MENG366

Bolar Plots and Nyquist Plots

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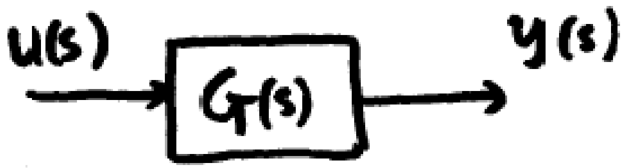
Frequency Response Method

Given:



- $G(j\omega)$ as a function of ω is called the freq. resp.
- For each ω, $G(j\omega) = x(\omega) + jy(\omega)$ is a point in the complex plane
- As ω varies from 0 to ∞, the plot of $G(j\omega)$ is called the Nyquist plot

Frequency Response Method

- Can rewrite in Polar Form:

$$G(j\omega) = |G(j\omega)| \cdot e^{j\angle G(j\omega)}$$

- $|G(j\omega)|$ as a function of ω is called the amplitude resp.
- $\angle G(j\omega)$ as a function of ω is called the phase resp.
- The two plots:

$$\begin{cases} 20\log_{10}|G(j\omega)| \sim \omega \\ \angle G(j\omega) \sim \omega \end{cases}$$

With log scale-ω, are Bode plot

Frequency Response Method

To obtain freq. Resp from G(s):

- Select $\omega = \left[\omega_1\ \omega_2 \cdots \omega_N\right]$
- Evaluate $G(j\omega)$ at those to get

$$G = \left[G_1\ G_2 \cdots G_N\right]$$

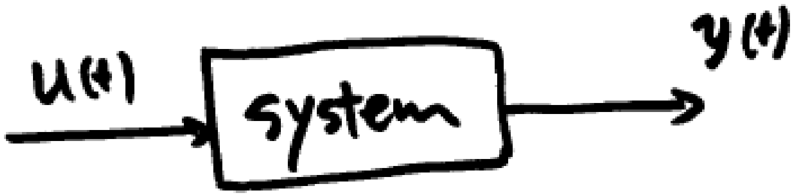
- Plot Imag(G) vs Real(G): Nyquist
- or plot $\begin{cases} 20 \cdot \log_{10}(abs(G)) & vs\ \omega \\ angle(G) & vs\ \omega \end{cases}$

With log scale ω

- Matlab command to explore: nyquist, bode

Frequency Response Method

To obtain freq. resp. experimentally:



- Select $\omega = \omega_1\ \omega_2 \cdots \omega_N$
- Given input to system as: $u(t) = A_1 \sin(\omega_1 t)$
- Adjust A_1 so that the output is not saturated or distorted.
- Measure amp B_1 and phase φ_1 of output: $y(t) = B_1 \sin(\omega_1 t + \varphi_1)$

Frequency Response Method

- Then $\begin{cases} \dfrac{B_1}{A_1}\ and\ \varphi_1 \\ or\ \dfrac{B_1}{A_1} e^{j\varphi_1} \end{cases}$ is the freq. resp. of the system at freq ω

- Repeat for all ω_K
- Either plot

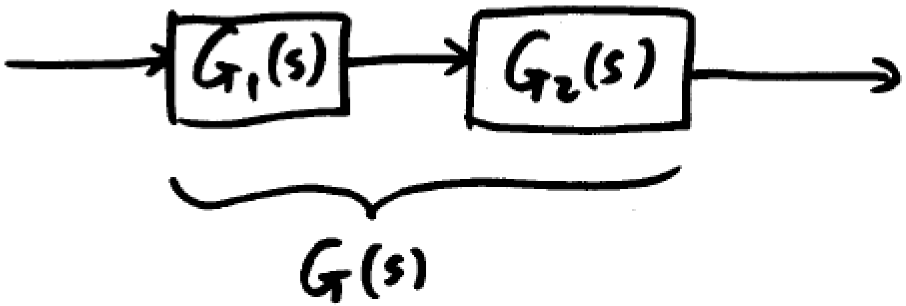
or plot $\begin{cases} \dfrac{B_K}{A_K} \sim \omega_K \\ \varphi_K \sim \omega_K \end{cases} \longleftarrow Bode$

$$\frac{B_K}{A_K}\sin\varphi_K \sim \frac{B_K}{A_K}\cos\varphi_K \longleftarrow Nyquist$$

$$\uparrow \qquad\qquad \uparrow$$

$$imag\left(\frac{B_K}{A_K} e^{j\varphi_K}\right) \qquad real\left(\frac{B_K}{A_K} e^{j\varphi_K}\right)$$

Frequency Response Method



Product of T.F.

$$G(s) = G_1(s)G_2(s)$$

$$G(j\omega) = G_1(j\omega)G_2(j\omega)$$

$$|G(j\omega)| = |G_1(j\omega)| \cdot |G_2(j\omega)|$$

$$20\log|G(j\omega)| = 20\log|G_1(j\omega)| + 20\log|G_2(j\omega)|$$

$\uparrow$ mag. Bode of G — $\uparrow$ mag. Bode of G_1 — $\uparrow$ mag. Bode of G_2

$\therefore$ add mag. Bode plots

$$\angle G(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

- $\angle G(j\omega)$: phase Bode plot of G, the product
- $\angle G_1(j\omega)$: Bode phase plot of G_1
- $\angle G_2(j\omega)$: Bode phase plot of G_2

$\therefore$ the phase plot of the product

$= \Sigma$ individual phase plot.

Overall: Bode plot of the product

$= \Sigma$ individual Bode plot.

Nyquist Diagram or Analysis

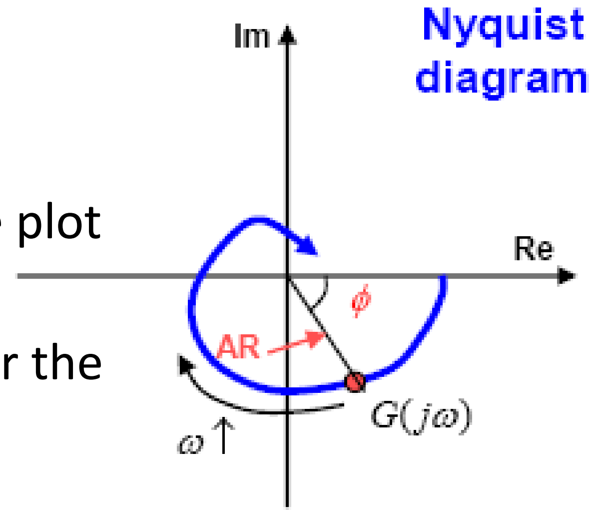
- The polar plot, or Nyquist diagram, of a sinusoidal transfer function G(jw) is a plot of the **magnitude** of G(jw) versus the **phase angle** of G(jw) on polar coordinates as **w is varied** from zero to infinity.
- Thus, the polar plot is the locus of vectors |G(jw)| LG(jw) as w is varied from zero to infinity.

Nyquist Diagram or Analysis

- The projections of G(jw) on the real and imaginary axis are its real and imaginary components.
- The **Nyquist Stability Criteria** is a test for system stability, just like the Routh-Hurwitz test, or the Root-Locus Methodology.

Nyquist is an alternative representation of frequency response

- Compact (one plot)
- Wider applicability of stability analysis than Bode plot
- High frequency characteristics will be shrunk near the origin.
- Inverse Nyquist diagram: polar plot of G(jw)
- Combination of different transfer function components is not easy as with Nyquist diagram as with Bode plot.



Nyquist Diagram or Analysis

- Note that in **polar plots, a positive** (negative) phase angle is measured counterclockwise (clockwise) from the positive real axis. In the polar plot, it is important to show the frequency graduation of the locus.
- **Routh-Hurwitz and Root-Locus** can tell us where the poles of the system are for particular **values of gain**.

Nyquist Diagram or Analysis

- By **altering the gain of the system**, we can determine if any of the poles move into the RHsP, and therefore become unstable.
- However, the **Nyquist Criteria** can also give us additional information about a system.
- The Nyquist Criteria, can tell us things about the **frequency characteristics** of the system.

Nyquist Diagram or Analysis

- For instance, some systems with constant gain might be *stable* for **low-frequency** inputs, but become *unstable* for **high-frequency** inputs.
- Also, the Nyquist Criteria can tell us things about the **phase of the input signals**, the **time-shift** of the system, and other important information.

Nyquist Kuo's View

- Kuo et al (2003) suggests that, ***the Nyquist criterion is a semi-graphical method that determines the stability of a closed loop system by investigating the properties of the frequency domain plot, the Nygmst plot of L(s) which is a plot of L(jw) in the polar coordinates of M [L(jw)] versus Re[L(jw)] as ω varies from 0 to ∞***.

Nyquist Xavier's View

- While, Xavier et al (2004) narrates that, ***the Nyquist criterion is based on "Cauchy's Residue Theorem" of complex variables which is referred to as "Principle of Argument".***

The Argument Principle

- If we have a **contour, Γ** (capital gamma), drawn in one plane (say the complex laplace plane, for instance), we can map that contour into another plane, the F(s) plane, by transforming the contour with the function F(s).
- The resultant contour, **Γ F(s)** will circle the origin point of the F(s) plane N times, where N is equal to the difference between Z and P (the number of zeros and poles of the function F(s), respectively).

Nyquist Criterion

- Let us first introduce the most **important equation** when dealing with the Nyquist criterion:

$$N = Z - P$$

- Where:
  - **N** is the number of encirclements of the (-1, 0) point.
  - **Z** is the number of zeros of the characteristic equation.
  - **P** is the number of poles of the open-loop characteristic equation.

Nyquist Stability Criterion

- A feedback control system is stable, if and only if the contour ΓF(s) in the F(s) plane does not encircle the (-1, 0) point when P is 0.
- A feedback control system is stable, if and only if the contour ΓF(s) in the F(s) plane encircles the (-1, 0) point a number of times equal to the number of poles of F(s) enclosed by Γ.

Nyquist Stability Criterion

- In other words, if P is zero then N must equal zero. Otherwise, N must equal P. Essentially, we are saying that Z must always equal zero, because Z is the number of zeros of the characteristic equation (and therefore the number of poles of the closed-loop transfer function) that are in the right-half of the s plane.

Nyquist Manke’s View

- While Manke (1997) outlines that, the Nyquist criterion is used to identify the presence of roots of a characteristic equation of a control system in a specified region of s-plane.
- He further adds that although the purpose of using Nyquist criterion is similar to RHC, the approach differs in the following respect:

Nyquist Manke's View Cont…

– The open loop transfer G(s) H(s) is considered instead of the closed loop characteristic equation 1 + G(s) H(s) = 0
– Inspection of graphical plots G(s) H(s) enables to get more than YES or NO answer of RHC pertaining to the stability of control systems.

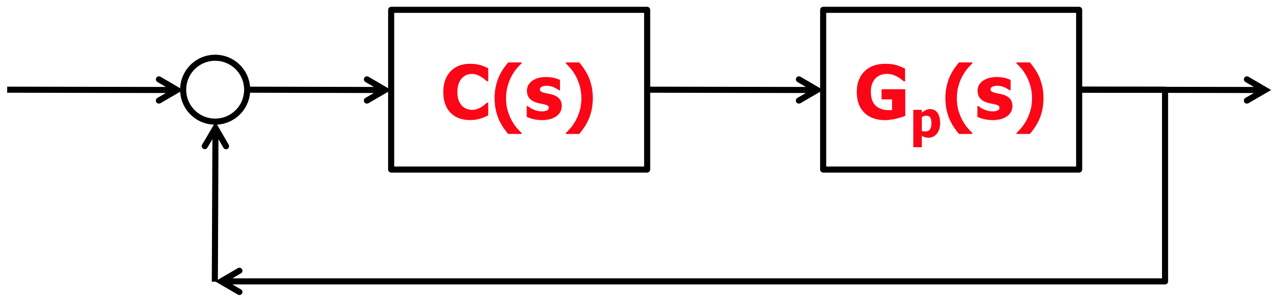
Kuo's Features of Nyquist Criterion

- Kuo also outlines the following as the features that make the Nyquist criterion an attractive alternative for the analysis and design of control systems:
  - In addition to providing the absolute stability, like the RHC, the NC also gives information on the relative of a stable system and the degree of instability.
  - The Nyquist plot of G(s) H(s) or of L (s) is very easy to obtain.

Kuo's Features of Nyquist Criterion

- The Nyquist plot of G(s) H(s) gives information on the frequency domain characteristics such as Mr, Wr, BW and others with ease.
- The Nyquist plot is useful for systems with pure time delay that cannot be treated with the RHC and are difficult to analyze with root locus method.

Nyquist plot

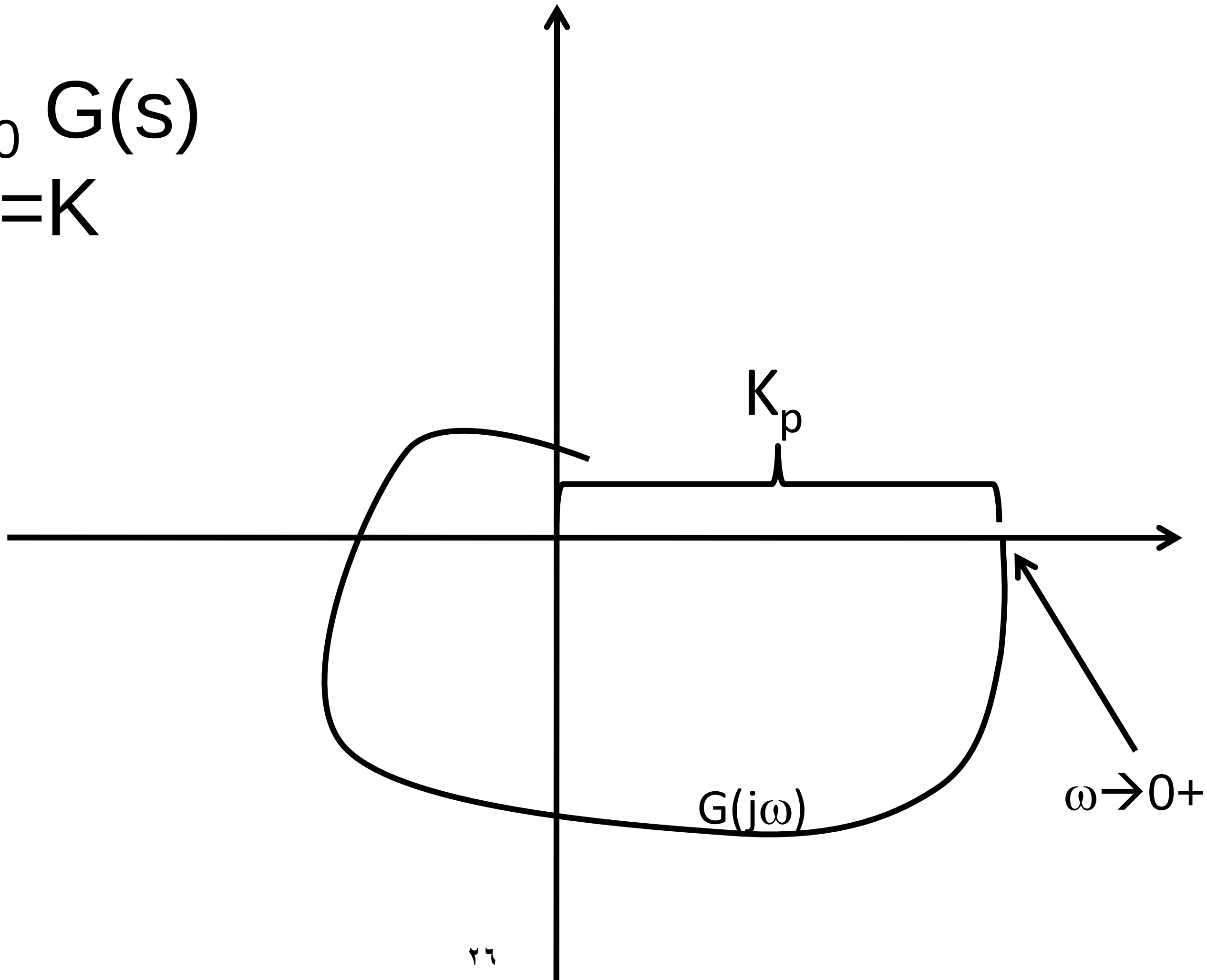


$$G(j\omega)=\frac{K(jT_a\omega+1)(jT_b\omega+1)\cdots}{(j\omega)^N(jT_1\omega+1)(jT_2\omega+1)\cdots}$$

