MENG366

Transfer Functions & Block Diagrams

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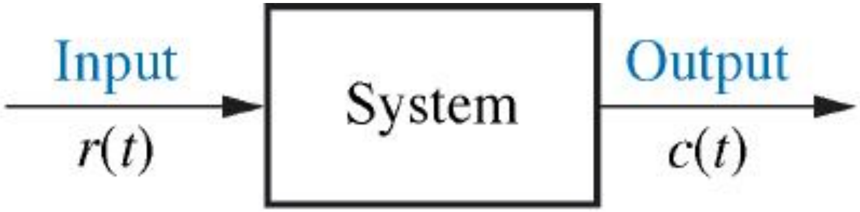
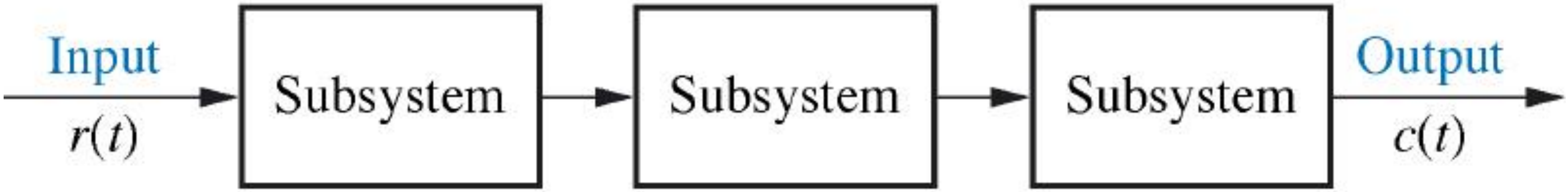
Linear System

Transfer Function



(*a*)



(*b*)

Note: The input, $r(t)$, stands for *reference input*.
The output, $c(t)$, stands for *controlled variable*.

Transfer Function

The output *c*(*t*) is related to the input *r*(*t*) by

$$a_n \frac{d^n c}{dt^n} + a_{n-1} \frac{d^{n-1} c}{dt^{n-1}} + \ldots + a_0 c = b_m \frac{d^m r}{dt^m} + b_{m-1} \frac{d^{m-1} r}{dt^{m-1}} + \ldots + b_0 r$$

where *a's* and *b's* are system parameters

Using Laplace transform with zero initial conditions

$$\left(a_n s^n + a_{n-1} s^{n-1} + .. + a_0\right) C(s) = \left(b_m r^m + b_{m-1} r^{m-1} + \ldots + b_0\right) R(s)$$

where *C*(*s*) & *R*(*s*) are the Laplace transforms of *c*(*t*) & *r*(*t*)

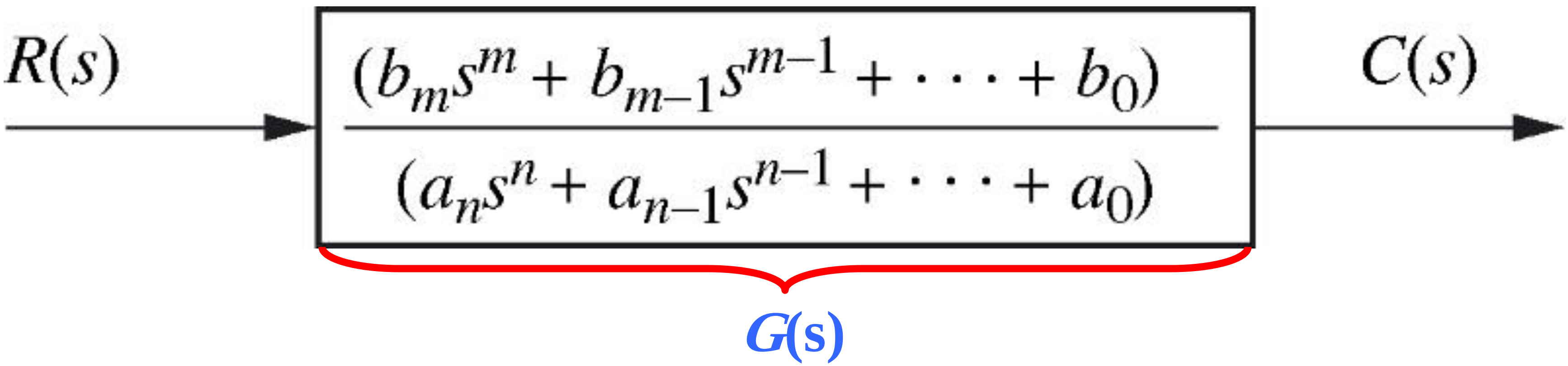
طباعة المذكرة مسبقا

- سلطان القحطاني
- هيثم
- عزام
- محمد المدني

Transfer Function

$$\frac{C(s)}{R(s)} = G(s) = \frac{\left(b_m r^m + b_{m-1} r^{m-1} + \ldots + b_0\right)}{\left(a_n s^n + a_{n-1} s^{n-1} + .. + a_0\right)}$$

where $G(s)$ = transfer function of the system



$$C(s) = G(s)\,R(s)$$

Poles & Zeros of Transfer Function

$$\frac{C(s)}{R(s)} = G(s) = \frac{\left(b_m r^m + b_{m-1} r^{m-1} + \ldots + b_0\right)}{\left(a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0\right)} = \frac{N(s)}{D(s)}$$

