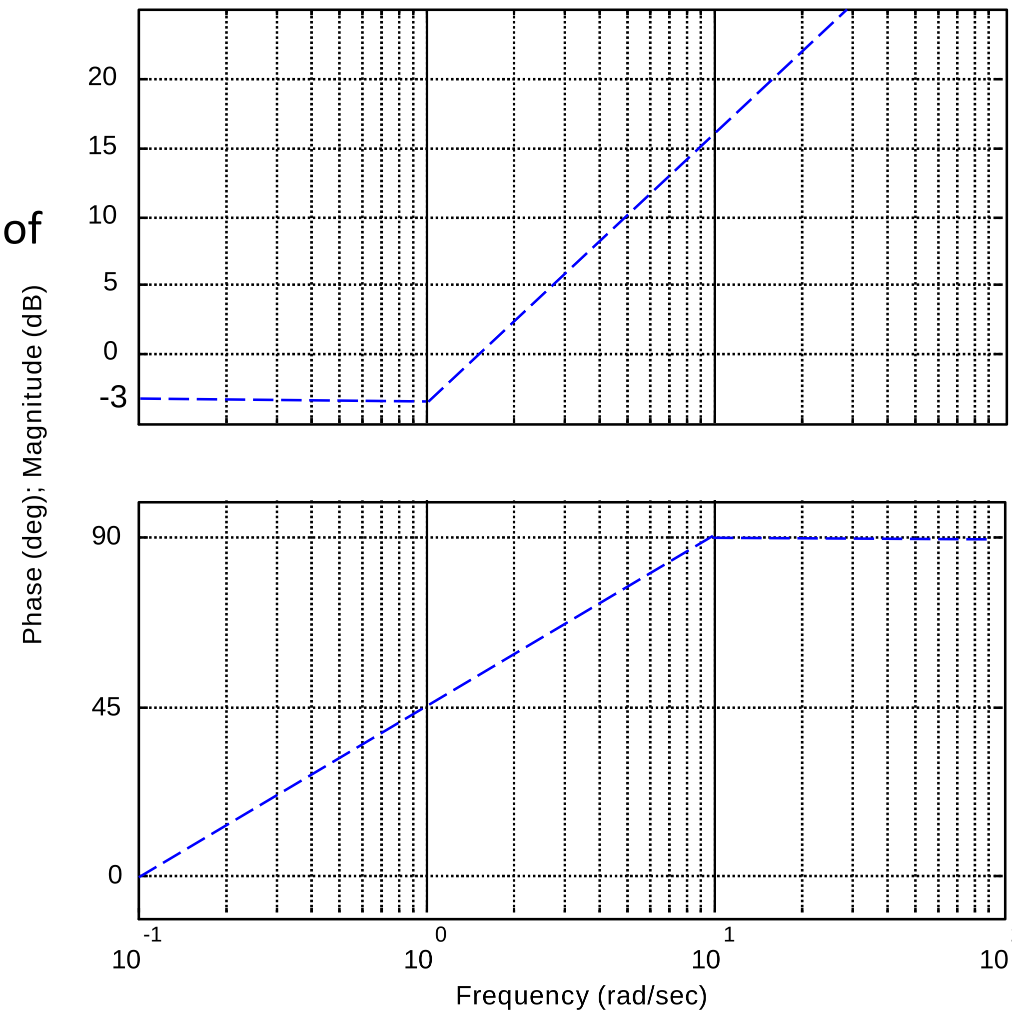
$$G(s) = 0.7s + 0.7$$

Plot the straight line approximation of *G*(*s*)'s Bode diagram:

$$G(j\omega) = 0.7\left(j\omega + 1\right)$$

$$20\log_{10}\left|G(j\omega)\right| = 20\log_{10} 0.7 + 20\log_{10}\left|j\omega + 1\right|$$

$$\angle G(j\omega) \quad = \angle\left(j\omega + 1\right)$$



Example

- Lead Compensator

Transfer Function:

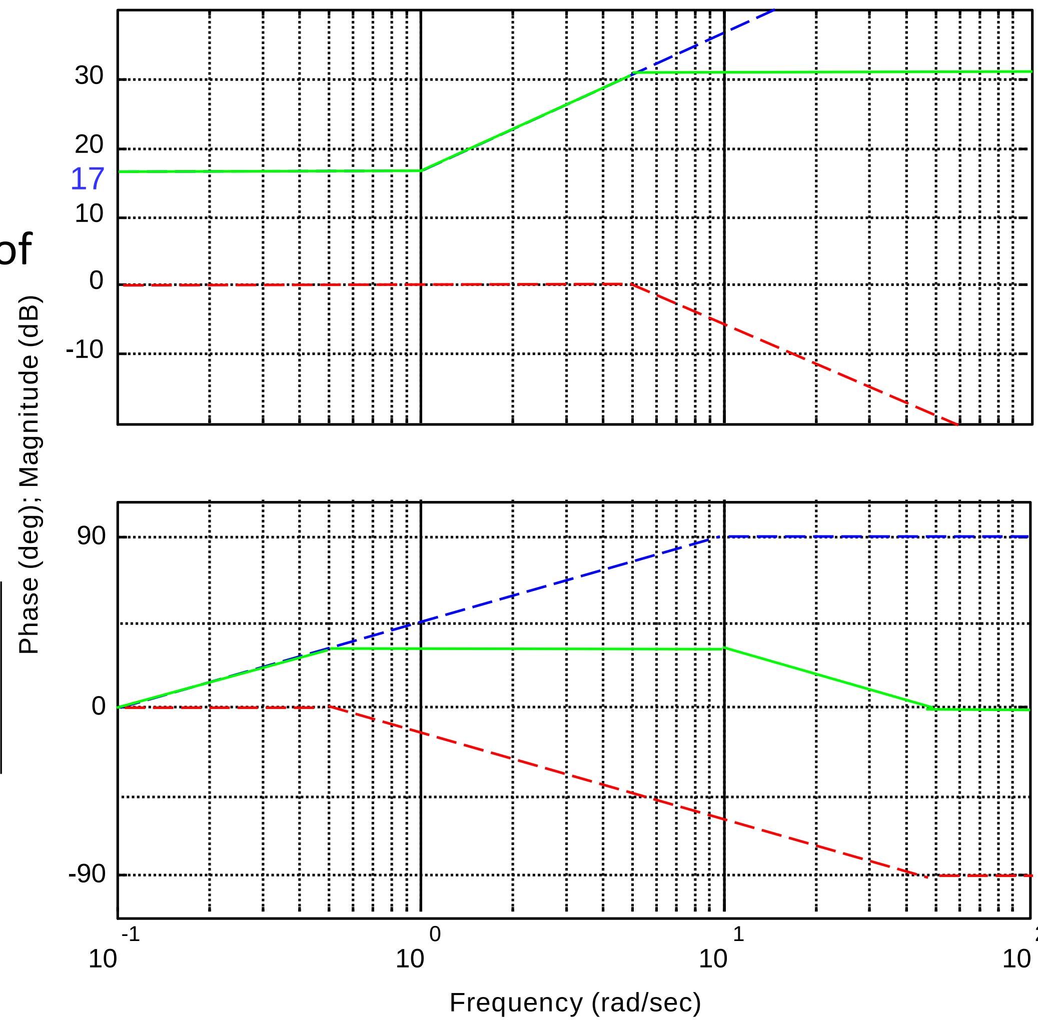
$$G(s) = \frac{35s+35}{s+5}$$

Plot the straight line approximation of *G*(*s*)'s Bode diagram:

$$G(j\omega) = \frac{35}{5}(j\omega+1)\cdot\frac{1}{\frac{1}{5}j\omega+1}$$

$$20\log_{10}|G(j\omega)| = \underbrace{16.9}_{20\log_{10}7} + 20\log_{10}|j\omega+1| + 20\log_{10}\left|\frac{1}{\frac{1}{5}j\omega+1}\right|$$

$$\angle G(j\omega) \;= \angle(j\omega+1) + \angle\left(\frac{1}{\frac{1}{5}j\omega+1}\right)$$



1st Order Bode Plots Summary

• 1st Order Poles

$$G_{p1}(s) = \frac{1}{\tau s + 1} \quad , \quad \tau > 0$$

– Break Frequency

$$\omega_b = \frac{1}{\tau} \quad [\text{rad/s}]$$

– Mag. Plot Approximation

0 dB from DC to ω_b and a straight line with –20 dB/decade slope after ω_b

– Phase Plot Approximation

0 deg from DC to $\frac{1}{10}\omega_b$. Between $\frac{1}{10}\omega_b$ and $10\omega_b$, a straight line from 0 deg to –90 deg (passing –45 deg at ω_b). For frequency higher than $10\omega_b$, straight line on –90 deg.

• 1st Order Zeros

$$G_{z1}(s) = \tau s + 1 \quad , \quad \tau > 0$$

– Break Frequency

$$\omega_b = \frac{1}{\tau} \quad [\text{rad/s}]$$

– Mag. Plot Approximation

0 dB from DC to ω_b and a straight line with 20 dB/decade slope after ω_b

– Phase Plot Approximation

0 deg from DC to $\frac{1}{10}\omega_b$. Between $\frac{1}{10}\omega_b$ and $10\omega_b$, a straight line from 0 deg to 90 deg (passing 45 deg at ω_b). For frequency higher than $10\omega_b$, straight line on 90 deg.

Note: By looking at Bode plots you should be able to determine the relative order of the system, its break frequency, and DC (steady-state) gain. This process should also be reversible, i.e., given a transfer function, be able to plot a straight line approximation of Bode plots.