As $\omega \rightarrow 0$ $\quad G(j\omega)\rightarrow \dfrac{K}{(j\omega)^N}$

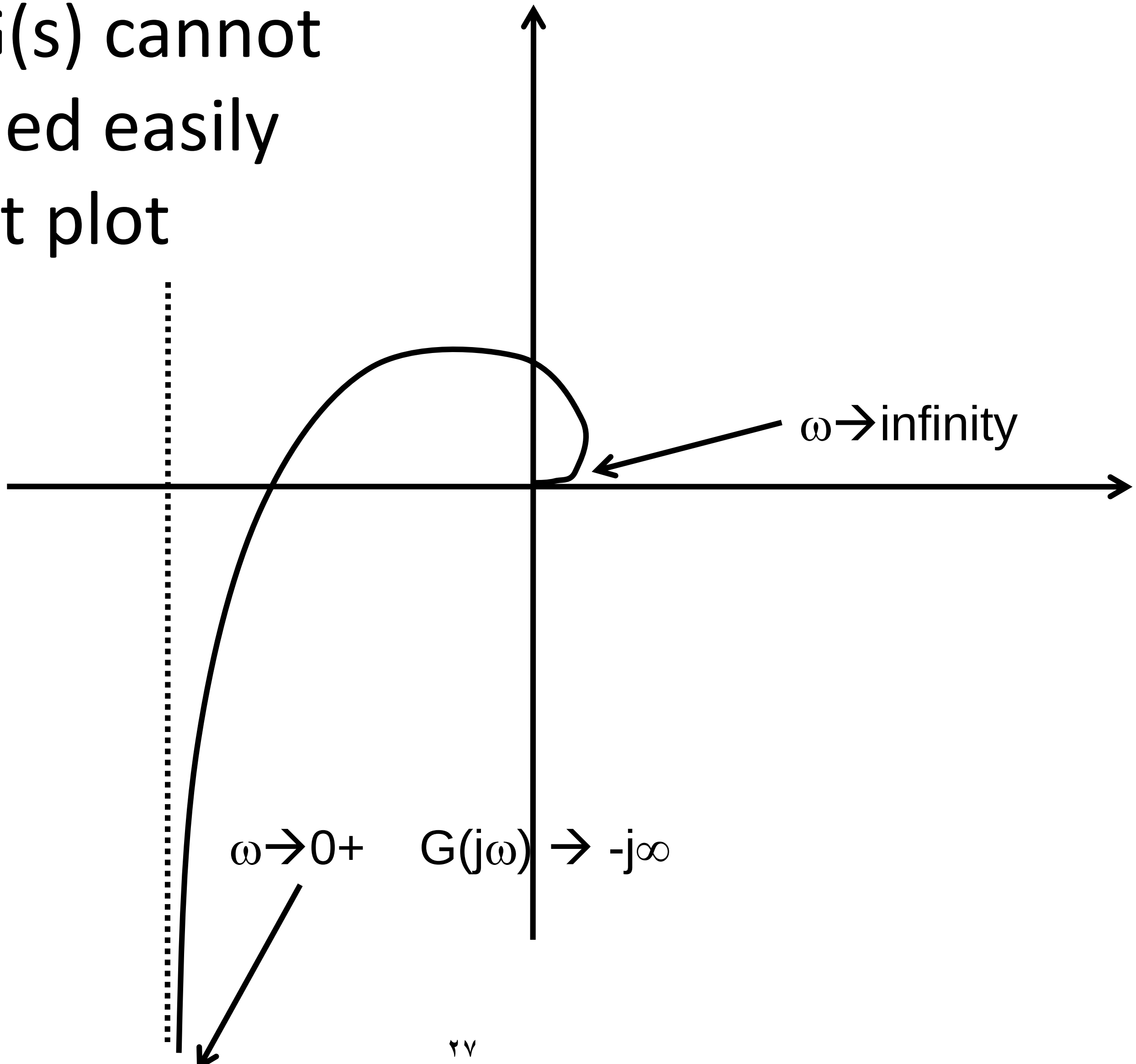
Nyquist Plot of Type 0 system, N=0

$K_p = \lim_{s \to 0} G(s)$
$= G(0) = K$



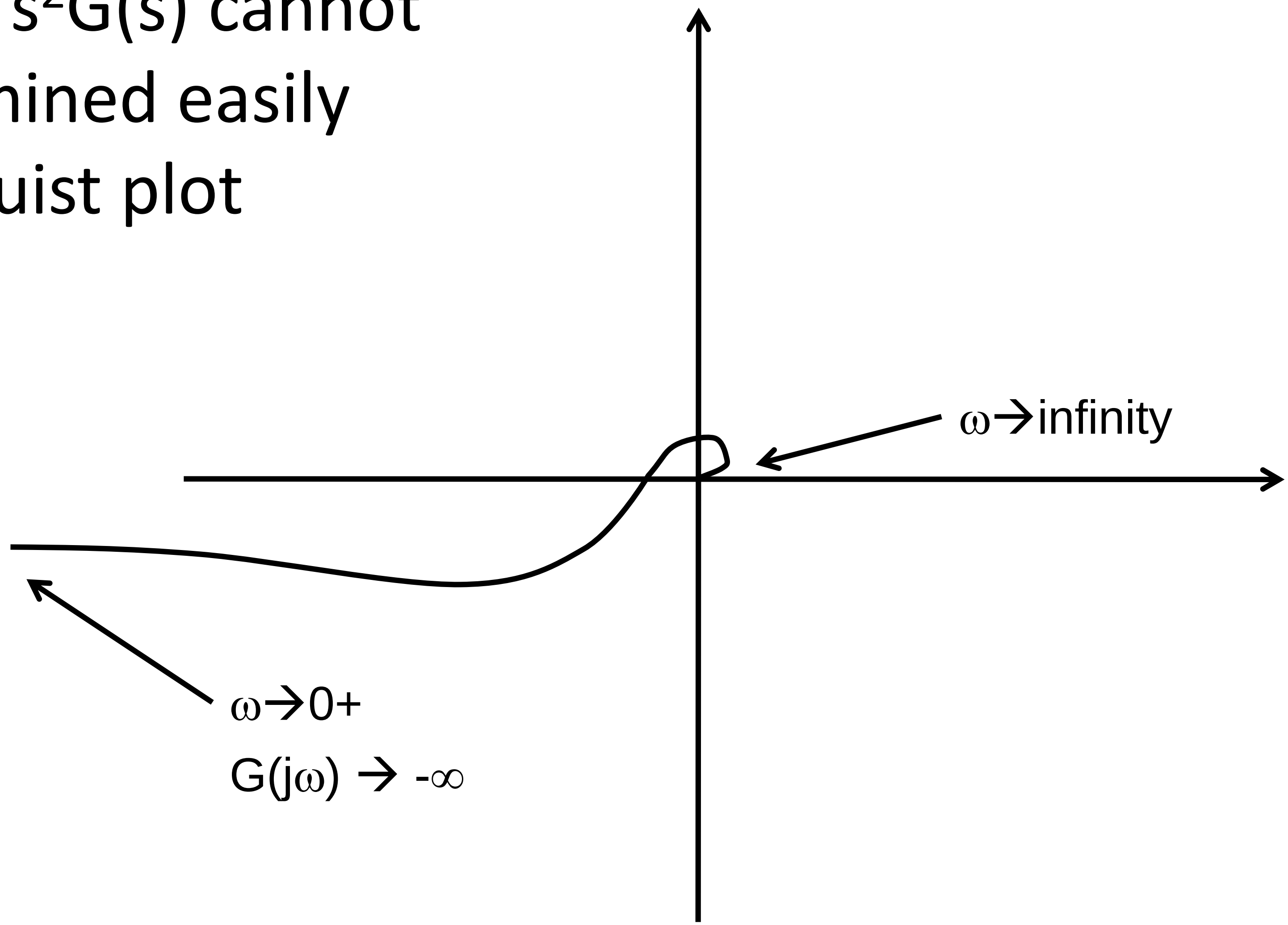
Nyquist Plot of Type 1 system, N=1

$K_v = \lim_{s \to 0} sG(s)$ cannot be determined easily from Nyquist plot

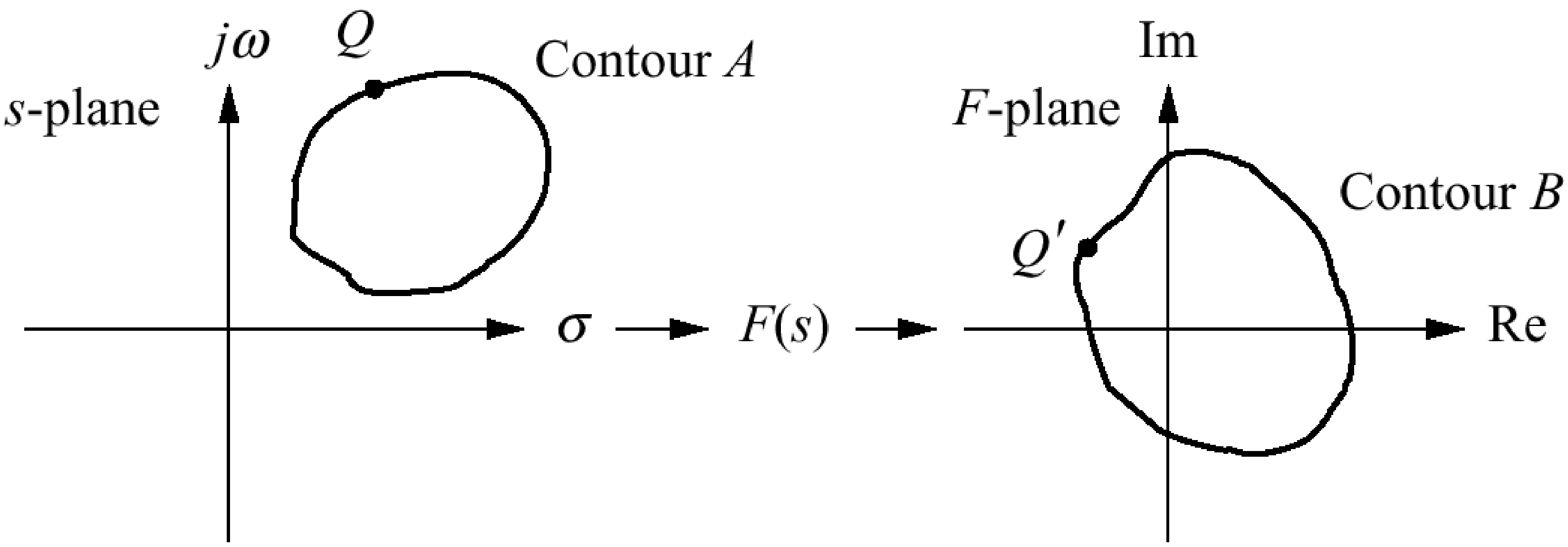


Nyquist Plot of Type 2 system, N=2

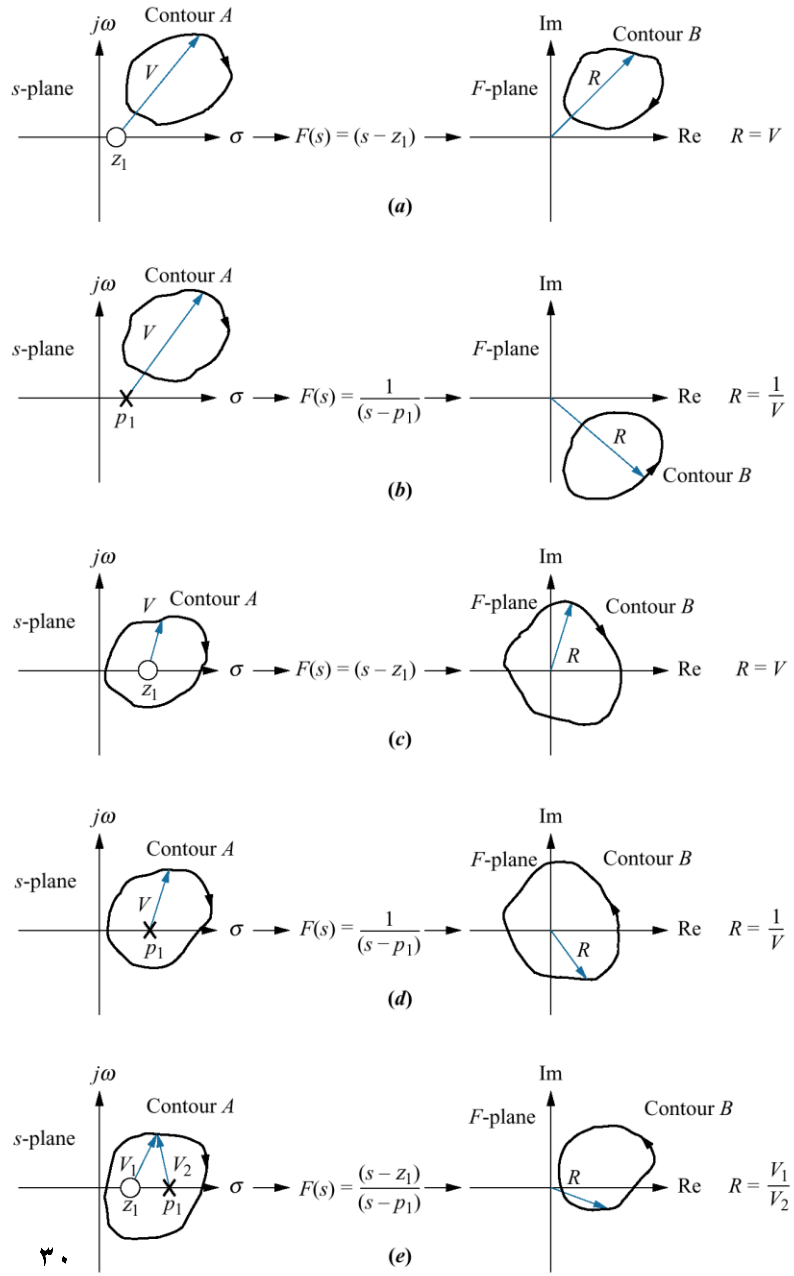
$K_a = \lim_{s \to 0} s^2 G(s)$ cannot be determined easily from Nyquist plot



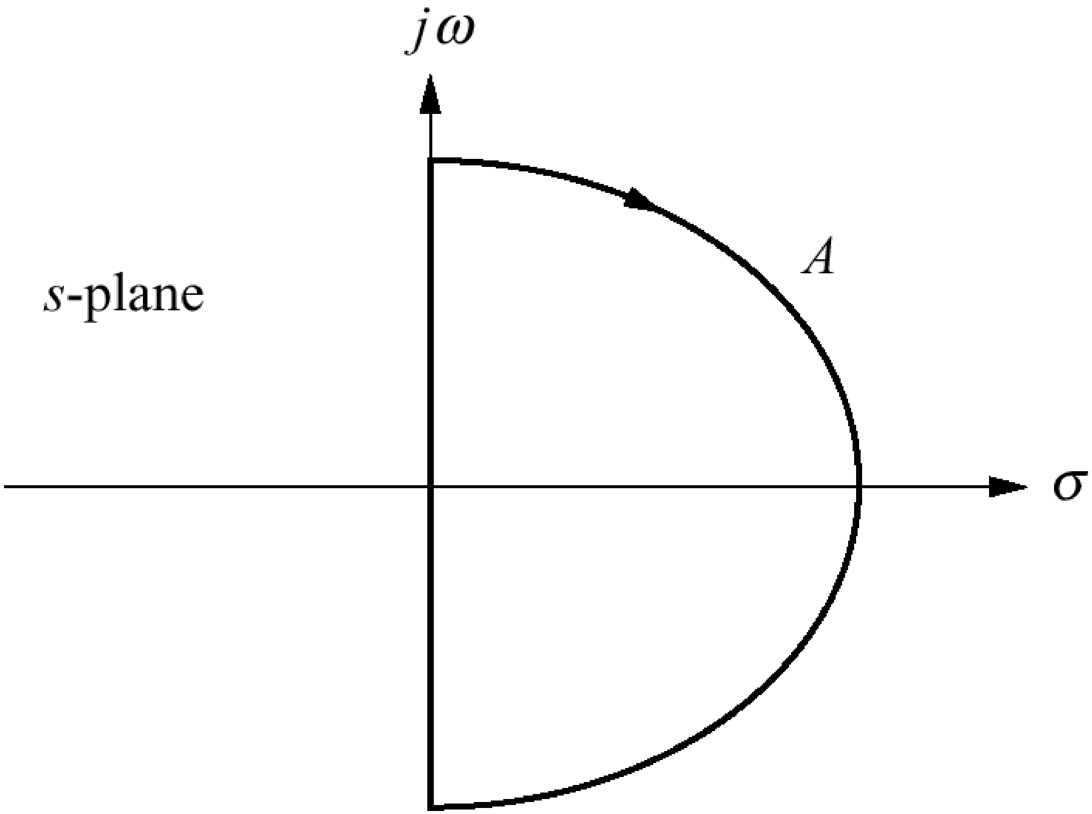
Mapping contour *A* through function *F*(*s*) to contour *B*



Examples of contour mapping

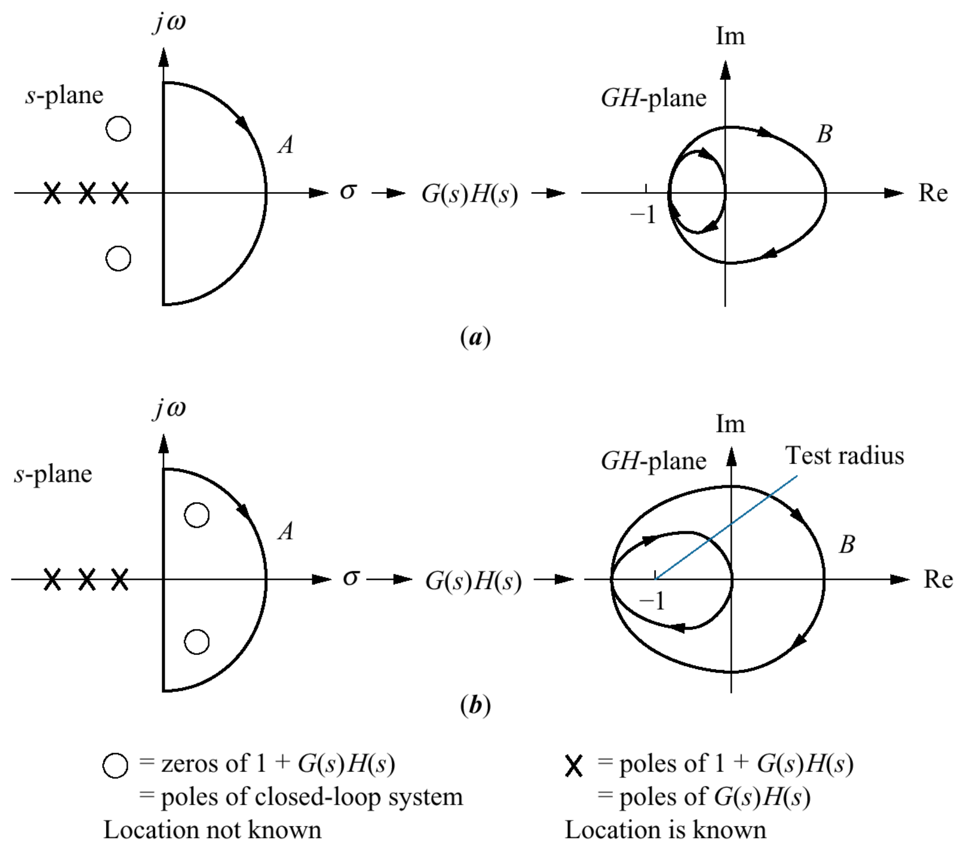


Contour enclosing right half-plane to determine stability



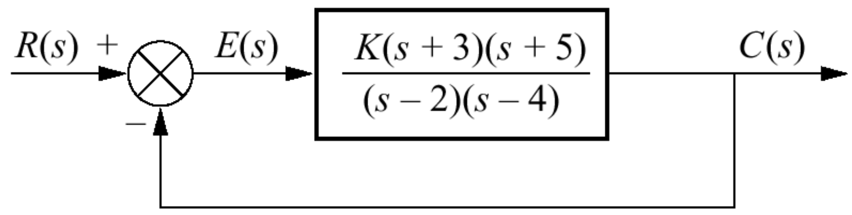
Mapping Examples

a. contour does not enclose closed-loop poles;
b. contour does enclose closed-loop poles

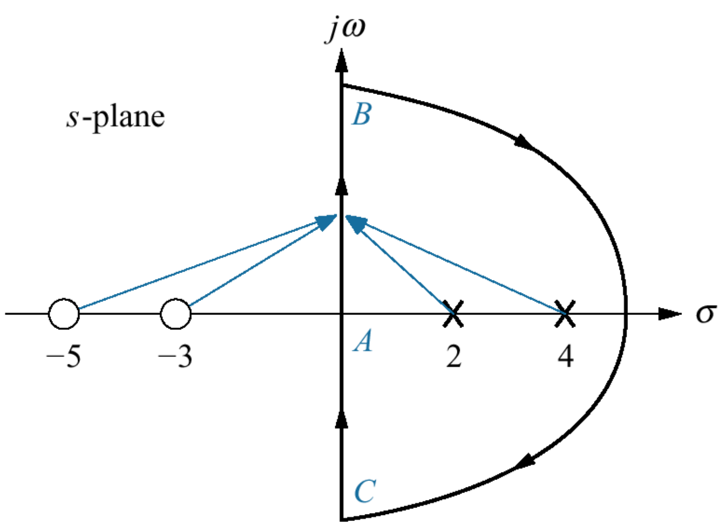


Demonstrating Nyquist Stability

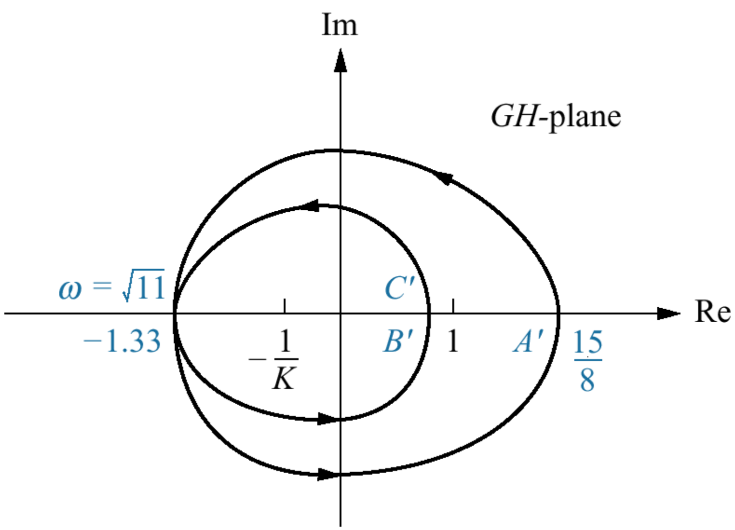
a. system;
b. contour;
c. Nyquist diagram



(a)



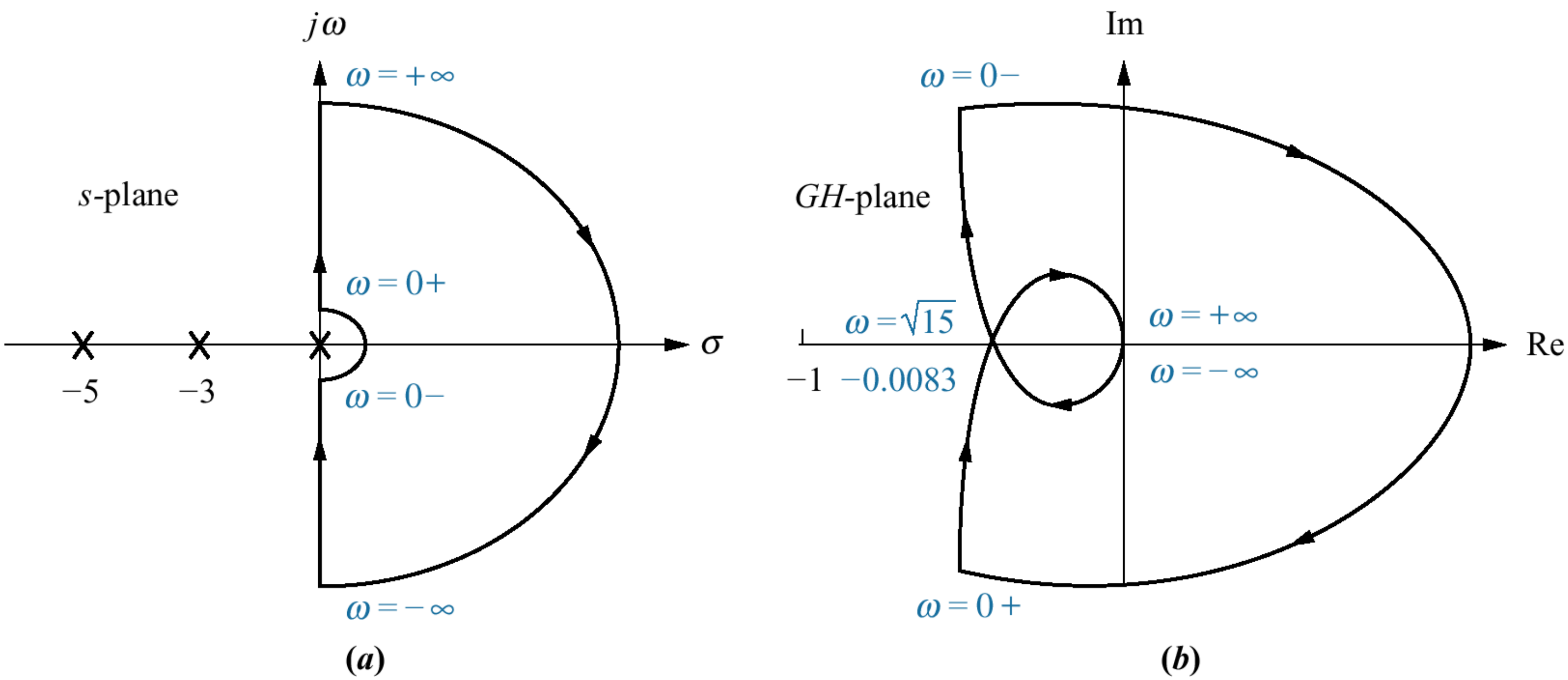
(b)



(c)

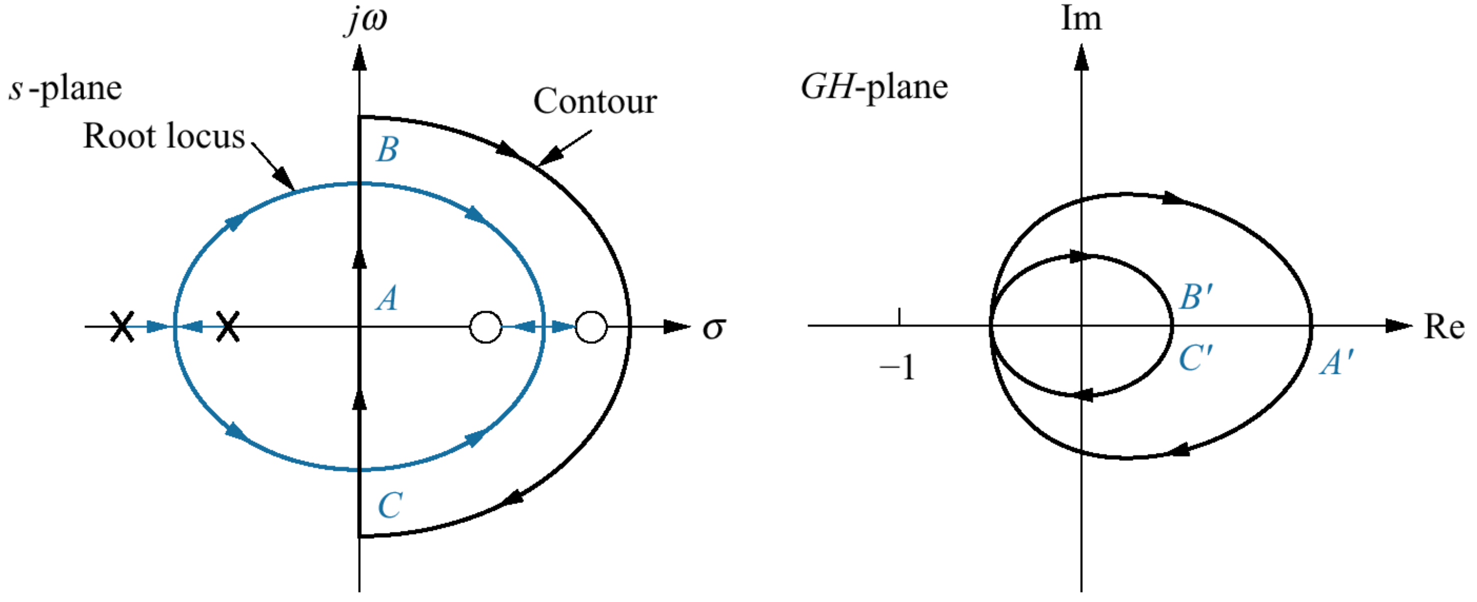
Demonstrating Nyquist Stability

a. Contour for Example 10.6;
b. Nyquist diagram



Demonstrating Nyquist Stability

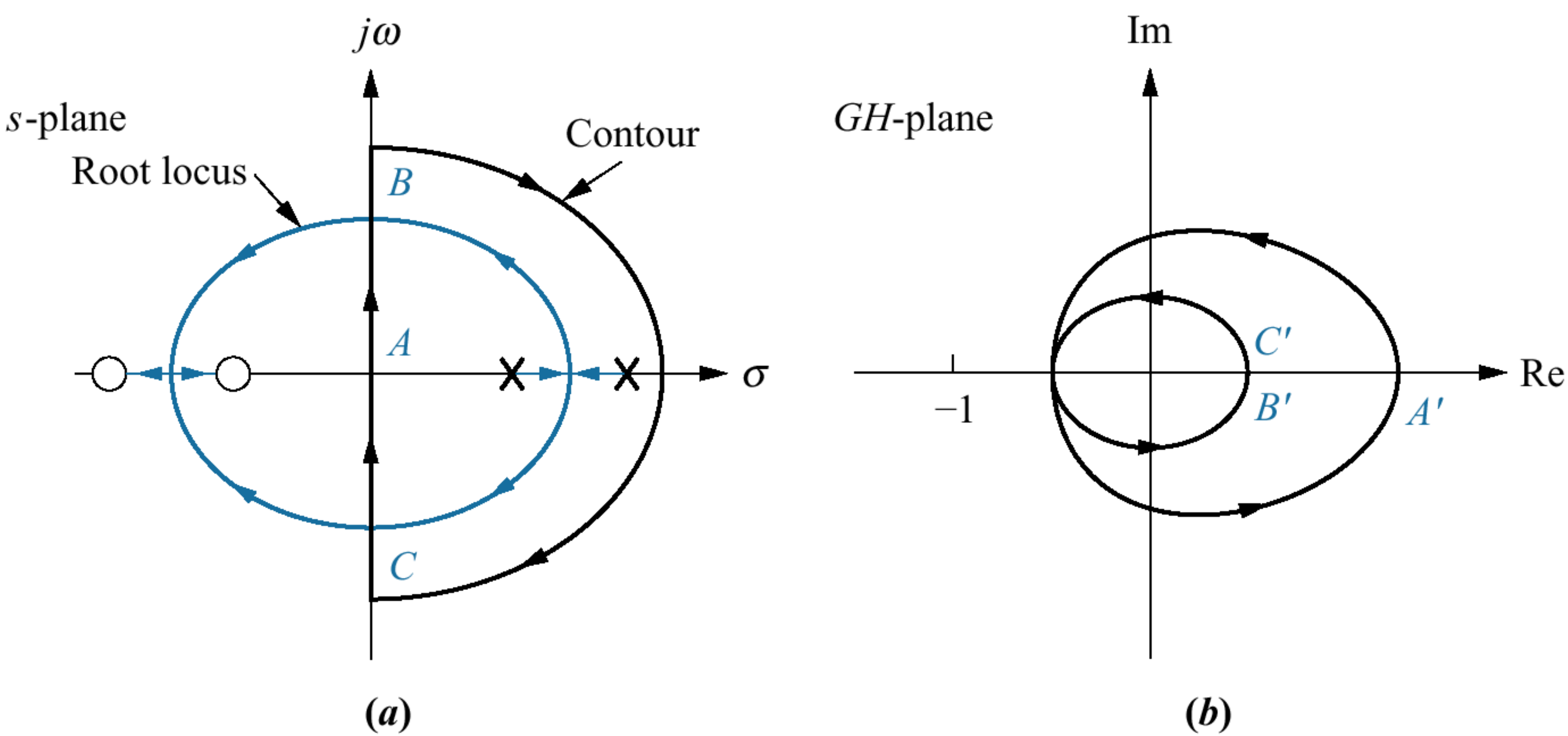
a. Contour and root locus of system that
is stable for small gain and unstable for large gain;
b. Nyquist diagram



(a) (b)