Bode Plots of Complex Poles

Standard Form of Transfer Function

$$G_{p2}(s)=\frac{\omega_n^{\;2}}{s^2+2\zeta\omega_n s+\omega_n^{\;2}}\;,\quad 1\ge\zeta\ge 0$$

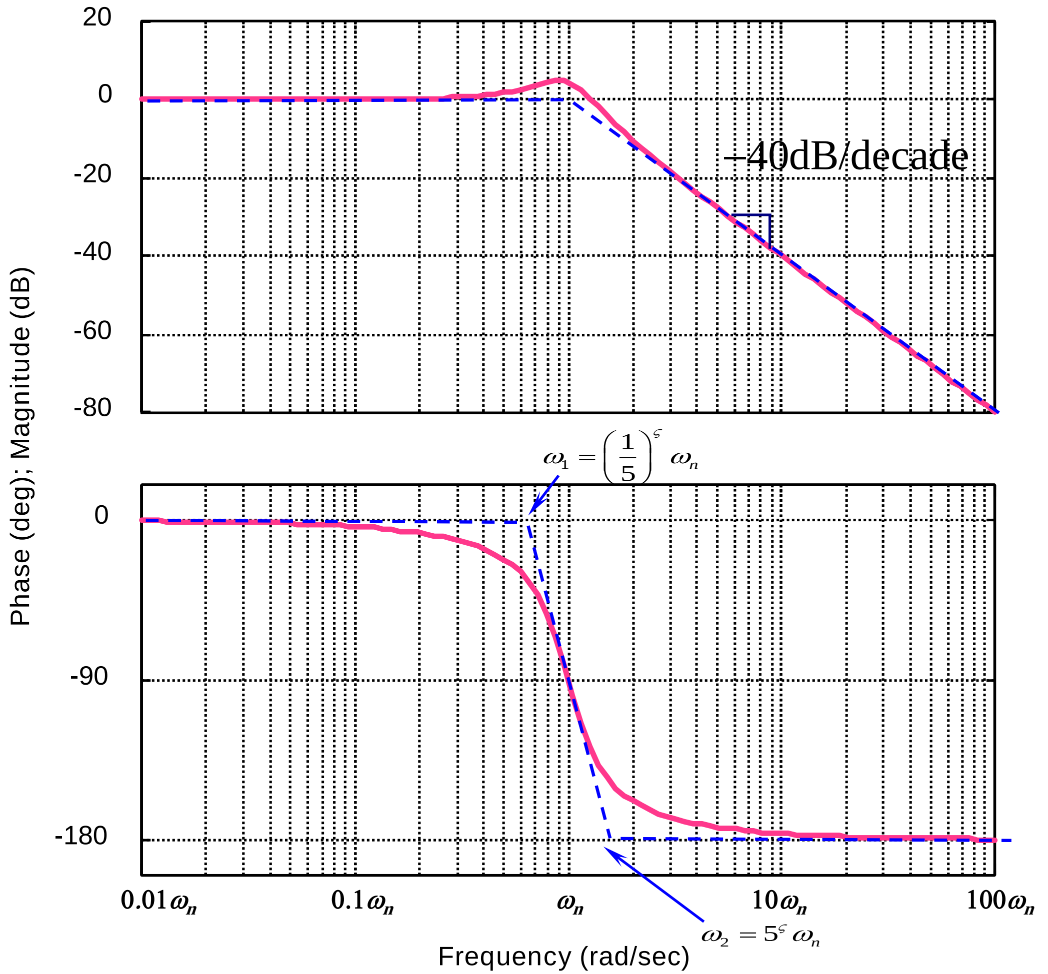
Frequency Response

$$G_{p2}(j\omega)=\frac{1}{\left(1-\frac{\omega^2}{\omega_n^{\;2}}\right)+j\frac{2\zeta\omega}{\omega_n}}$$

$$\left|G_{p2}(j\omega)\right|=\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^{\;2}}\right)^2+\frac{4\zeta^2\omega^2}{\omega_n^{\;2}}}}$$

$$\angle G_{p2}(j\omega)=-\angle\left(\left(1-\frac{\omega^2}{\omega_n^{\;2}}\right)+j\frac{2\zeta\omega}{\omega_n}\right)$$

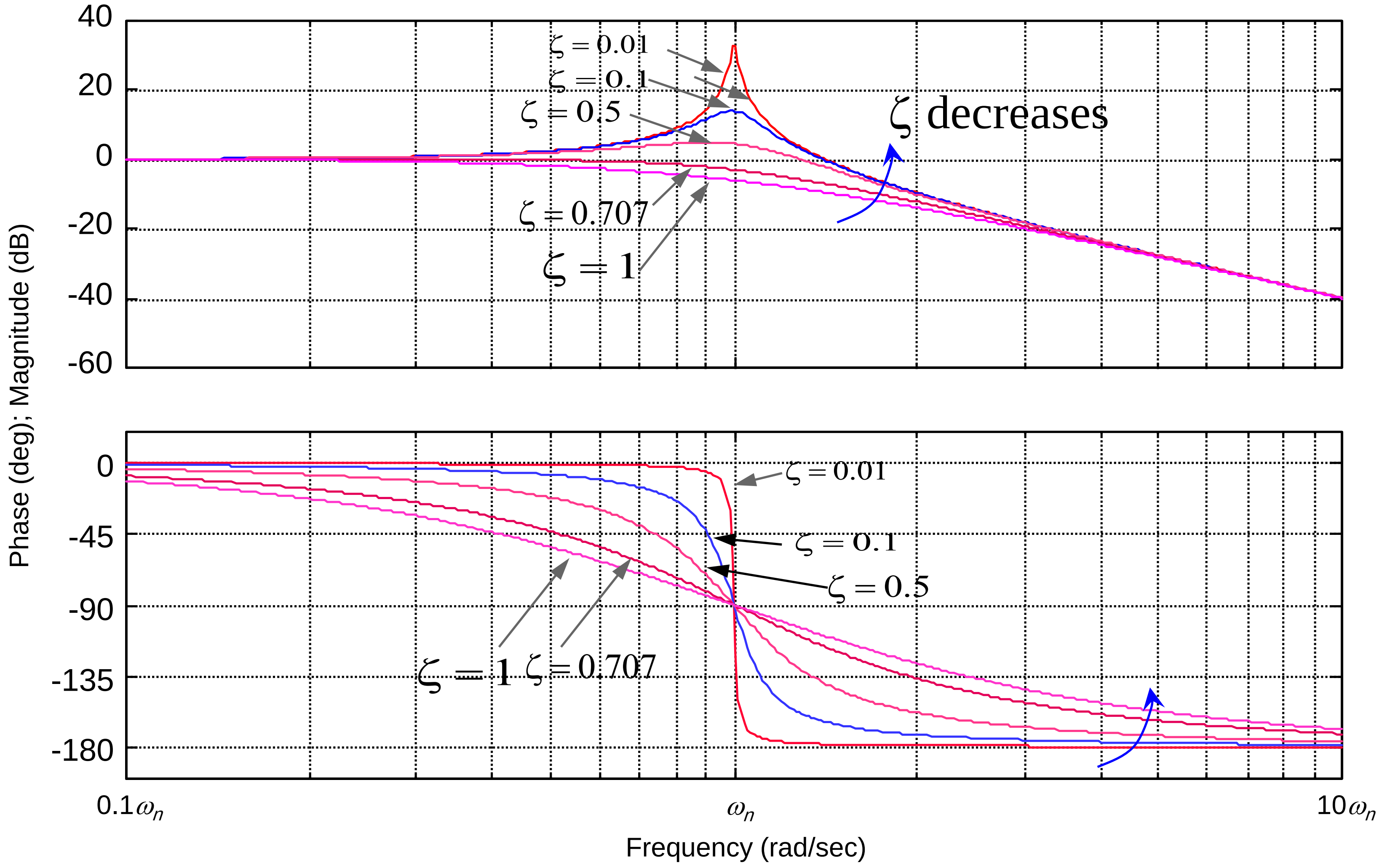
$$=-\operatorname{atan2}\left(\frac{2\zeta\omega}{\omega_n},\quad\left(1-\frac{\omega^2}{\omega_n^{\;2}}\right)\right)$$



Peak (Resonant) Frequency and Magnitude for $\zeta\le 1/\sqrt{2}=0.707$

$$\omega_r=\omega_n\sqrt{1-2\zeta^2}\quad\text{and}\quad\left|G_{p2}(j\omega_r)\right|=\frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

2nd Order System Frequency Response



A Few Observations:

- Three *different* characteristic frequencies:
  - Natural Frequency (ω_n)
  - Damped Natural Frequency (ω_d): $\omega_d = \omega_n \sqrt{1-\zeta^2}$
  - Resonant (Peak) Frequency (ω_r): $\omega_r = \omega_n \sqrt{1-2\zeta^2}$

$$\omega_r \leq \omega_d \leq \omega_n$$

- When the damping ratio $\zeta > 0.707$, there is no peak in the Bode magnitude plot. DO NOT confuse this with the condition for over-damped and under-damped systems: when $\zeta < 1$ the system is under-damped (has overshoot) and when $\zeta > 1$ the system is over-damped (no overshoot).

- As ζ ® 0 , ω_r ® ω_n and $\left| G(j\omega) \right|_{\mathrm{MAX}}$ increases; also the phase transition from 0 deg to $-$180 deg becomes sharper.