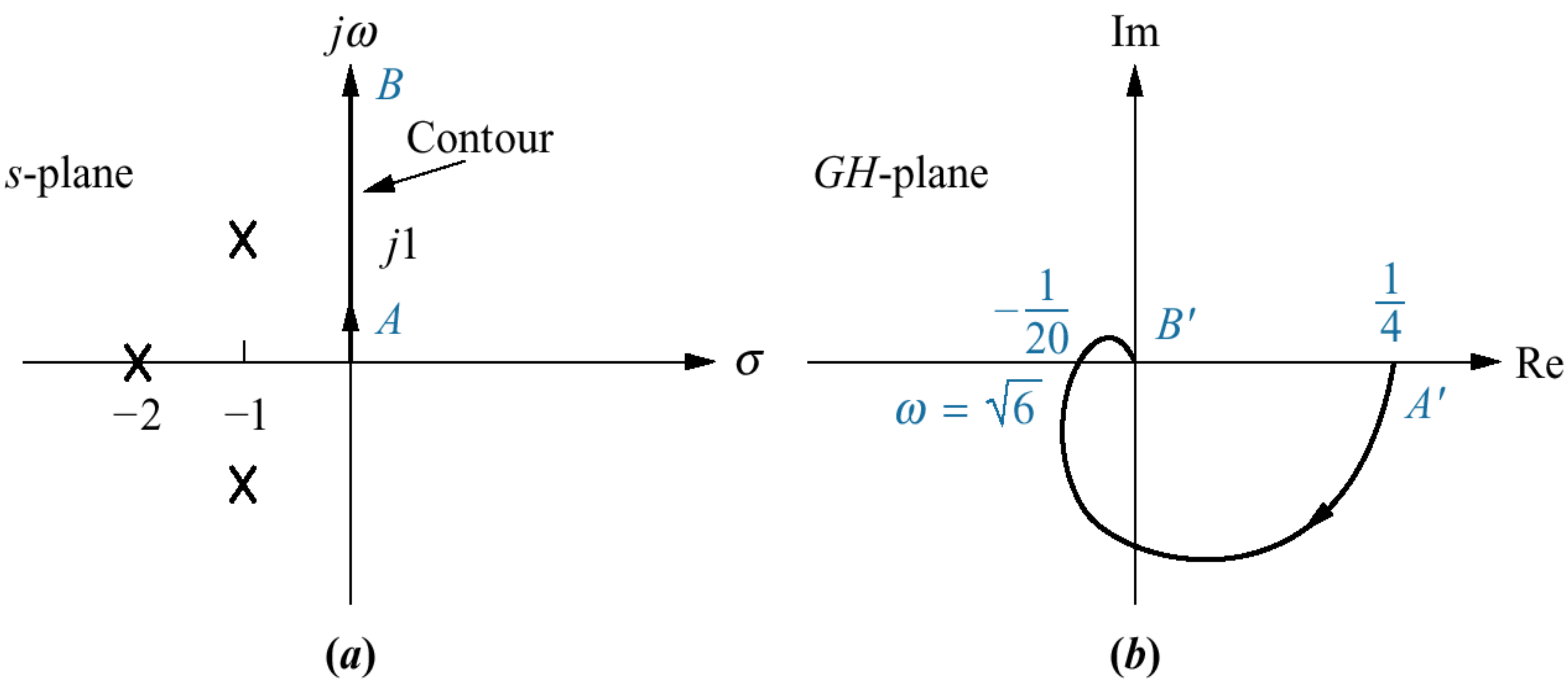
Demonstrating Nyquist Stability

a. Contour and root locus of system that is unstable for small gain and stable for large gain;
b. Nyquist diagram



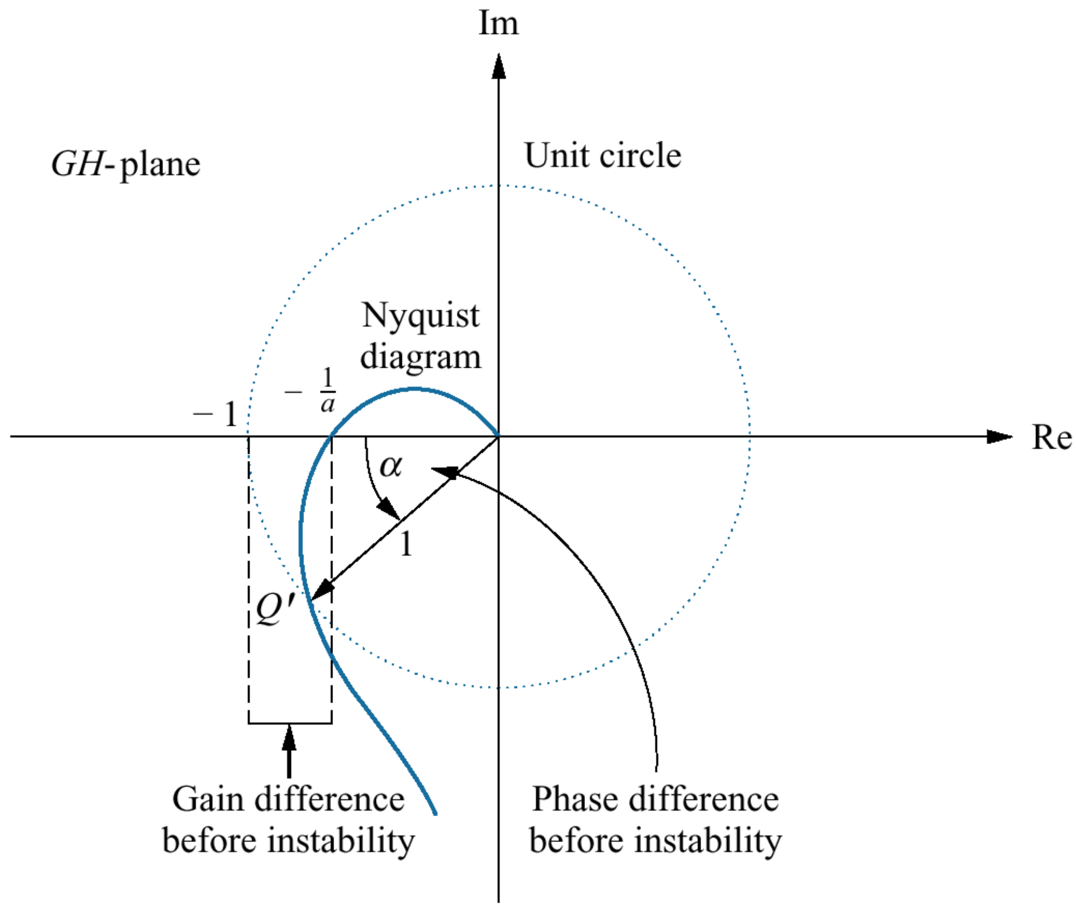
Demonstrating Nyquist Stability

a. Portion of contour to be mapped for Example 10.7;
b. Nyquist diagram of mapping of positive imaginary axis



Margins on Nyquist plot

Nyquist diagram showing gain and phase margins



Gain margin = G_M = 20 log a Phase margin = $\Phi_M = \alpha$

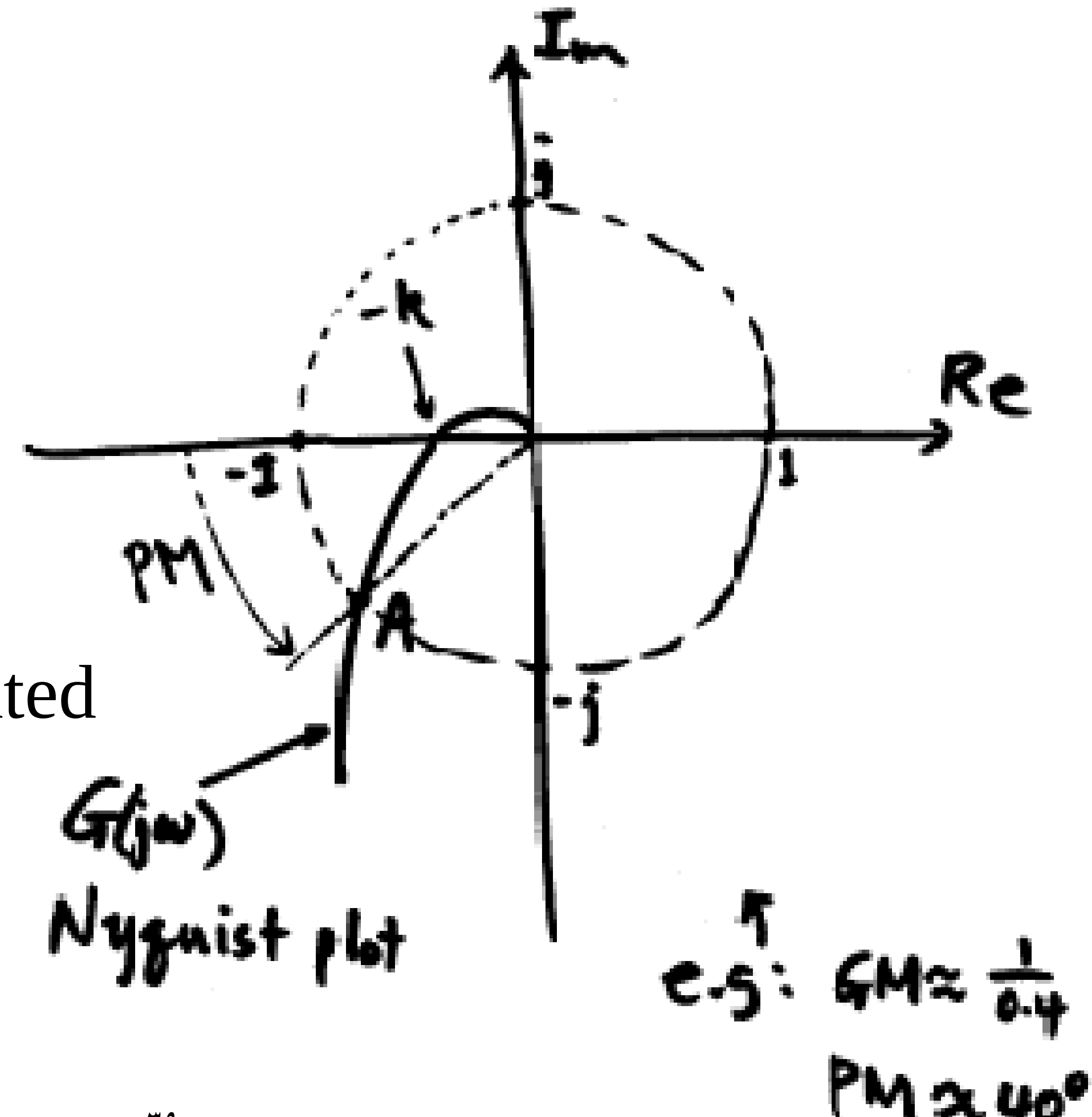
Margins on Nyquist plot

Suppose:

- Draw Nyquist plot $G(j\omega)$ & unit circle
- They intersect at point A
- Nyquist plot cross neg. real axis at $-k$

Then : PM = angle indicated

$$GM = 1/k \text{ in value}$$



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Stability using Nyquist Plots

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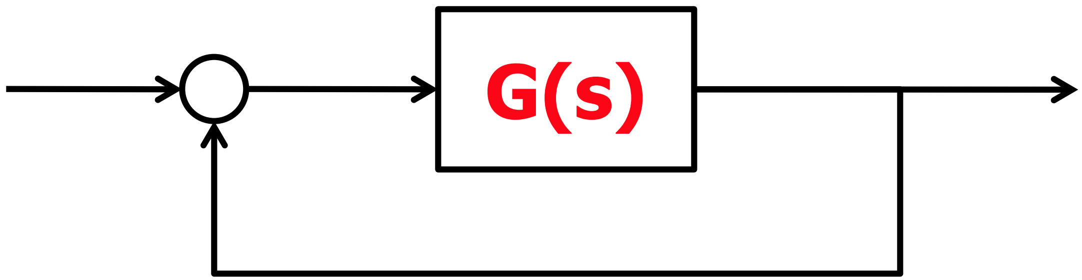
Stability from Nyquist plot



1. Get complete Nyquist plot
2. Obtain the # of encirclement of "-1"
3. # (unstable poles of closed-loop) Z
   = # (unstable poles of open-loop) P
   + # encirclement N

To have closed-loop stable:

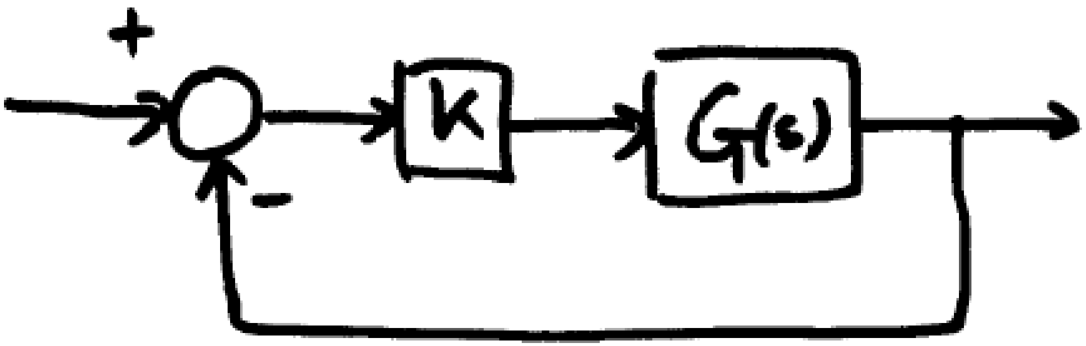
need $Z = 0$, i.e. $N = -P$

Stability from Nyquist plot

- Here we are counting only poles with positive real part as "unstable poles"
  - jω-axis poles are excluded
- Completing the NP when there are jω-axis poles in the open-loop TF G(s):
  - If $j\omega_o$ is a non-repeated pole, NP sweeps 180 degrees in clock-wise direction as ω goes from ω_o- to ω_o+.
  - If $j\omega_o$ is a double pole, NP sweeps 360 degrees in clock-wise direction as ω goes from ω_o- to ω_o+.

Stability from Nyquist plot

Example:



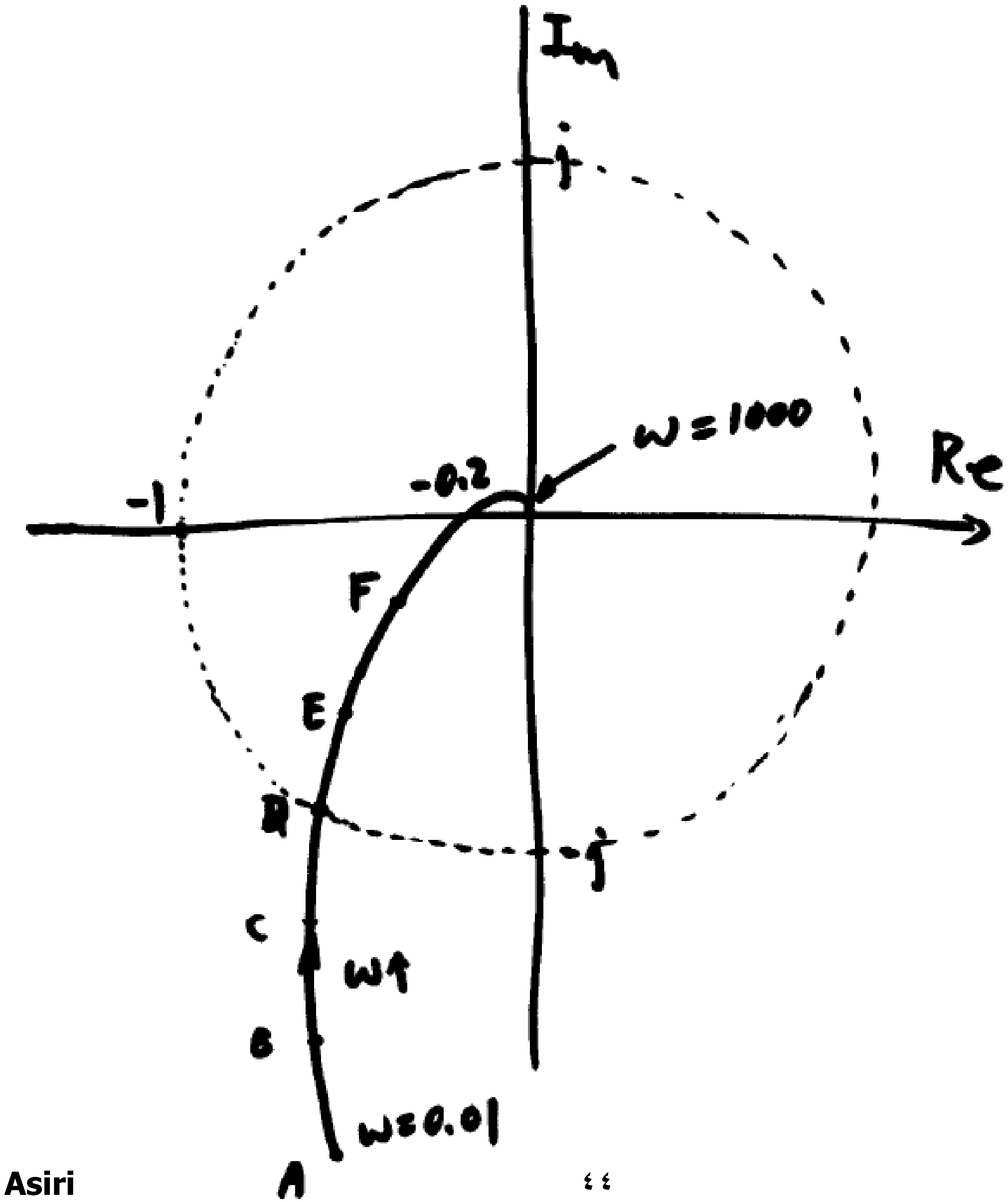
Given:

1. $G(s)$ is stable
2. With $K = 1$, performed open-loop sinusoidal tests, and $G(j\omega)$ is on next page

Q: 1. Find stability margins

2. Find Nyquist criterion to determine closed-loop stability

Stability from Nyquist plot



Stability from Nyquist plot

Solution:

1. Where does $G(j\omega)$ cross the unit circle?

   ____

   $\therefore$ Phase margin $\approx$ ________

   Where does $G(j\omega)$ cross the negative real axis? ________

   $\therefore$ Gain margin $\approx$ ________

   Is closed-loop system stable with K = 1? ________

Stability from Nyquist plot

Note that the total loop T.F. is $KG(s)$.

If K is not = 1, Nyquist plot of $KG(s)$ is a scaling of $G(j\omega)$.

e.g. If $K = 2$, scale $G(j\omega)$ by a factor of 2 in all directions.

Q: How much can K increase before GM becomes lost? ________

How much can K decrease? _______

Stability from Nyquist plot

$\therefore$ Some people say the gain margin is 0 to 5 in this example

Q: As K is increased from 1 to 5, GM is lost, what happens to PM?

What's the max PM as K is reduced to 0 and GM becomes ∞?

Stability from Nyquist plot

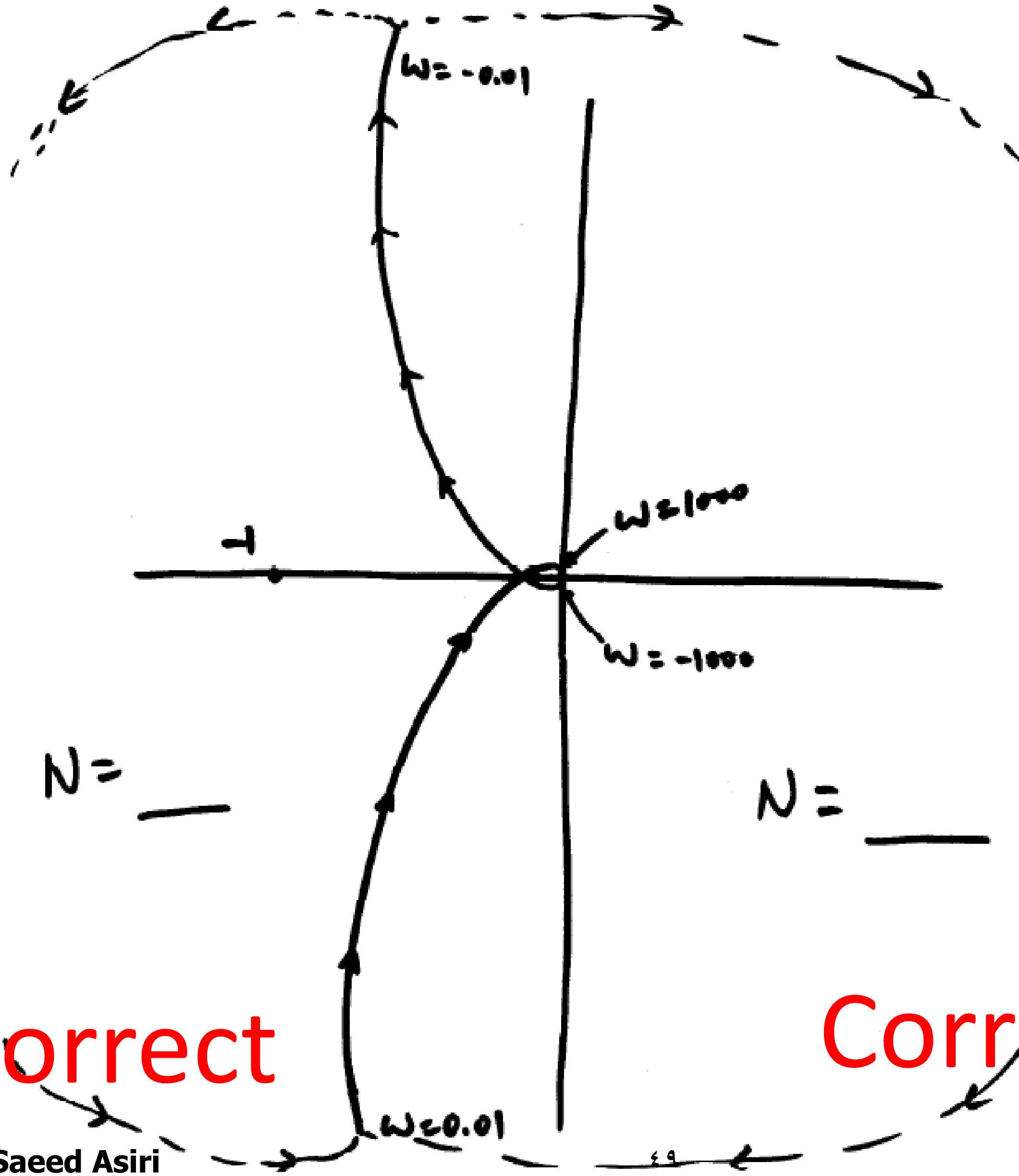
2. To use Nyquist criterion, need complete Nyquist plot.
   a) Get complex conjugate
   b) Connect $\omega = 0^{-}$ to $\omega = 0^{+}$ through an infinite circle
   c) Count # encirclement N
   d) Apply: $Z = P + N$

o.l. stable, $\therefore P =$ ________

$\therefore Z =$ ________

$\therefore$ c.l. stability: ________

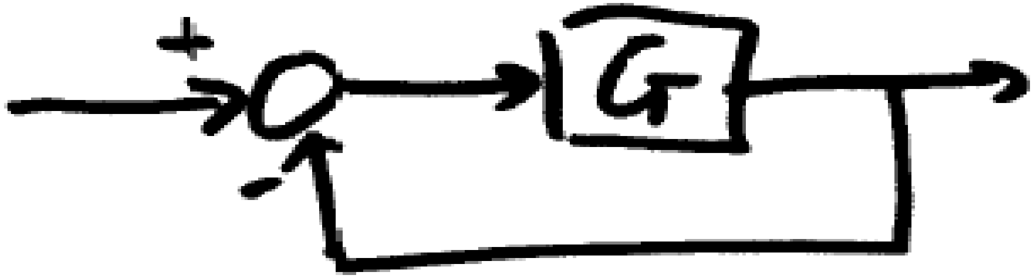
Stability from Nyquist plot



Incorrect

Correct

Stability from Nyquist plot



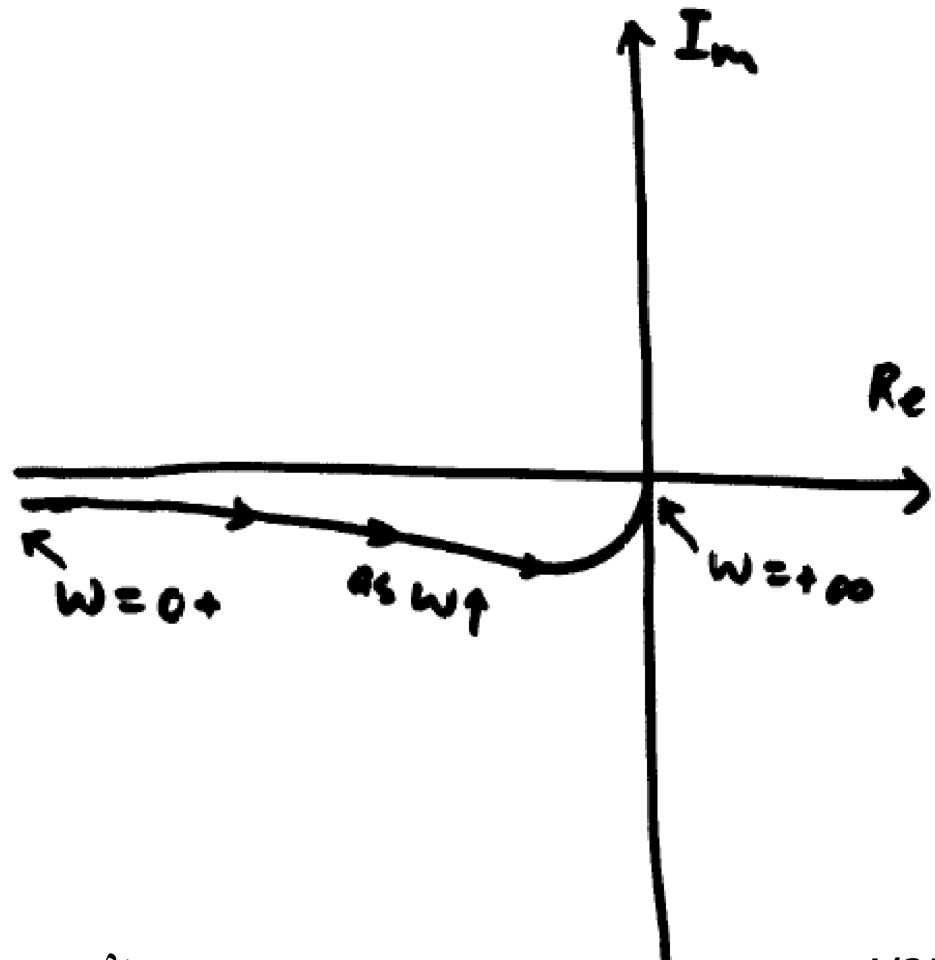
Example:

$G(s)$ stable, $P = 0$

$G(j\omega)$ for $\omega > 0$ as given.

1. Get $G(j\omega)$ for $\omega < 0$ by conjugate
2. Connect $\omega = 0^-$ to $\omega = 0^+$.

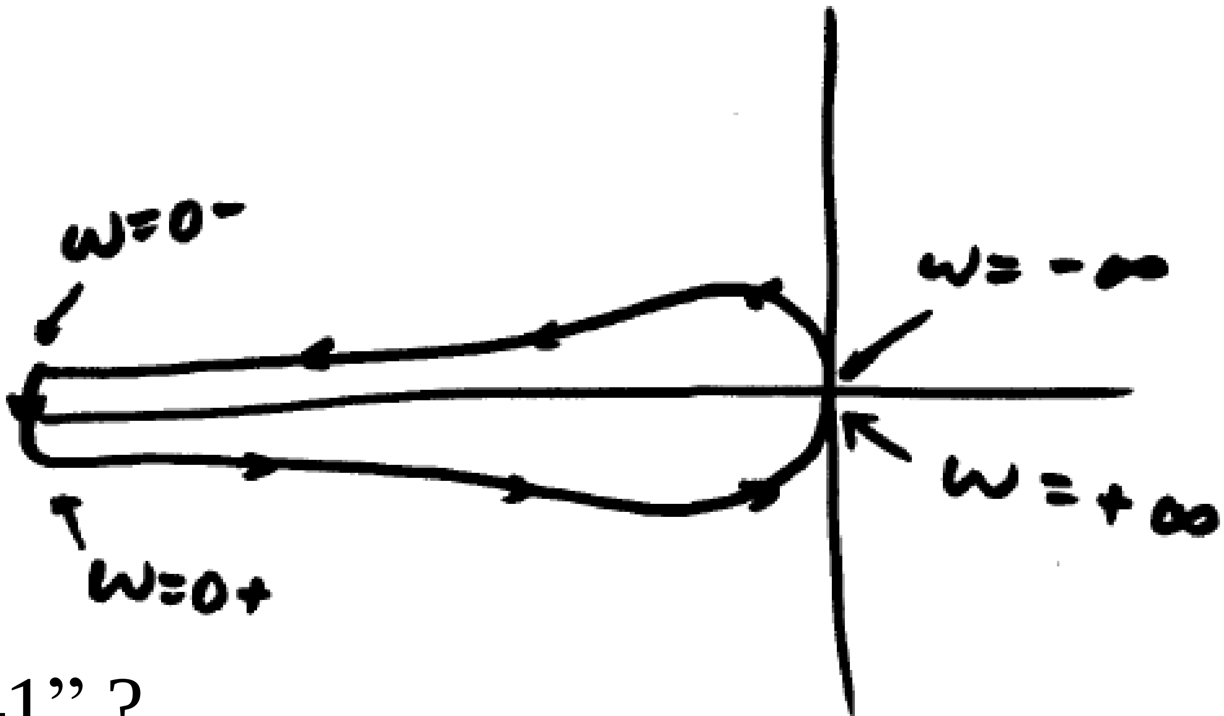
   But how?



Stability from Nyquist plot

Choice a) :

Incorrect



Where's "–1" ?

∴ # encirclement $N =$ ________

∴ $Z = P + N =$ ________

Make sense? ________

Stability from Nyquist plot

Choice b) :

Where is

“–1” ?

encir.

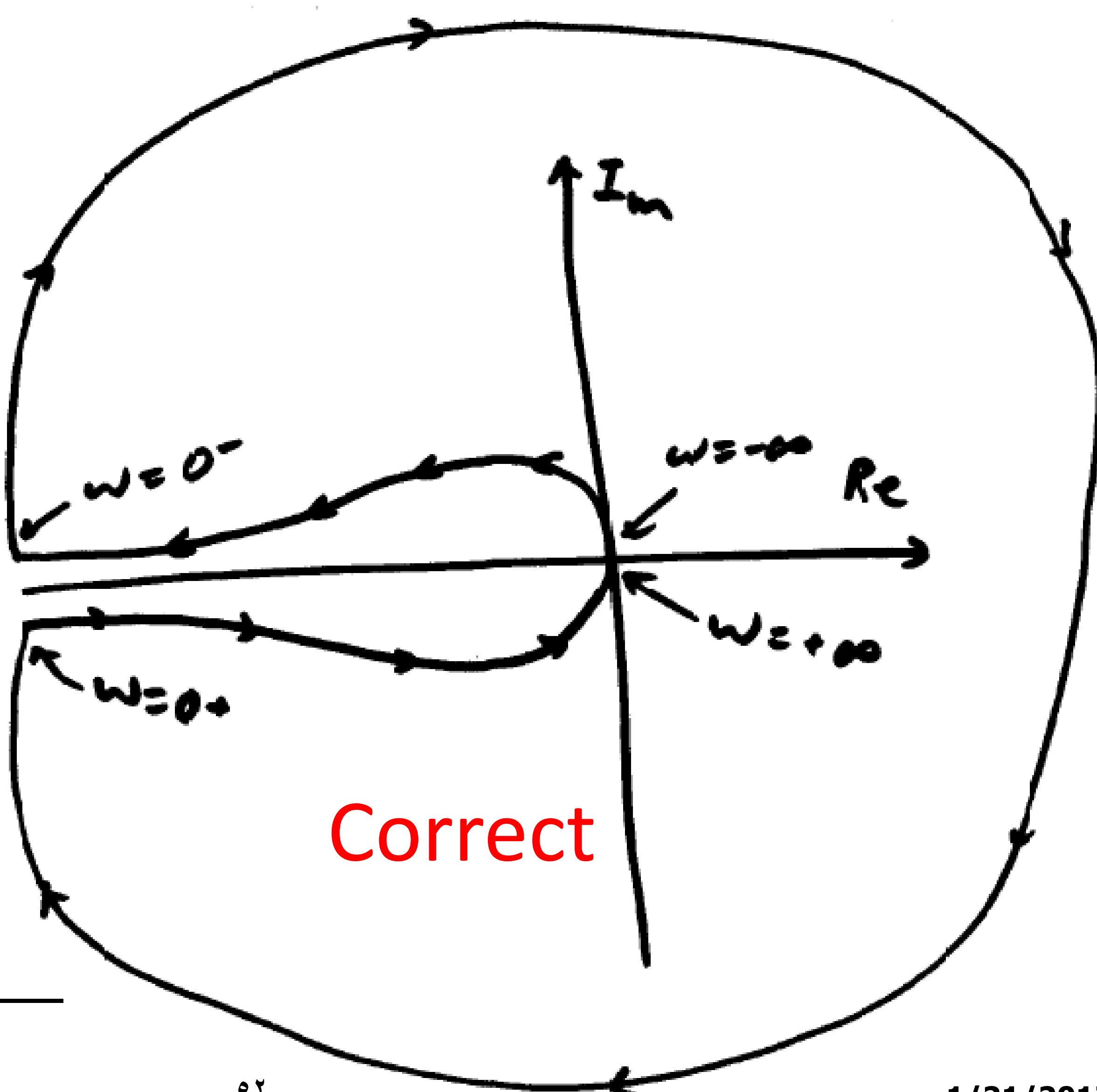
$N =$ ______

$Z = P + N$

$=$ ________

closed-loop

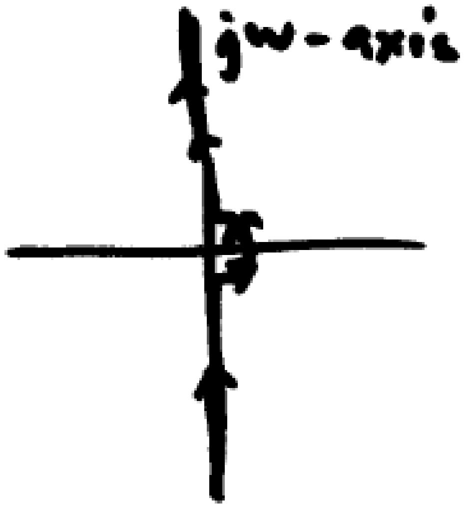
stability ________



Stability from Nyquist plot

Note: If $G(j\omega)$ is along –Re axis to ∞ as $\omega \to 0^+$, it means $G(s)$ has $\frac{1}{s^2}$ in it.