Example

- **Second-Order System**

  Transfer Function:

$$G(s) = \frac{2500}{s^2 + 10s + 2500}$$

  Plot the straight line approximation of *G*(*s*)'s Bode diagram:

$$G_{p2}(s) = \frac{\omega_n^{\,2}}{s^2 + 2\zeta\omega_n s + \omega_n^{\,2}} \;,\;\; 1 \geq \zeta \geq 0$$

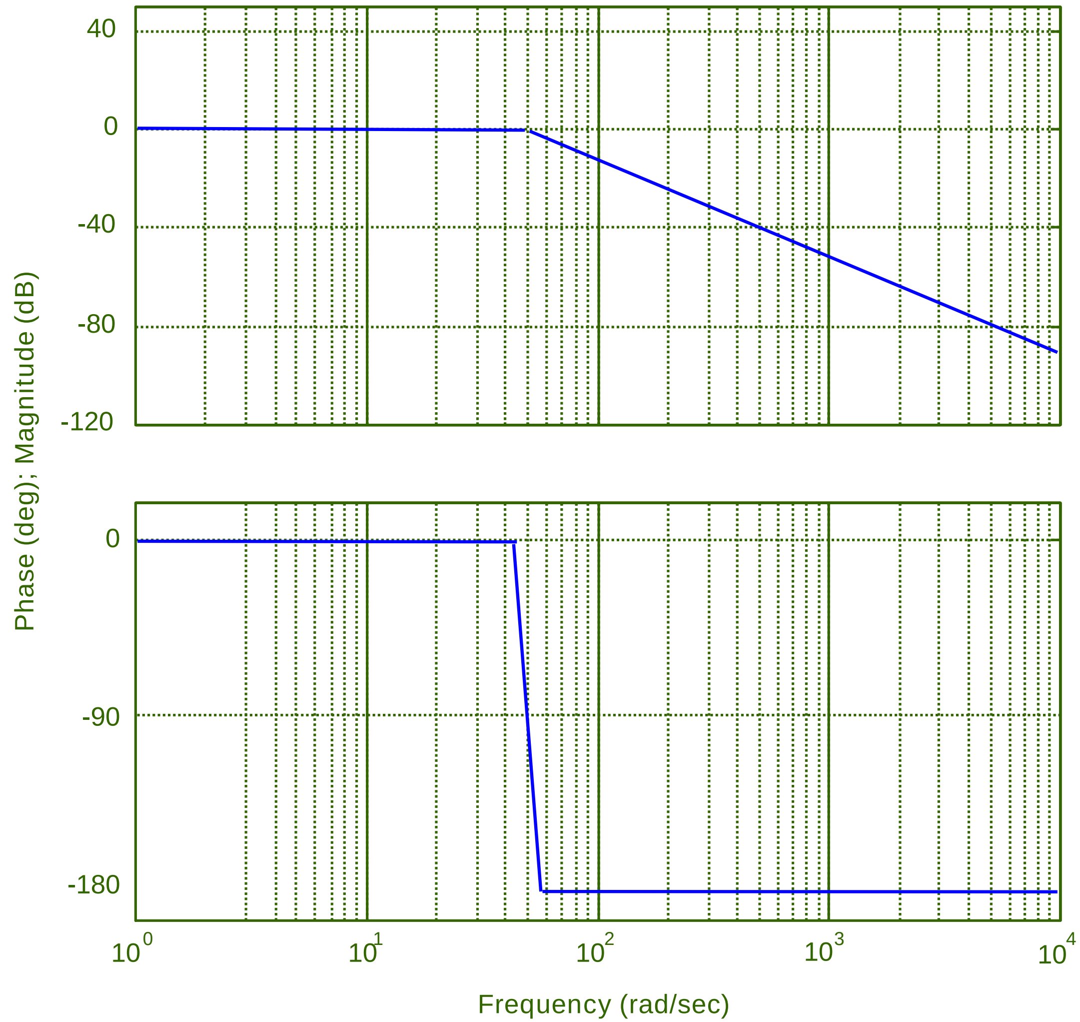
$$\omega_n^{\,2} = 2500, \Rightarrow \omega_n = 50$$

$$2\varsigma\omega_n = 10, \Rightarrow \varsigma = \frac{10}{2\omega_n} = 0.1$$

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2} \approx \omega_n$$

$$\omega_1 = \left(\frac{1}{5}\right)^{0.1} \omega_n = 42.6$$

$$\omega_2 = 5^{0.1}\omega_n = 58.7$$



Bode Plots of Complex Zeros

Standard Form of Transfer Function

$$G_{z2}(s) = \frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{\omega_n^2}, \quad 1 \geq \zeta \geq 0$$

Frequency Response

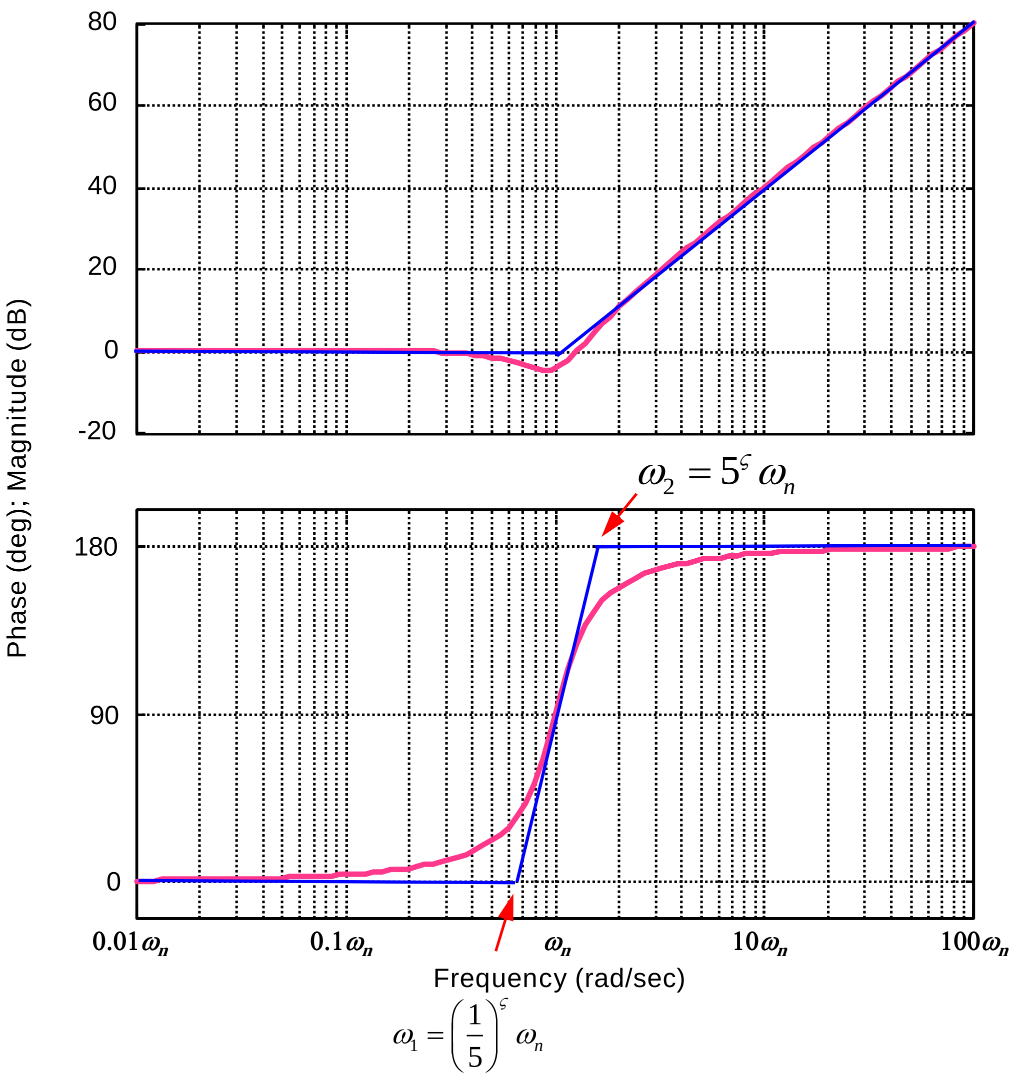
$$G_{z2}(j\omega) = \left(1 - \frac{\omega^2}{\omega_n^2}\right) + j\frac{2\zeta\omega}{\omega_n}$$

$$\left|G_{z2}(j\omega)\right| = 1/\left|G_{p2}(j\omega)\right|$$

$$= \sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \frac{4\zeta^2\omega^2}{\omega_n^2}}$$

$$\angle G_{z2}(j\omega) = -\angle G_{p2}(j\omega)$$

$$= \operatorname{atan2}\left(\frac{2\zeta\omega}{\omega_n}, \quad 1 - \frac{\omega^2}{\omega_n^2}\right)$$



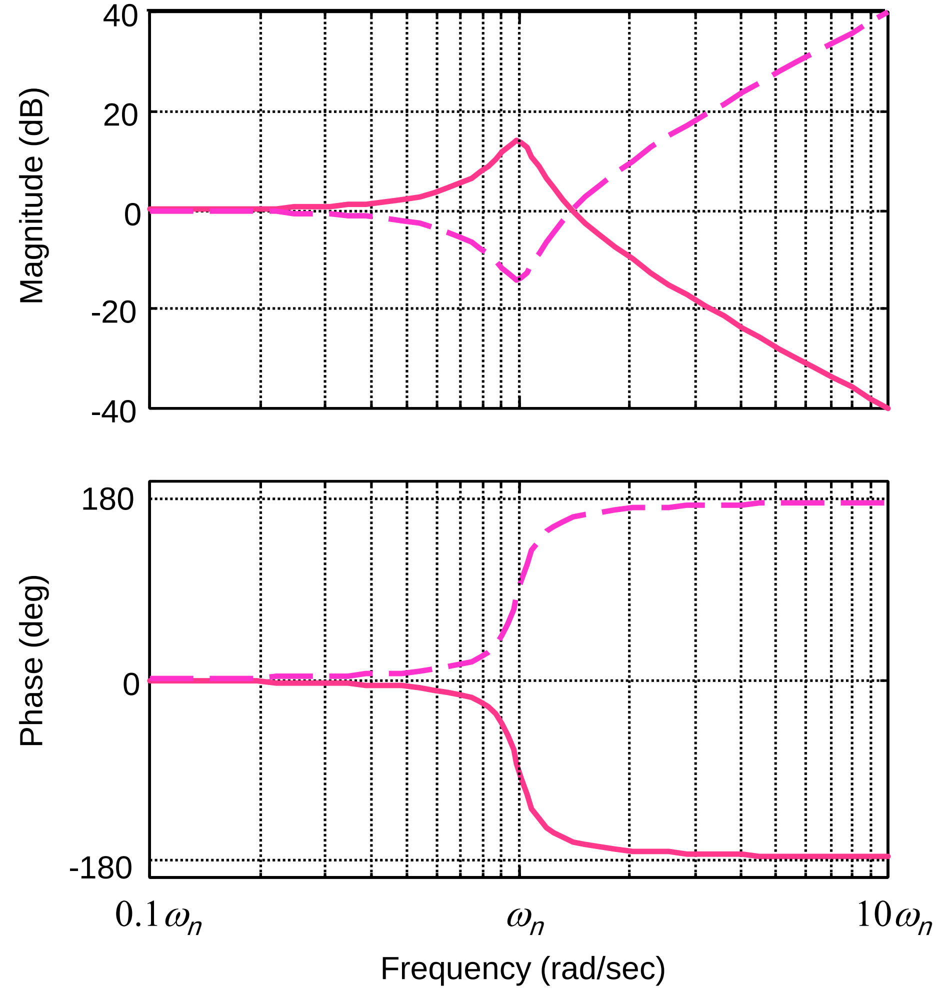
Bode Plots of Poles and Zeros

Bode plots of zeros are the mirror images of the Bode plots of the identical poles w.r.t. the 0 dB line and the 0 deg line, respectively:

$$\text{Let} \quad G_p(s) = \frac{1}{G_z(s)}$$

$$\Rightarrow \begin{cases} |G_p(j\omega)| = \dfrac{1}{|G_z(j\omega)|} \\ \angle G_p(j\omega) = -\angle G_z(j\omega) \end{cases}$$

$$\Rightarrow \begin{cases} 20\log_{10}|G_p(j\omega)| = -20\log_{10}|G_z(j\omega)| \\ \angle G_p(j\omega) = -\angle G_z(j\omega) \end{cases}$$



2nd Order Bode Diagram Summary

- **2nd Order Complex Poles**

$$G_{p2}(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2},\ 1\geq\zeta>0$$

  - Break Frequency

$$\omega_b=\omega_n \quad [\text{rad/s}]$$

  - Mag. Plot Approximation

    0 dB from DC to ω_n and a straight line with –40 dB/decade slope after ω_n. Peak value occurs at:

$$\omega_r=\omega_n\sqrt{1-2\zeta^2}$$

$$\Rightarrow\ \left|G_{p2}(j\omega_r)\right|_{MAX}=\frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

  - Phase Plot Approximation

    0 deg from DC to $(1/5)^{\zeta}\,\omega_n$ Between $(1/5)^{\zeta}\,\omega_n$ and $5^{\zeta}\,\omega_n$, a straight line from 0 deg to –180 deg (passing –90 deg at ω_n). For frequency higher than $5^{\zeta}\,\omega_n$, straight line on –180 deg.

- **2nd Order Complex Zeros**

$$G_{z2}(s)=\frac{s^2+2\zeta\omega_n s+\omega_n^2}{\omega_n^2},\ 1\geq\zeta>0$$

  - Break Frequency

$$\omega_b=\omega_n \quad [\text{rad/s}]$$

  - Mag. Plot Approximation

    0 dB from DC to ω_n and a straight line with 40 dB/decade slope after ω_n.

  - Phase Plot Approximation

    0 deg from DC to $(1/5)^{\zeta}\,\omega_n$. Between $(1/5)^{\zeta}\,\omega_n$ and $5^{\zeta}\,\omega_n$, a straight line from 0 deg to 180 deg (passing 90 deg at ω_n). For frequency higher than $5^{\zeta}\,\omega_n$, straight line on 180 deg.

Example

- **Combination of Systems**

Transfer Function:

$$G(s) = \frac{35s + 35}{s(s+5)}$$

Plot the straight line approximation of $G(s)$'s Bode plots:

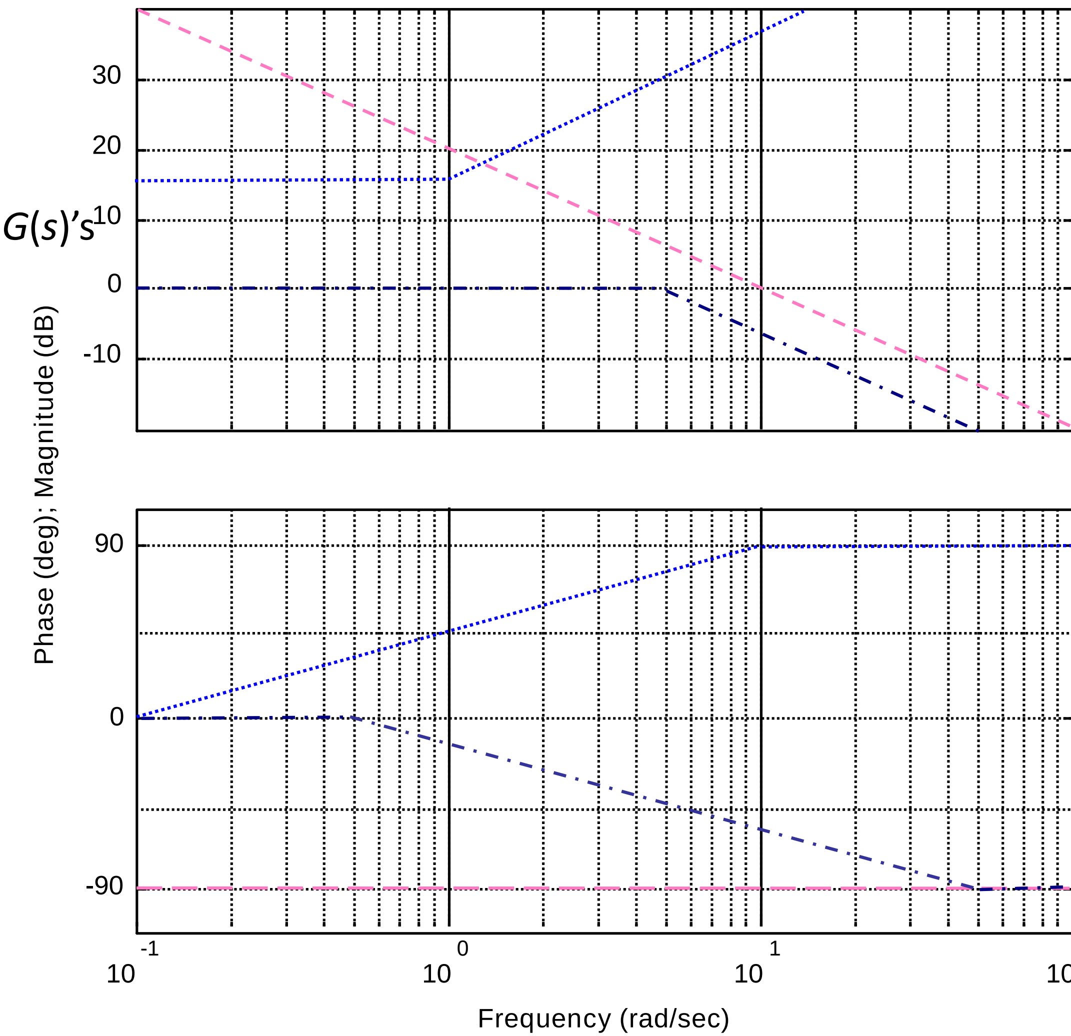
$$G(s) = \frac{35}{5}(s+1) \cdot \frac{1}{s} \cdot \frac{1}{\frac{1}{5}s+1}$$

$$G(j\omega) = 7(j\omega+1) \cdot \frac{1}{j\omega} \cdot \frac{1}{\frac{1}{5}j\omega+1}$$

$$20\log_{10}|G(j\omega)| = \underbrace{16.9}_{20\log_{10}7} + 20\log_{10}|j\omega+1|$$

$$+20\log_{10}\left|\frac{1}{j\omega}\right| + 20\log_{10}\left|\frac{1}{\frac{1}{5}j\omega+1}\right|$$

$$\angle G(j\omega) = \angle(j\omega+1) - 90^{o} + \angle\left(\frac{1}{\frac{1}{5}j\omega+1}\right)$$



SUM THEM

Example

- **Combination of Systems**

Transfer Function:

$$G(s) = \frac{2500}{s(s^2 + 55s + 250)}$$

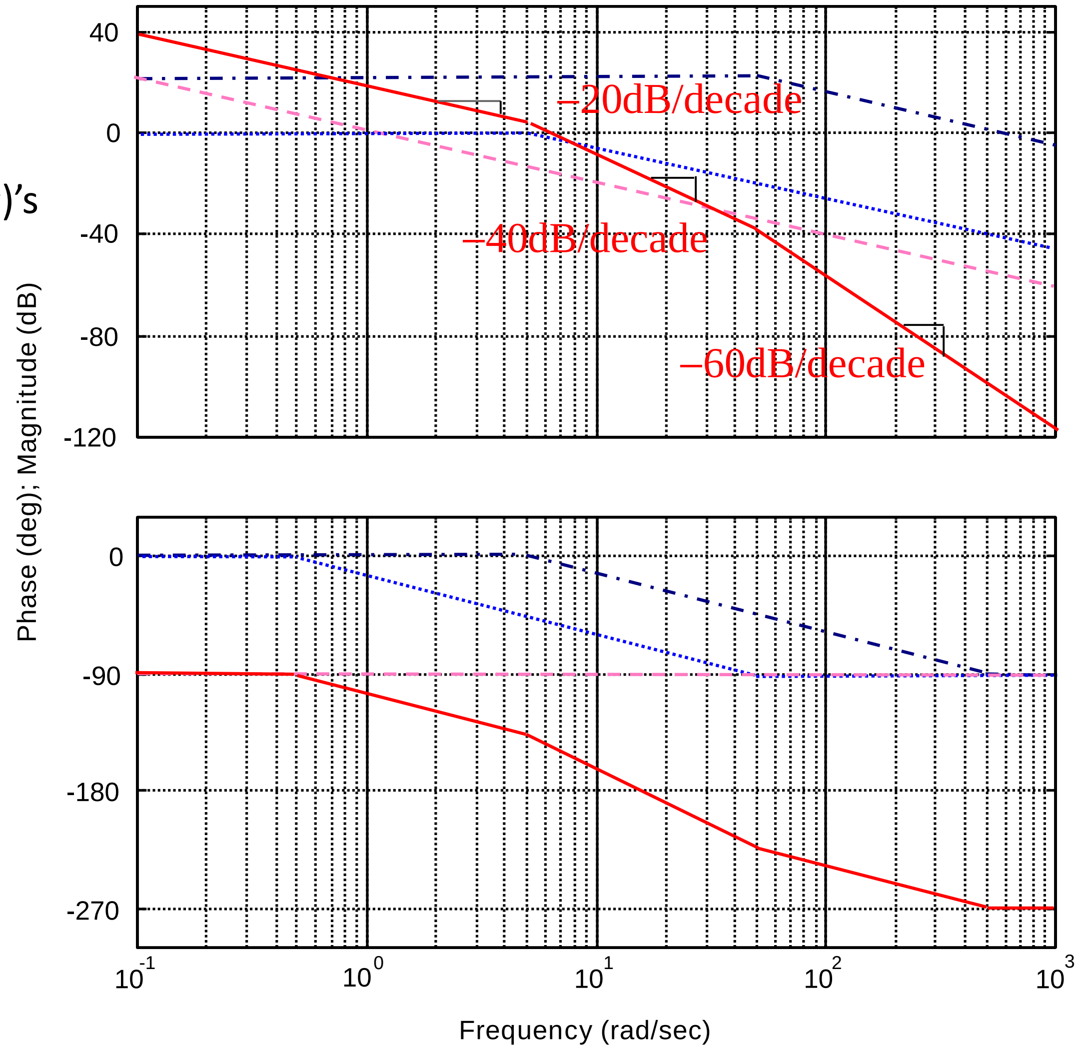
Plot the straight line approximation of *G*(*s*)'s Bode plots:

$$G(s) = \frac{2500}{s(s+50)(s+5)} = \frac{2500}{50 \times 5} \frac{1}{\frac{1}{50}s+1} \square \frac{1}{s} \square \frac{1}{\frac{1}{5}s+1}$$

$$G(j\omega) = 10 \times \frac{1}{\frac{1}{50}j\omega+1} \square \frac{1}{j\omega} \square \frac{1}{\frac{1}{5}j\omega+1}$$

$$20\log_{10}|G(j\omega)| = \underbrace{20}_{20\log_{10}10} + 20\log_{10}\left|\frac{1}{\frac{1}{50}j\omega+1}\right|$$

$$+20\log_{10}\left|\frac{1}{j\omega}\right| + 20\log_{10}\left|\frac{1}{\frac{1}{5}j\omega+1}\right|$$



Teamwork Example

- **Combination of Systems**

  Transfer Function:

$$G(s) = \frac{2000(s^2 + s + 25)}{s(s + 200)(s^2 + 10s + 2500)}$$

Plot the straight line approximation of *G*(*s*)'s Bode diagram:

$$G(s) = \frac{2000\times 25}{200\times 2500} \square \frac{s^2+s+25}{25} \square \frac{1}{s} \square \frac{1}{\frac{1}{200}s+1} \square \frac{2500}{s^2+10s+2500}$$

$$20\log_{10}\left|G(j\omega)\right| = \underset{20\log_{10}0.1}{-20} + 20\log_{10}\left|\frac{\left(25-\omega^2\right)+j\omega}{25}\right| + 20\log_{10}\left|\frac{1}{j\omega}\right|$$

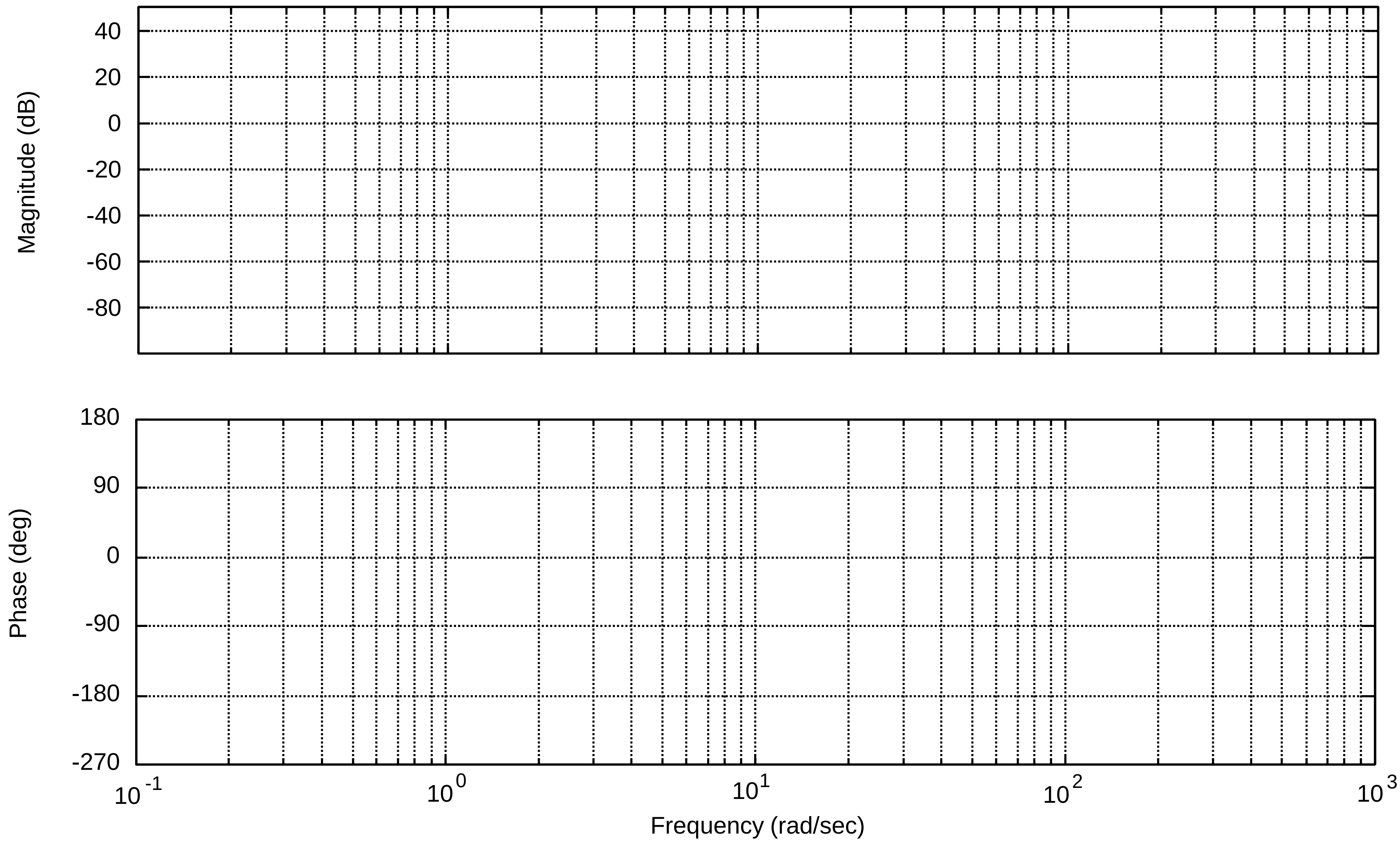
$$+20\log_{10}\left|\frac{1}{\frac{1}{200}j\omega+1}\right| + 20\log_{10}\left|\frac{2500}{\left(2500-\omega^2\right)+10j\omega}\right|$$