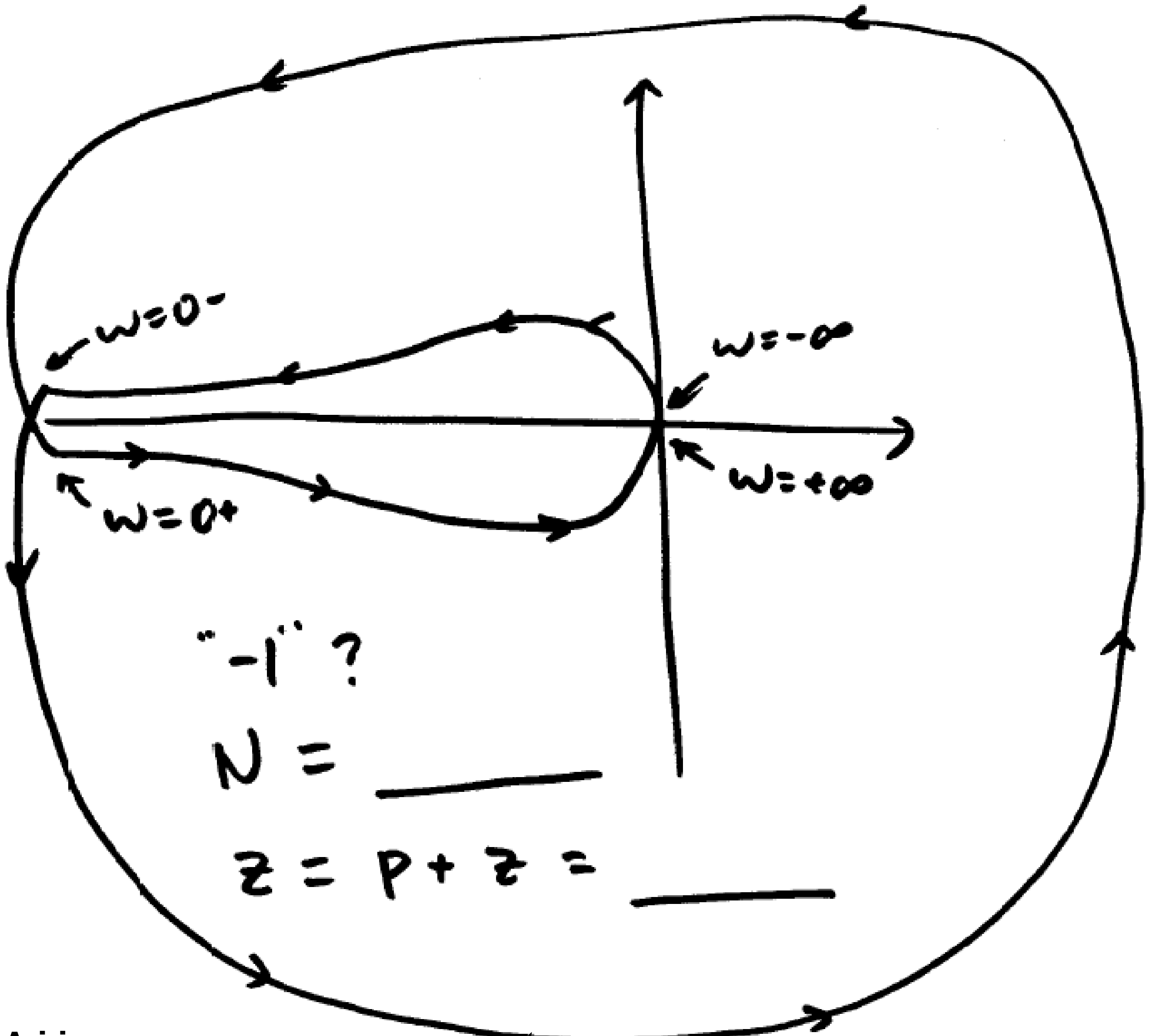
$\therefore$ when s makes a half circle near $\omega = 0$, $G(s)$ makes a full circle near ∞.



$\therefore$ choice a) is impossible, but choice b) is possible.

Stability from Nyquist plot

Incorrect

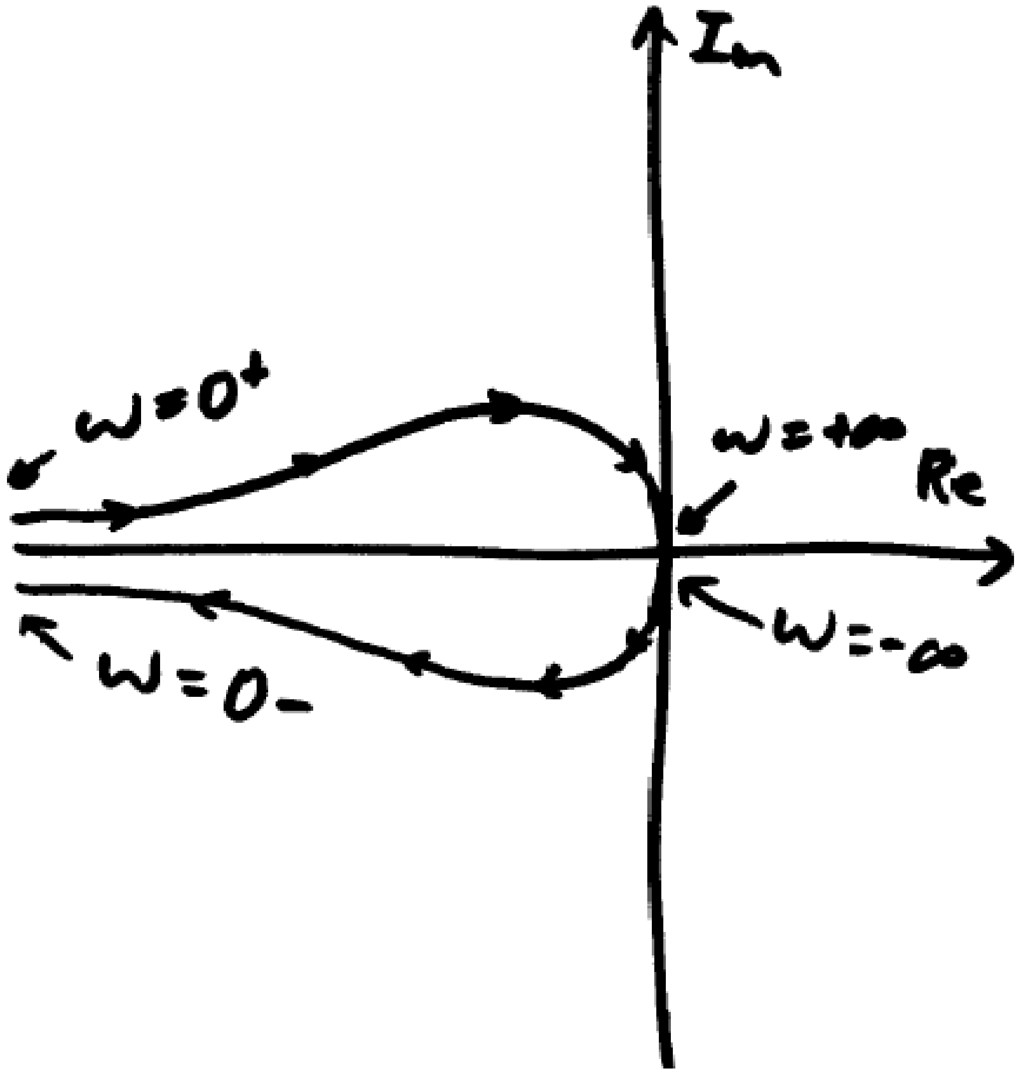


Stability from Nyquist plot

Example: $G(s)$

1. Get conjugate for $\omega < 0$
2. Connect $\omega = 0^-$ to $\omega = 0^+$.

Needs to go one full circle with radius ∞. Two choices.



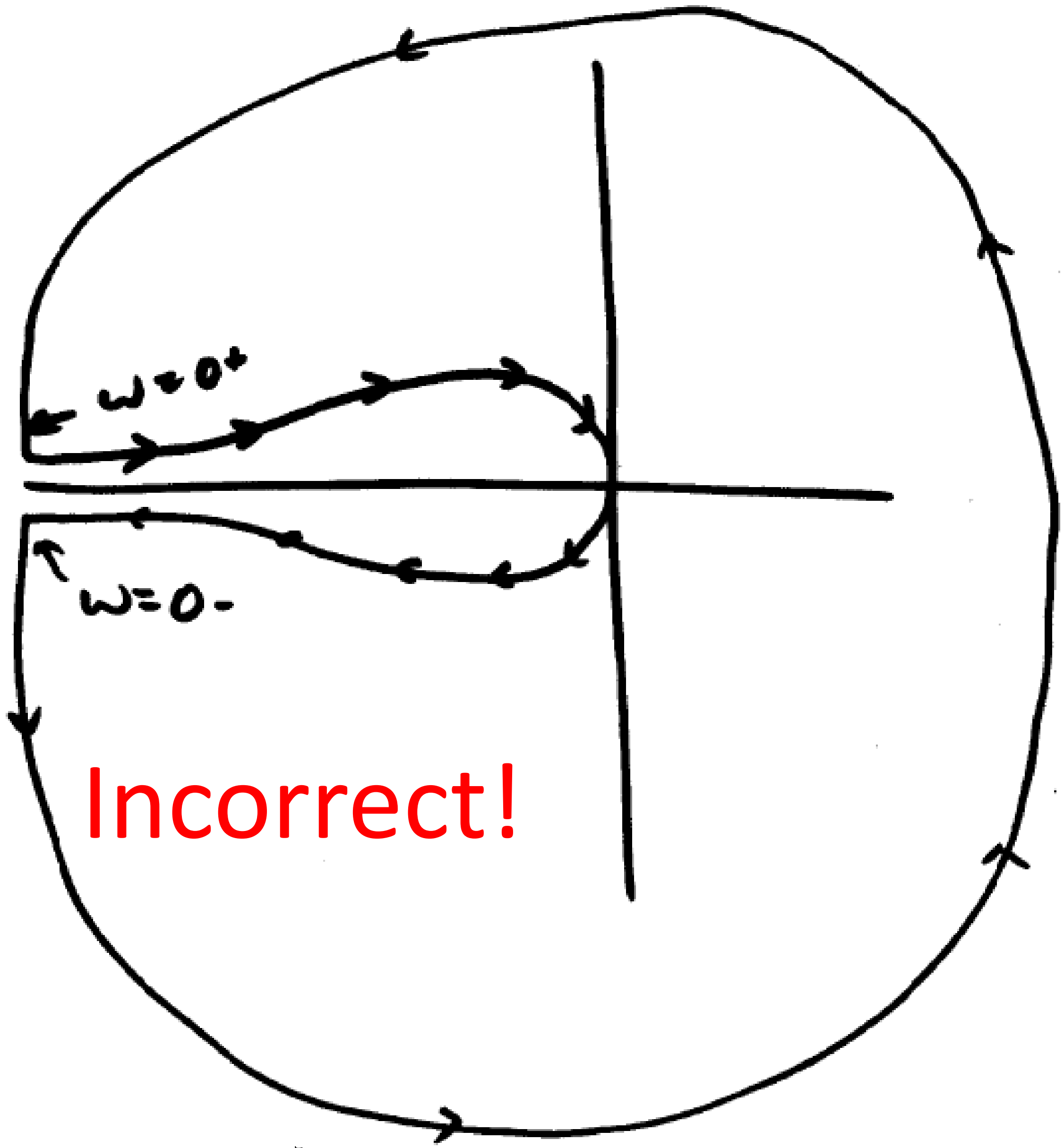
Stability from Nyquist plot

Choice a) :

$N = 0$

$Z = P + N = 0$

closed-loop stable



Stability from Nyquist plot

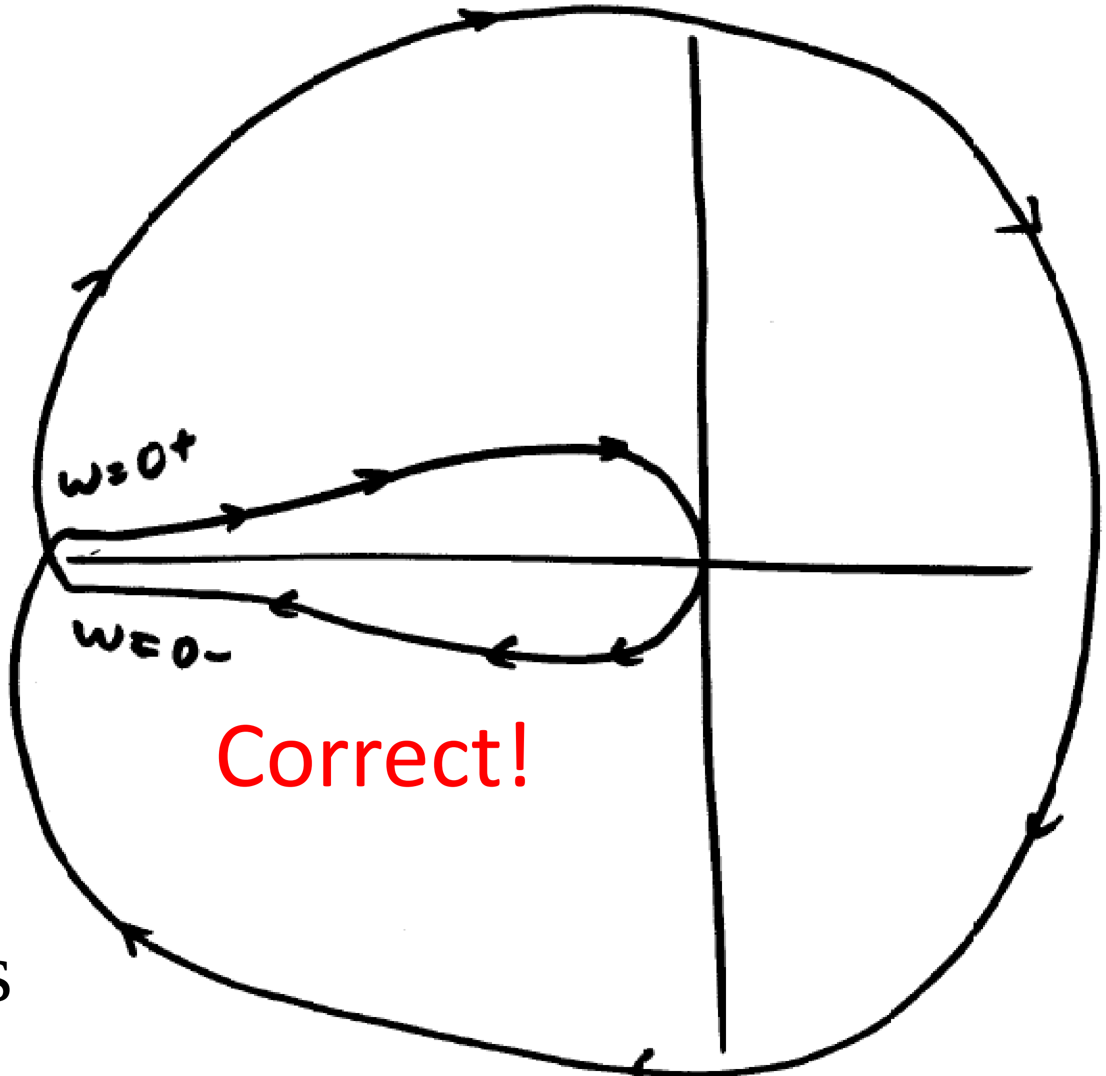
Choice b) :

$N = 2$

$Z = P + N$

$= 2$

Closed loop has two unstable poles



Stability from Nyquist plot

Which way is correct?

near $s=0$, $G(s) \approx \frac{K_0}{s^2}$ in this case

$\approx \frac{K_0}{s^N}$ in general

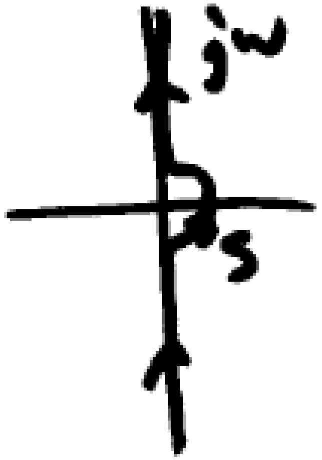
For stable & non-minimum phase systems,

$$K_0 > 0$$

when s circles in c.c.w.

$\frac{1}{s}$ circles in c.w.

$\therefore G(s)$ circles in c.w.



Stability from Nyquist plot

Example: $G(s)$ has one unstable pole

$\therefore P = 1$, no unstable zeros

1. Get conjugate
2. Connect

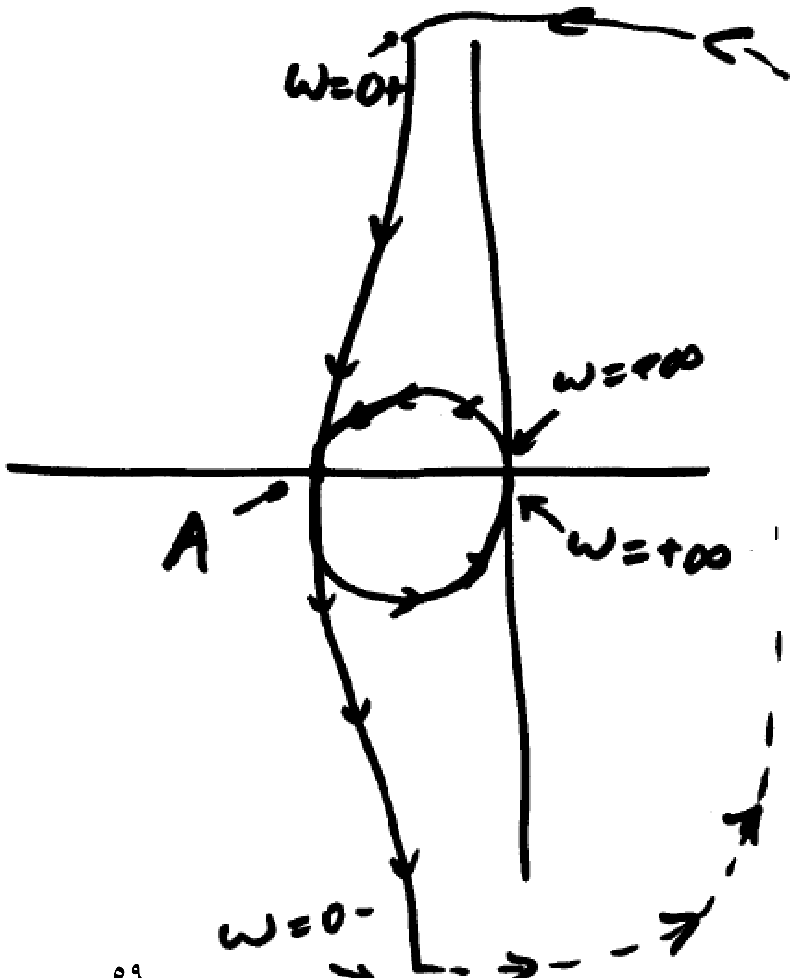
   $\omega = 0^-$

   to $\omega = 0^+$.

   How?

   One unstable pole/zero

   If connect in c.c.w.



Stability from Nyquist plot

\# encirclement $N = ?$

If "–1" is to the left of A

i.e. $A > -1$

then $N = 0$

$$Z = P + N = 1 + 0 = 1$$

but if a gain is increased, "–1" could be inside,

$$N = -2$$

$$Z = P + N = -1$$

$\therefore$ c.c.w. is impossible

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Stability from Nyquist plot

If connect c.w.:

For $A > -1$

$N =$ ______

$Z = P + N$

$=$ ______

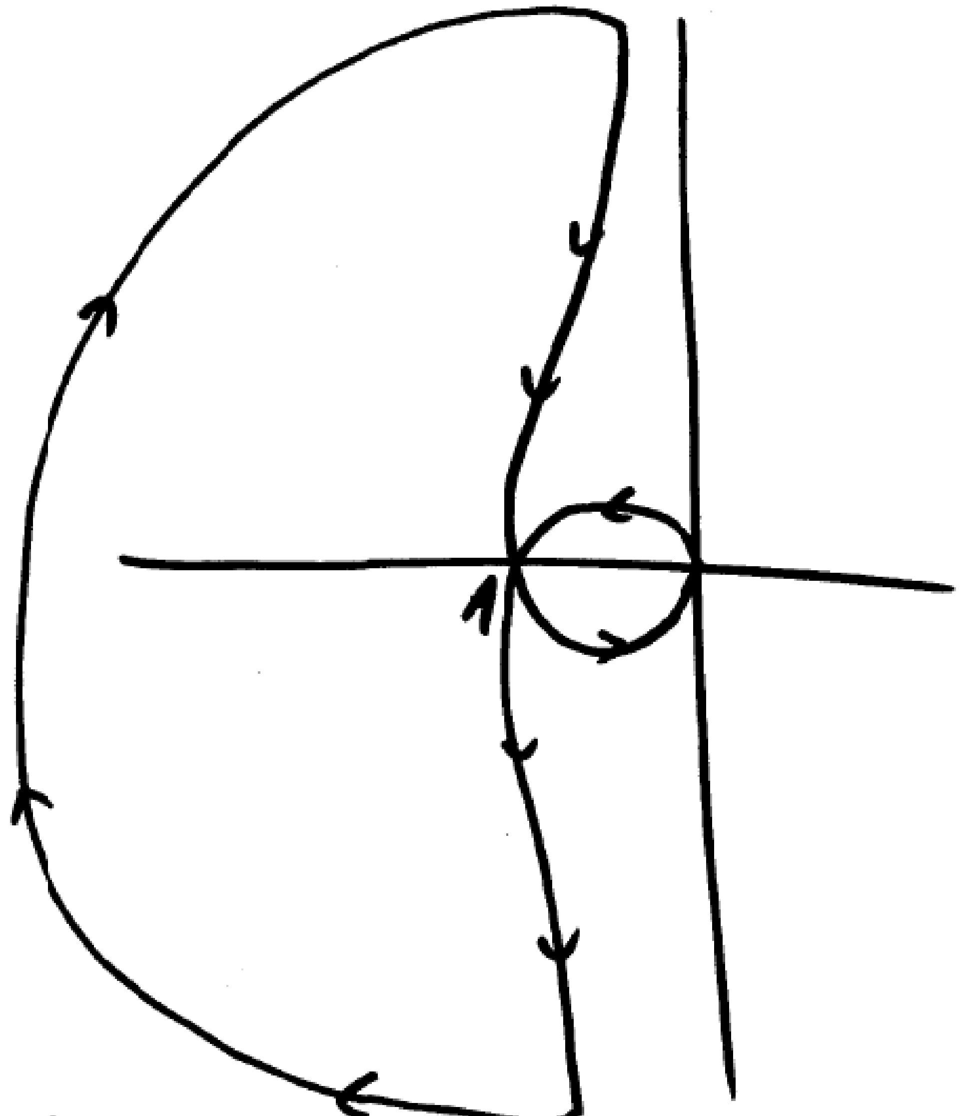
For $A < -1$

$N =$ ______

$Z =$ ______

No contradiction.

$\therefore$ This is the correct way.



Stability from Nyquist plot

Example: $G(s)$ stable, minimum phase

$$P = 0$$

$G(j\omega)$ as given:

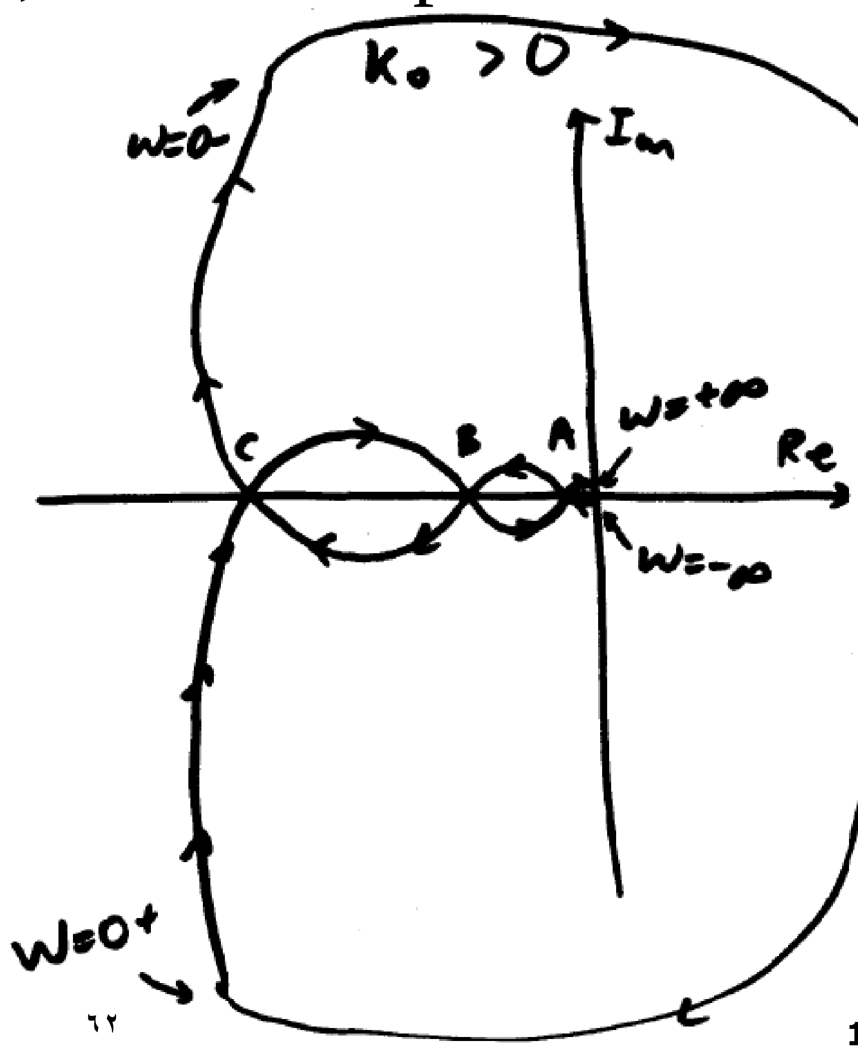
get conjugate.

Connect $\omega = 0^-$

to $\omega = 0^+$,

$\because K_0 > 0$

$\therefore$ c.w. direction



Stability from Nyquist plot

If $A < -1 < 0$:

$N =$ ______

$Z = P + N =$ ______

stability of c.l. : ______

If $B < -1 < A$: A=-0.2, B=-4, C=-20

$N =$ ______

$Z = P + N =$ ______

closed-loop stability: ______

Gain margin: gain can be varied between (-1)/(-0.2) and (-1)/(-4),

or can be less than (-1)/(-20)

Stability from Nyquist plot

If $C < -1 < B$:

$$N = \underline{\qquad\qquad}$$

$$Z = P + N = \underline{\qquad\qquad}$$

closed-loop stability: ______

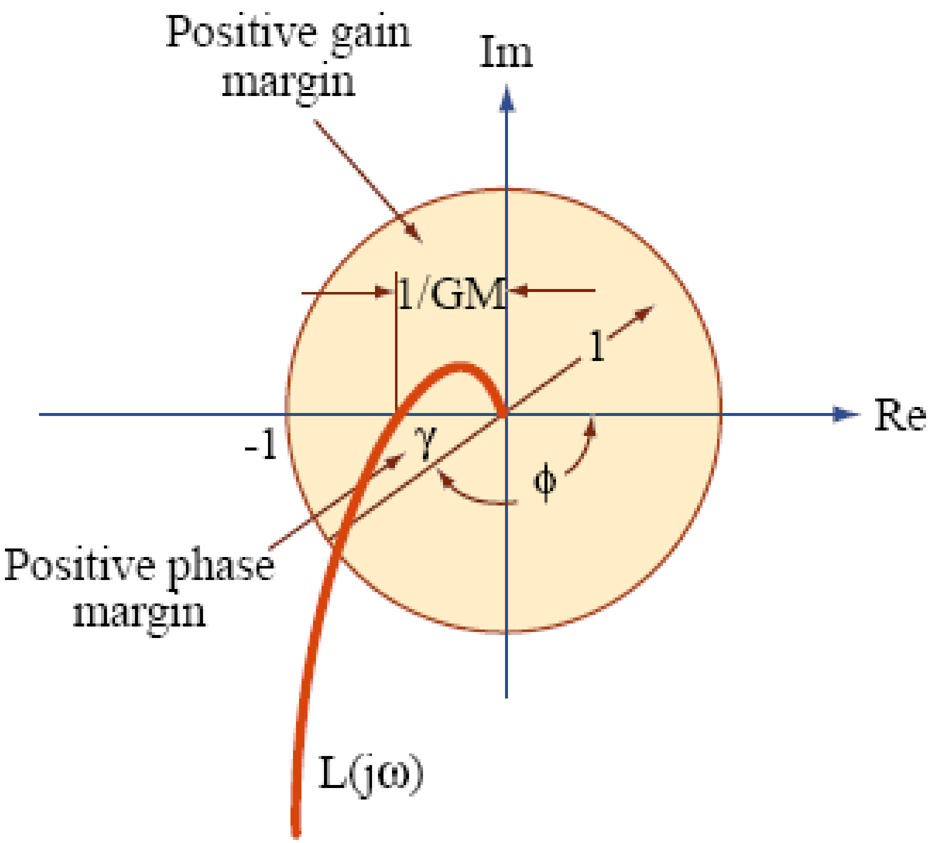
If $-1 < C$:

$$N = \underline{\qquad\qquad}$$

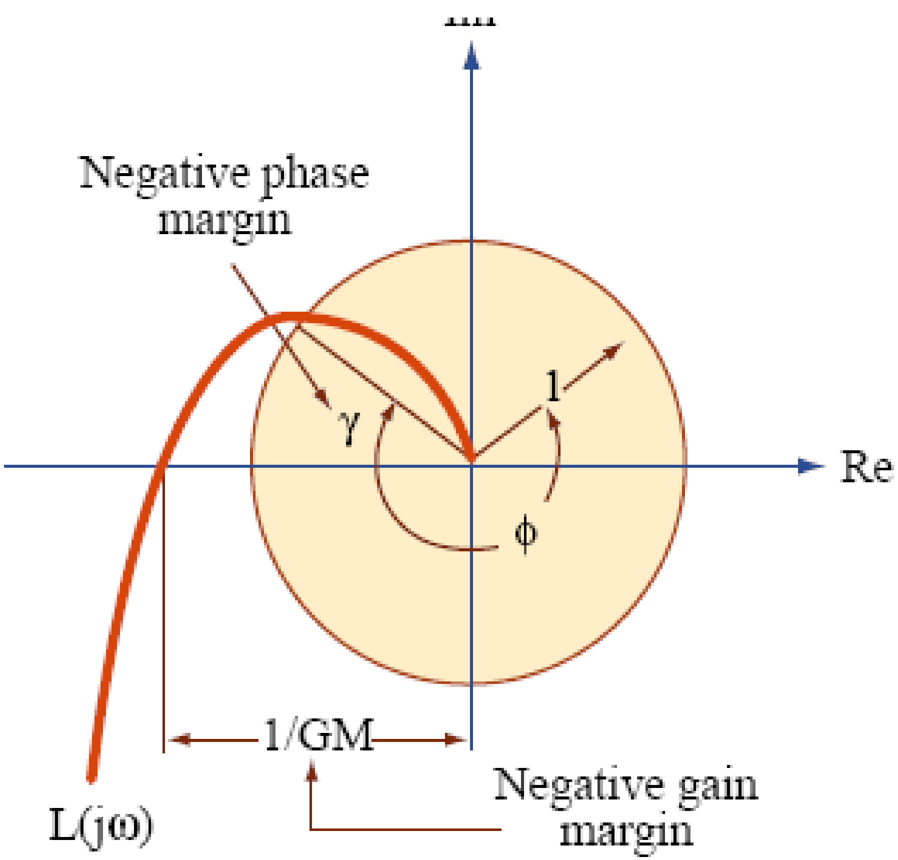
$$Z = P + N = \underline{\qquad\qquad}$$