POP. Quiz

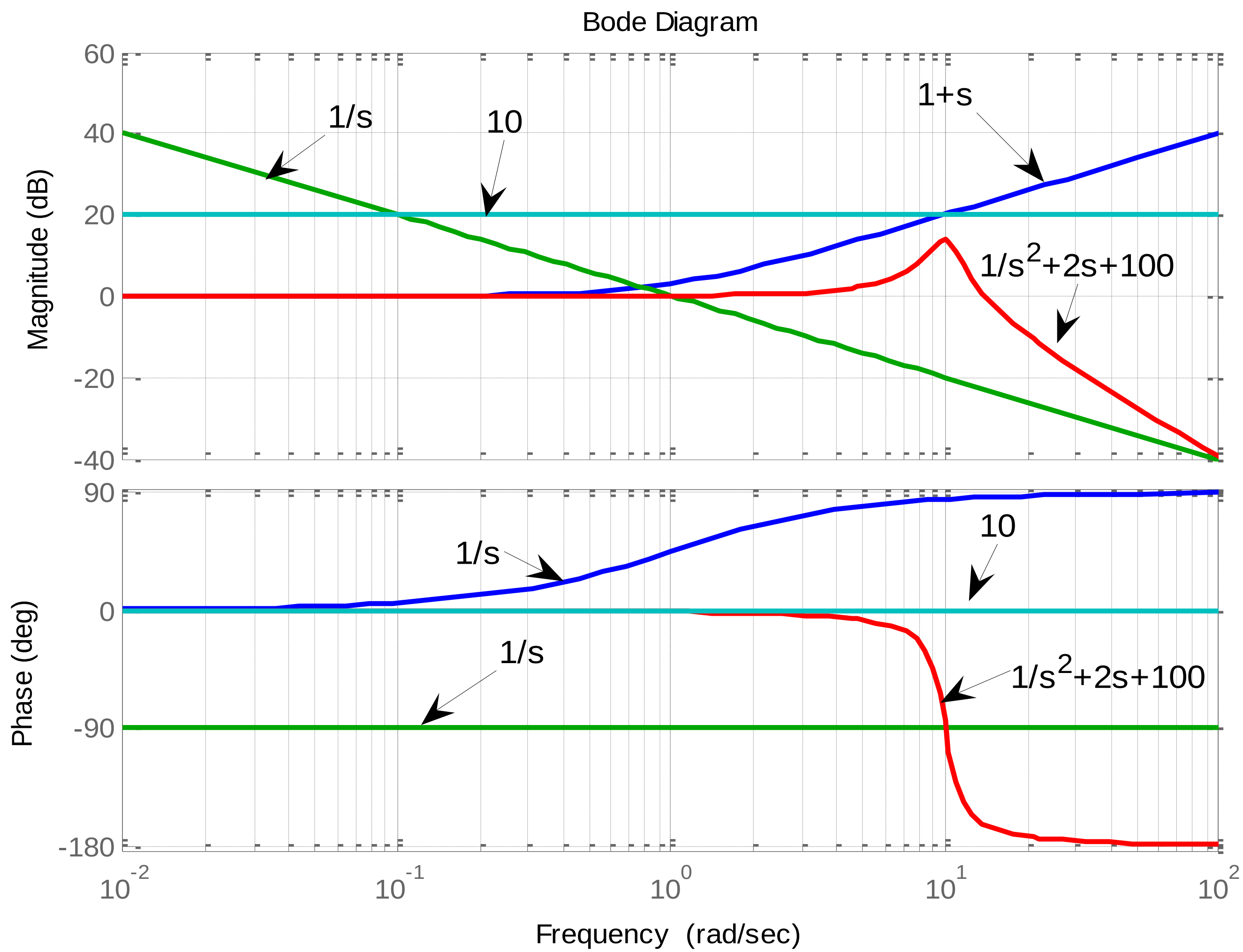
Draw Bode diagram of G(s) for the frequency range of $10^{-2} < \omega < 10^{2}$ rad/s.

$$G(s) = \frac{1000(s+1)}{s(s^2+2s+100)}$$

Margins on Bode Plots

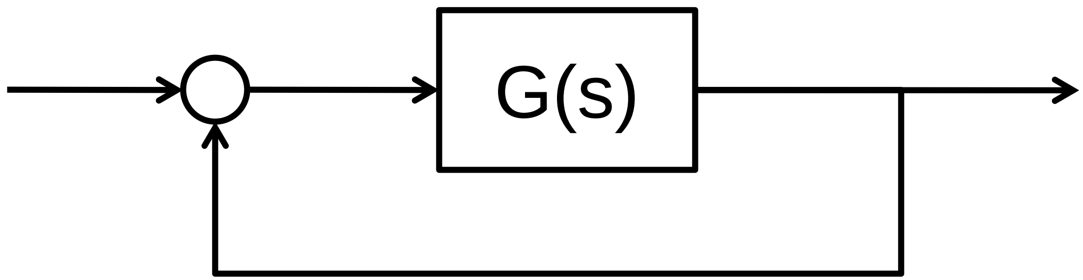


POP. Quiz



Bode Diagram
60
40
20
0
-20
-40
Magnitude (dB)
1/s
10
1+s
1/s^2+2s+100
90
0
-90
-180
Phase (deg)
1/s
10
1/s
1/s^2+2s+100
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/sec)

Margins on Bode Plots

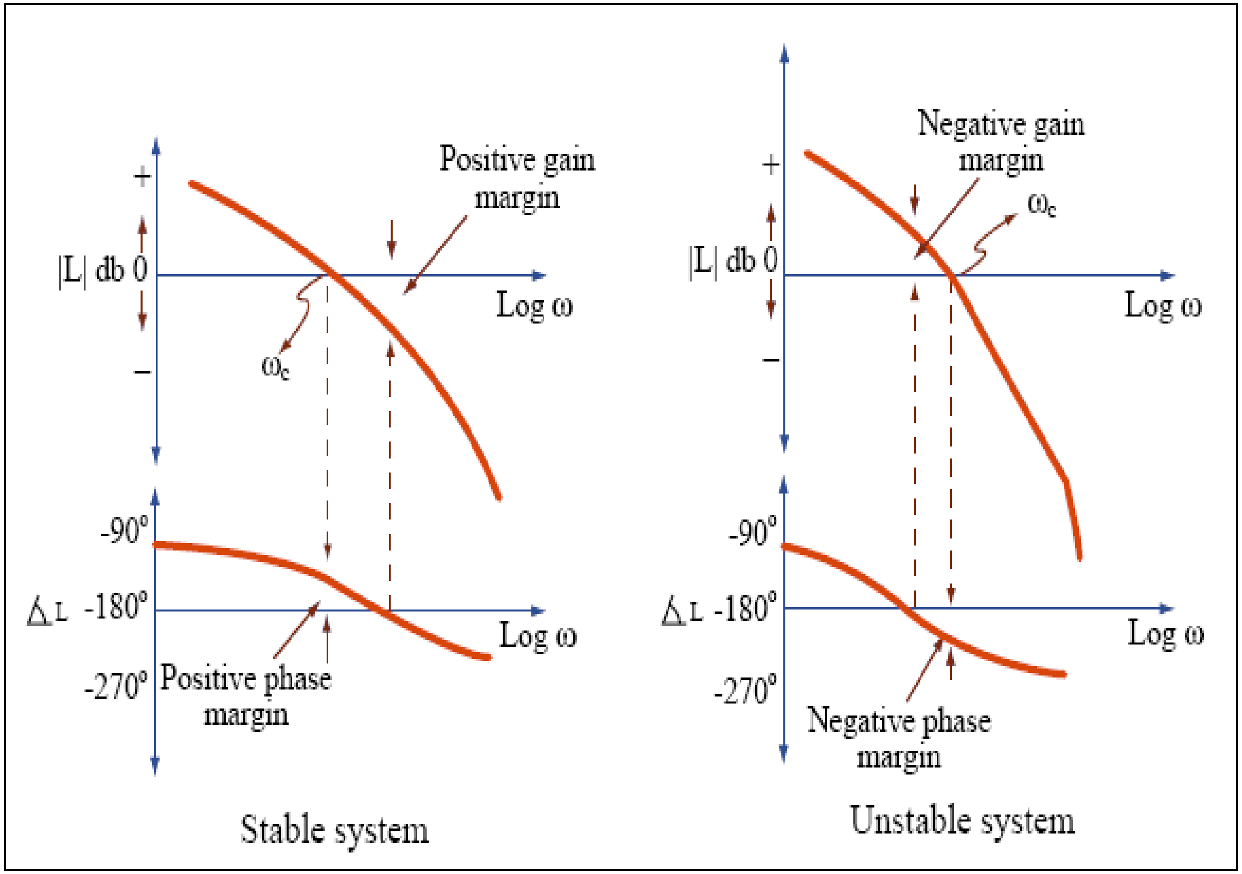


In most cases, stability of this closed-loop can be determined from the Bode plot of G:

– Phase margin > 0

– Gain margin > 0

Margins on Bode Plots



Margins on Bode Plots

$$\omega_{gc}: \quad \text{gain cross-over freq.}$$

$$\text{at } \omega_{gc}, |G(j\omega)| = 1 \text{ or } 0\,\text{dB}$$

$$PM: \quad \text{phase margin}$$

$$= 180^\circ + \angle G(j\omega_{gc})$$

$$\omega_{pc}: \quad \text{phase cross-over freq.}$$

$$\angle G(j\omega_{pc}) = 180^\circ$$

$$GM: \quad \text{gain margin} = -20\log|G(j\omega_{pc})|\,\text{dB}$$

$$= 1/|G(j\omega_{pc})| \text{ in value}$$

Margins on Bode Plots

If $\left|G(j\omega)\right|$ never cross 0 dB line (always below 0 dB line), then $PM = \infty$.

If $\angle G(j\omega)$ never cross –180° line (always above –180°), then $GM = \infty$.

If $\angle G(j\omega)$ cross –180° several times, then there are several GM's.

If $\left|G(j\omega)\right|$ cross 0 dB several times, then there are several PM's.

Margins on Bode Plots

Example:

$$G(s)=\frac{100(s+1)}{(s+2)(s+5)}$$

$$=10\,\frac{s+1}{\left(\frac{1}{2}s+1\right)\left(\frac{1}{5}s+1\right)}$$

Bode plot on next page.

1. $|G(j\omega)|$ cross 0 dB line near $\omega=100$

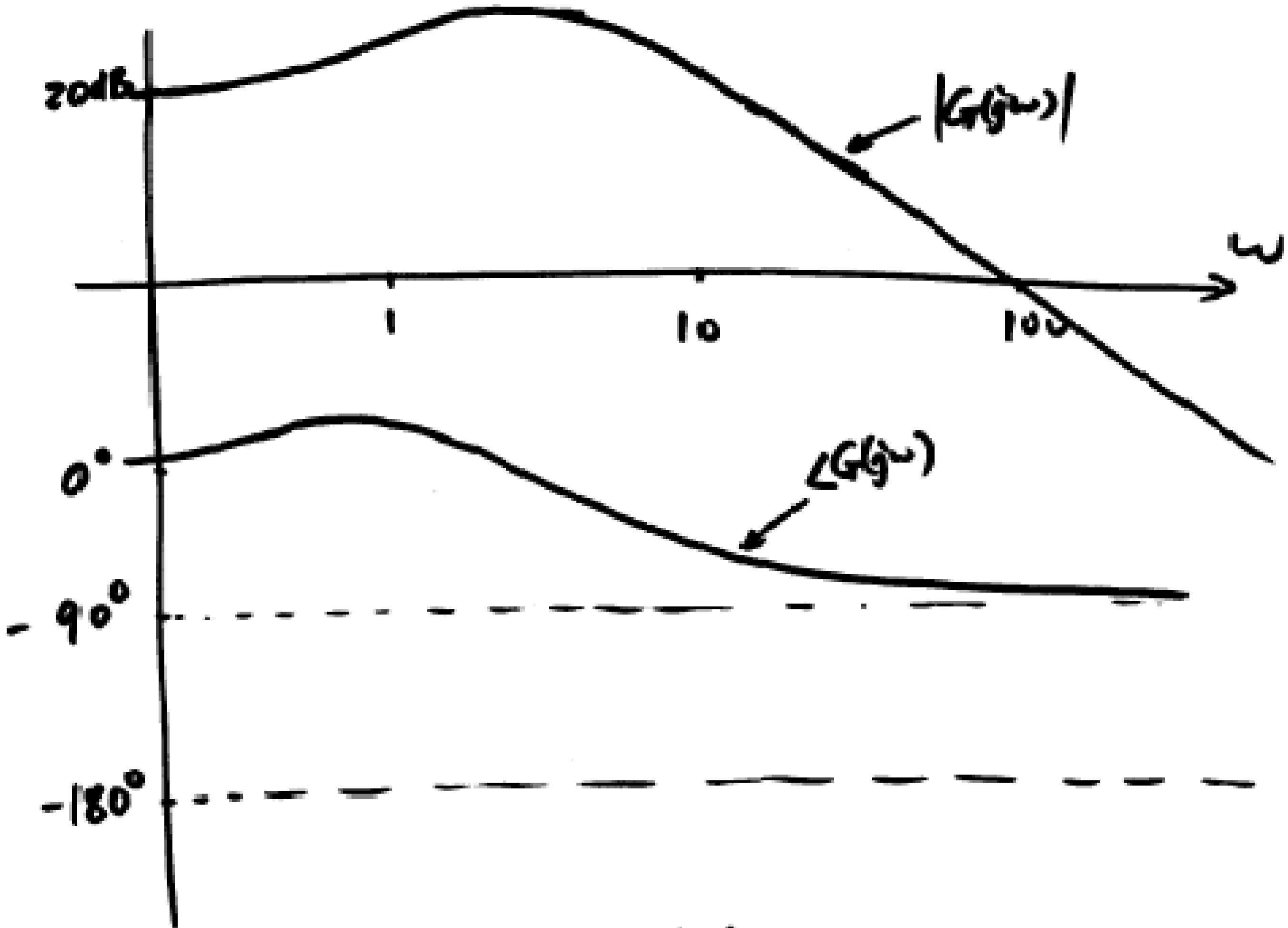
   $\therefore\ \omega_{gc}\approx 100$

   $PM\approx$ ________

2. $\angle G(j\omega)$ cross $-180°$ at $\omega_{pc}=$ ________

   $\therefore\ GM=$ ________

Margins on Bode Plots



Closed-loop stability: ______

Margins on Bode Plots

Example:

$$G(s) = \frac{25}{s(s^2 + 4s + 25)}$$

$$= \frac{1}{s\left(\frac{1}{25}s^2 + \frac{4}{25}s + 1\right)}$$

Bode plot on next page.

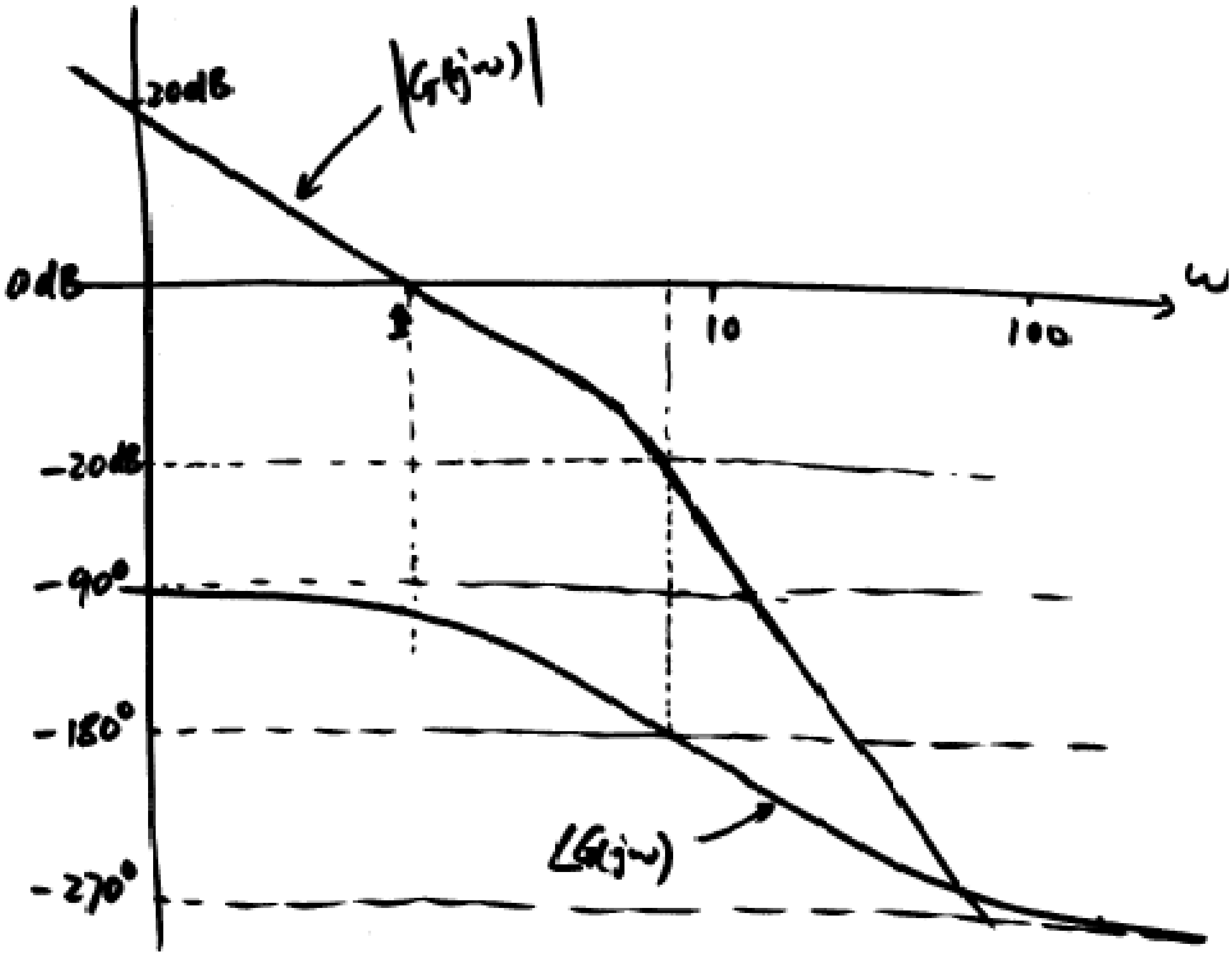
1. $|G(j\omega)|$ cross 0 dB line near ______

   $\therefore \omega_{gc} \approx$ ______

   $\angle G(j\omega)$ at ω_{gc} is about ______

   $\therefore PM \approx$ ______

Margins on Bode Plots



Margins on Bode Plots

1. Where does $\angle G(j\omega)$ cross the $-180°$ line
   Answer: ___________

   $\therefore \omega_{pc} \approx$ __________

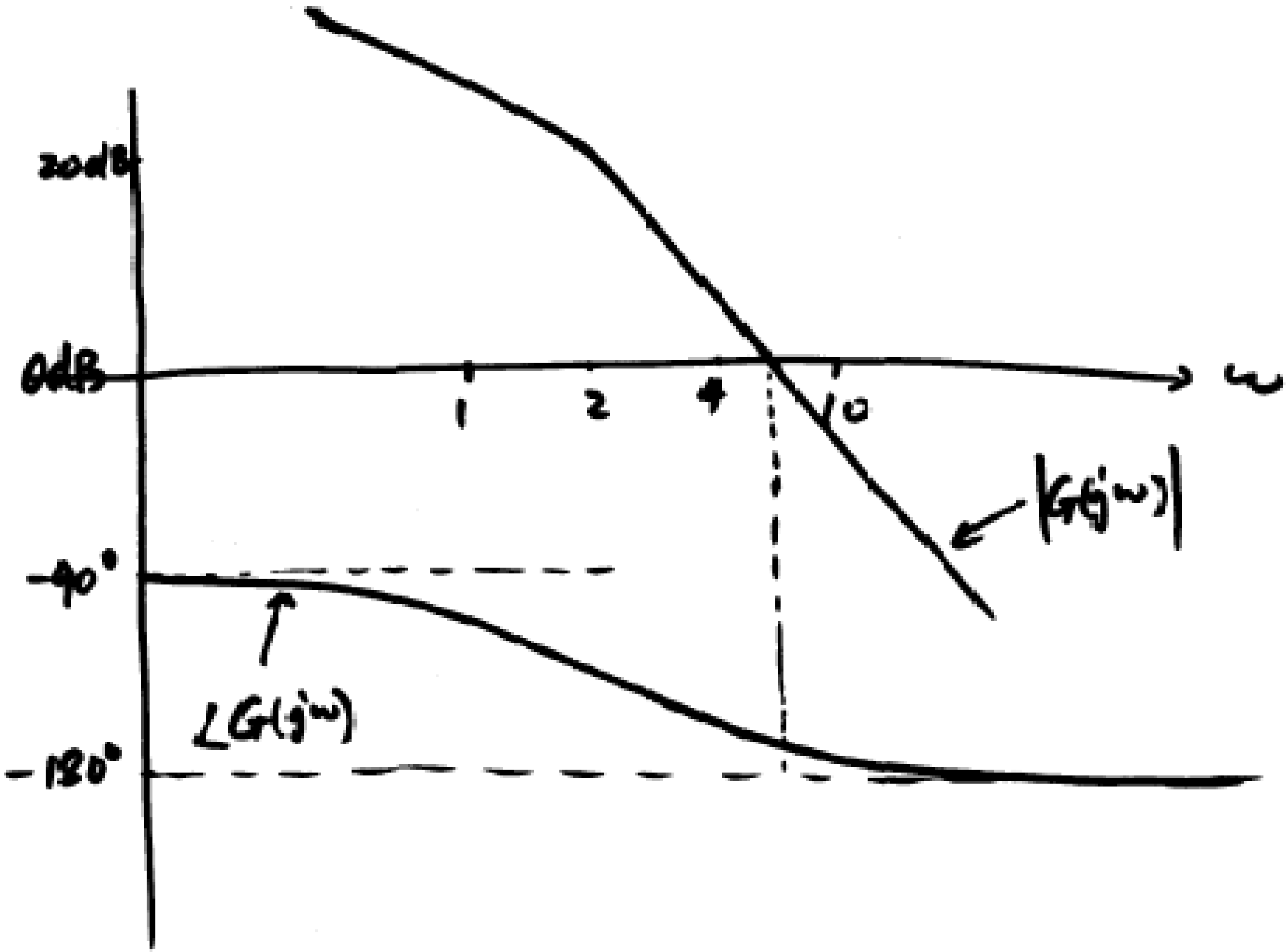
   at ω_{pc}, how much is $|G(j\omega)| \approx$ _________