closed-loop stability: ______

Stability from Nyquist plot



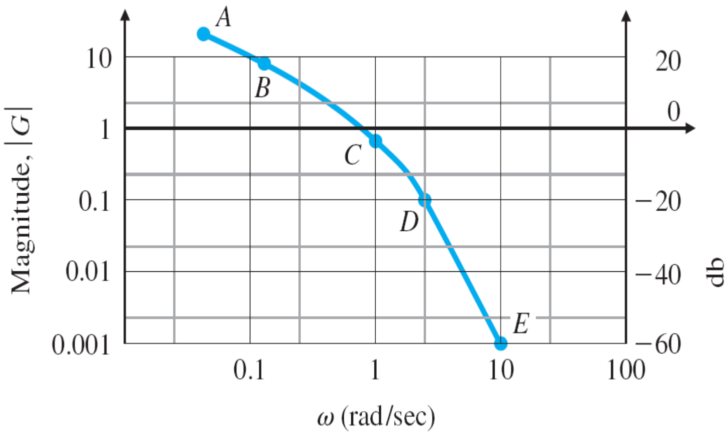
Stable system

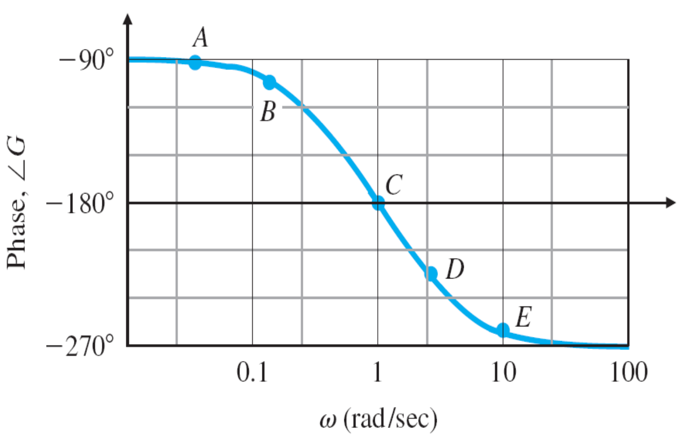


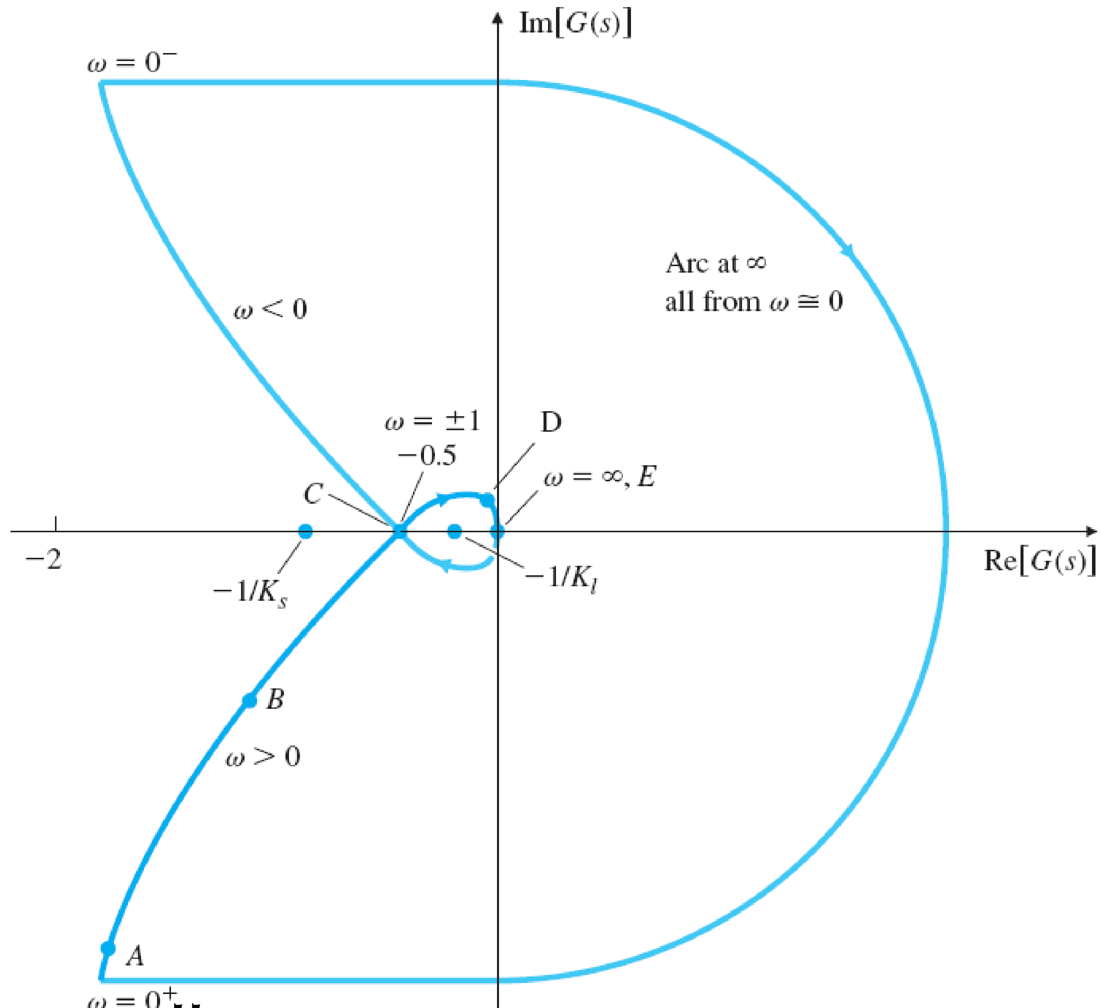
Unstable system

Nyquist-Bode

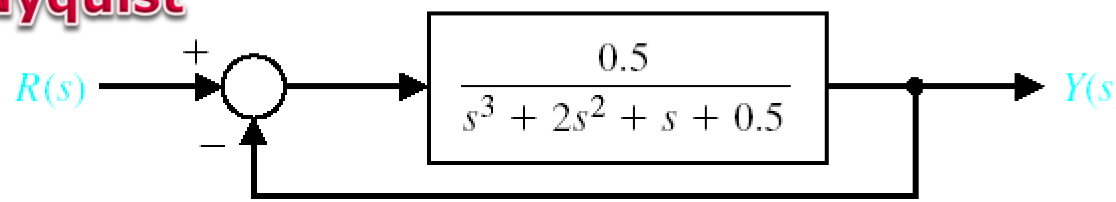
$$L(s) = \frac{1}{s(s+1)^2}$$







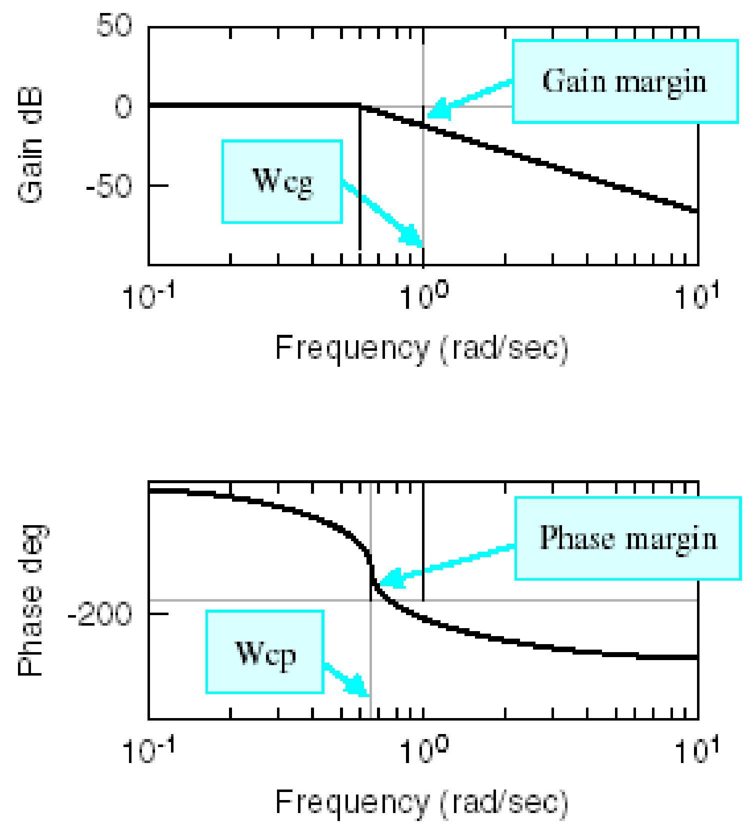
Example: Bode and Nyquist




```
[mag,phase,w]=bode(sys);
[Gm,Pm,Wcg,Wcp]=margin(mag,phase,w);
```

or

```
[Gm,Pm,Wcg,Wcp]=margin(sys);
```

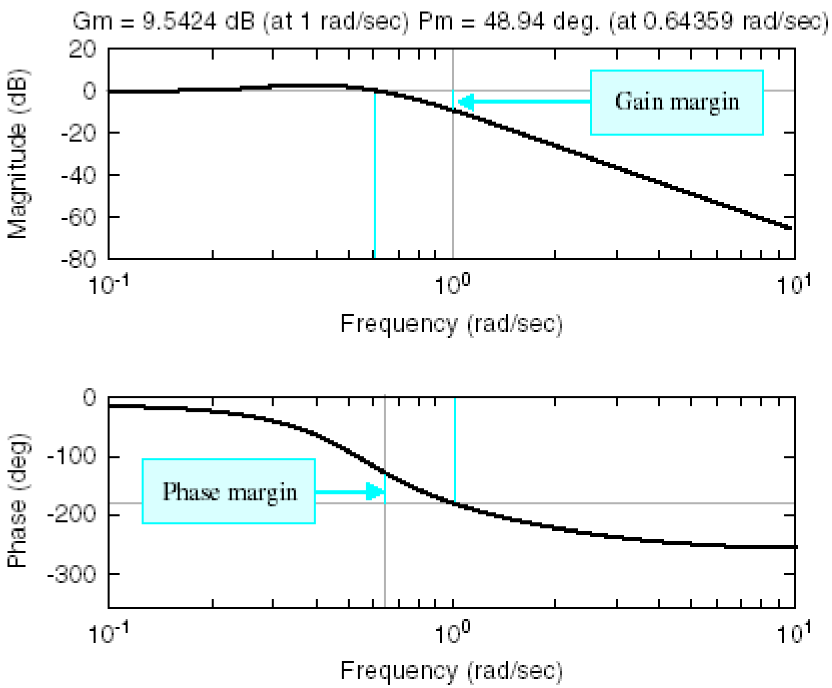


Example

```
num=[0.5]; den=[1 2 1 0.5];
sys=tf(num,den);
margin(sys);
```

Gm = gain margin (dB)
Pm = phase margin (deg)
Wcg = freq. for phase = −180
Wcp = freq. for gain = 0 dB

Example: Bode and Nyquist

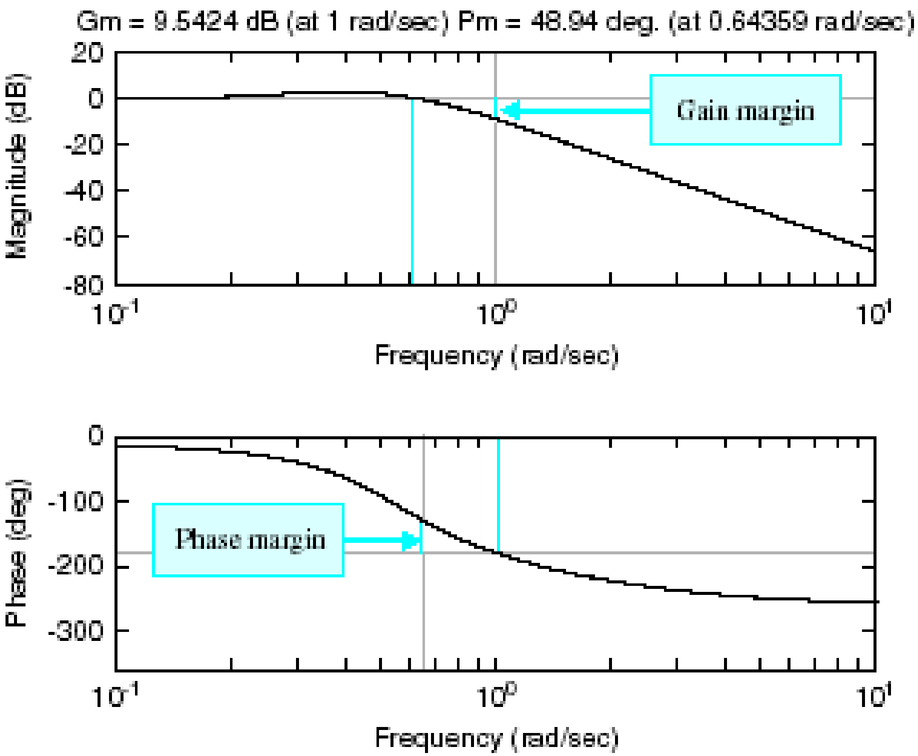


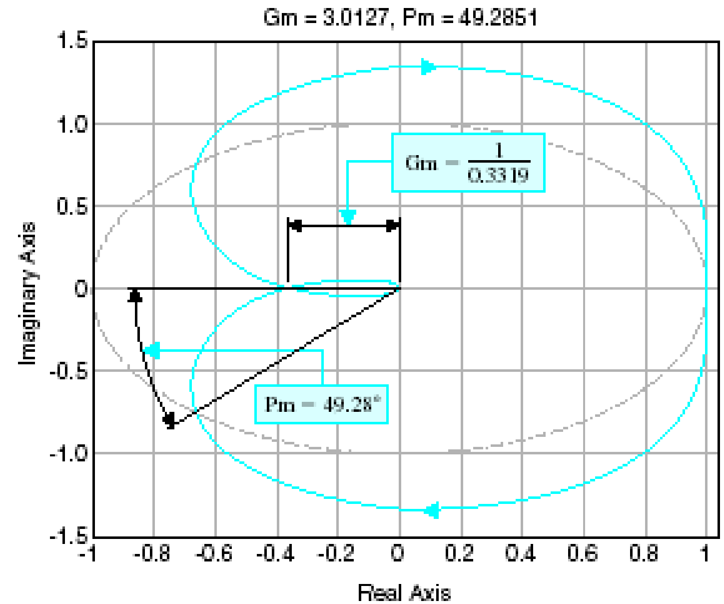

```
num=[0.5];
den=[1 2 1 0.5 ];
sys=tf(num,den);
%
w=logspace(-1,1,200);
%
[mag,phase,w]=bode(sys,w);
%
margin(mag,phase,w);
```

Open-loop system

Specify frequency range

Example: Bode and Nyquist






```
num=[0.5];
den=[1 2 1 0.5 ];
sys=tf(num,den);
%
w=logspace(-1,1,200);
%
[mag,phase,w]=bode(sys,w);
%
margin(mag,phase,w);
```

Open-loop system

Specify frequency range

```
%    The Nyquist plot of
%
%                      0.5
%    G(s) = ------------------------------
%             s^3 + 2 s^2 + s + 0.5
%
%    with gain and phase margin calculation.
%
num=[0.5]; den=[1 2 1 0.5]; sys=tf(num,den);
%
[mag,phase,w]=bode(sys);
[Gm,Pm,Wcg,Wcp]=margin(mag,phase,w);
%
nyquist(sys);
title(['Gm = ',num2str(gm),' Pm = ',num2str(Pm)])
```

Compute gain and phase margins.

Nyquist plot

Label gain and phase margins on plot.