   $\therefore GM \approx$ __________

2. Closed-loop stability: ___________

Margins on Bode Plots

$$\text{Example:} \qquad G(s) = \frac{40}{s(s+2)} = 20\frac{1}{s\left(\frac{1}{2}s+1\right)}$$



Margins on Bode Plots

1. $\left|G(j\omega)\right|$ crosses 0 dB at ____________

   $\therefore \omega_{gc} \approx$ __________

   at this freq, $\angle G(j\omega) \approx$ _________

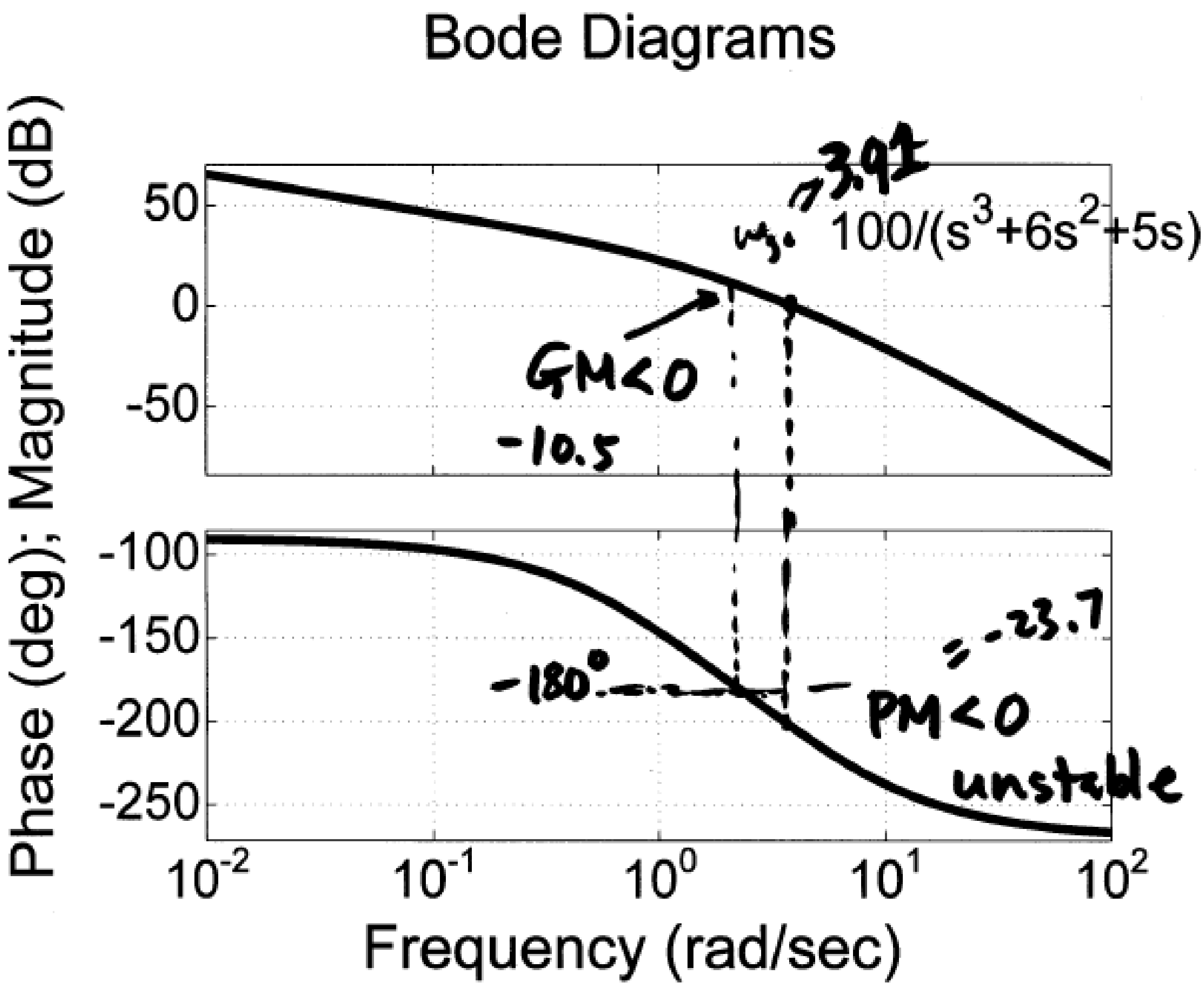
   $\therefore PM \approx$ __________

2. Does $\angle G(j\omega)$ cross $-180°$ line? _________

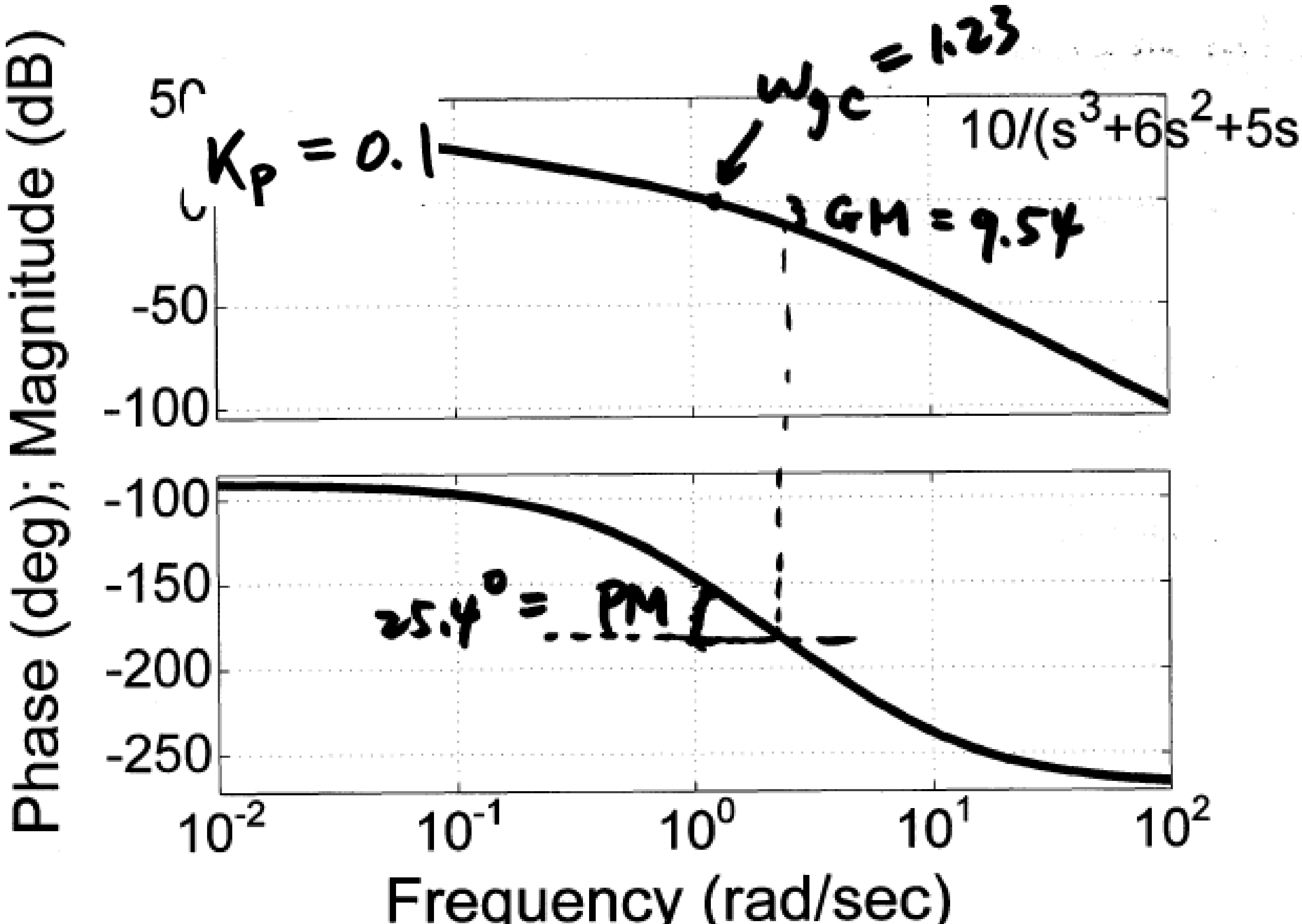
   $\therefore GM \approx$ __________

3. Closed-loop stability: ____________

Margins on Bode Plots



Margins on Bode Plots



Margins on Bode Plots