Definitions

Zeros = roots of $N(s)$, *i.e.* the values at which $N(s)=0$. At these values the numerator becomes ZERO

Poles = roots of $D(s)$, *i.e.* the values at which $D(s)=0$. At these values the denominator vanishes & the system output (for any finite input) goes to infinity.

Transfer f^n from DE

Example.1:

Find the transfer function represented by:

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Solution **with zero initial conditions**

$$\Rightarrow \quad sC(s) + 2C(s) = R(s)$$

or $$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

Example.2:

Find the time response of :

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

For a unit step input $r(t)$ (i.e. $r(t)=u(t)$).

Solution **as the transfer function is**

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

As $r(t)=u(t) \rightarrow R(s)=1/s$

Response from T. f^n

$$C(s) = G(s)R(s) = \frac{1}{s(s+2)}$$

or by expanding into partial fraction

$$C(s) = \frac{1}{2s} - \frac{1}{2(s+2)}$$

Taking the inverse Laplace transform

$$c(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

Response from T. f^n

With MATLAB

```
>> syms t s

>> ilaplace(1/(s*(s+2)),s,t)

ans =

-1/2*exp(-2*t)+1/2
```

Response from T. f^n

The Transfer Function

$$G(s)=\frac{C(s)}{R(s)}=\frac{1}{s+2}$$

has Poles at s=-2, No Zeros

MATLAB Code

```
>> n=1;
>> d=[1  2];
>>
[z,p,k]=tf2zp(n,d)
```

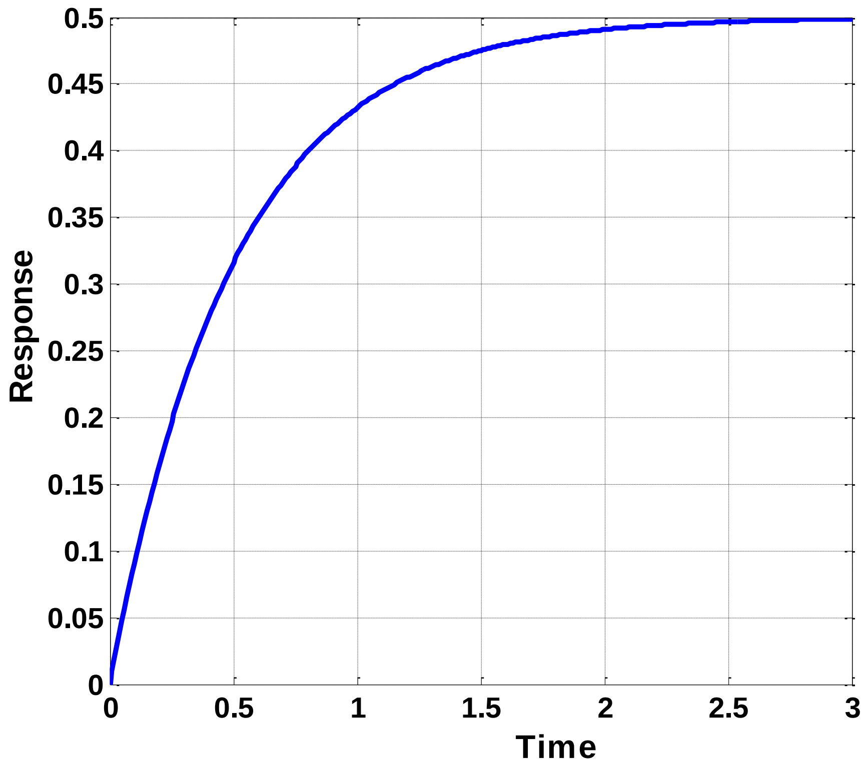
OUTPUT

```
z =
   Empty matrix: 0-by-1
p =     -2
k =      1
```

Response from T. f^n

MATLAB Time Response

```
>> t=0:0.01:3;
>> plot(t,(1/2-1/2*exp(-2*t)))
>> grid
>> xlabel('Time')
>> ylabel('Response')
```

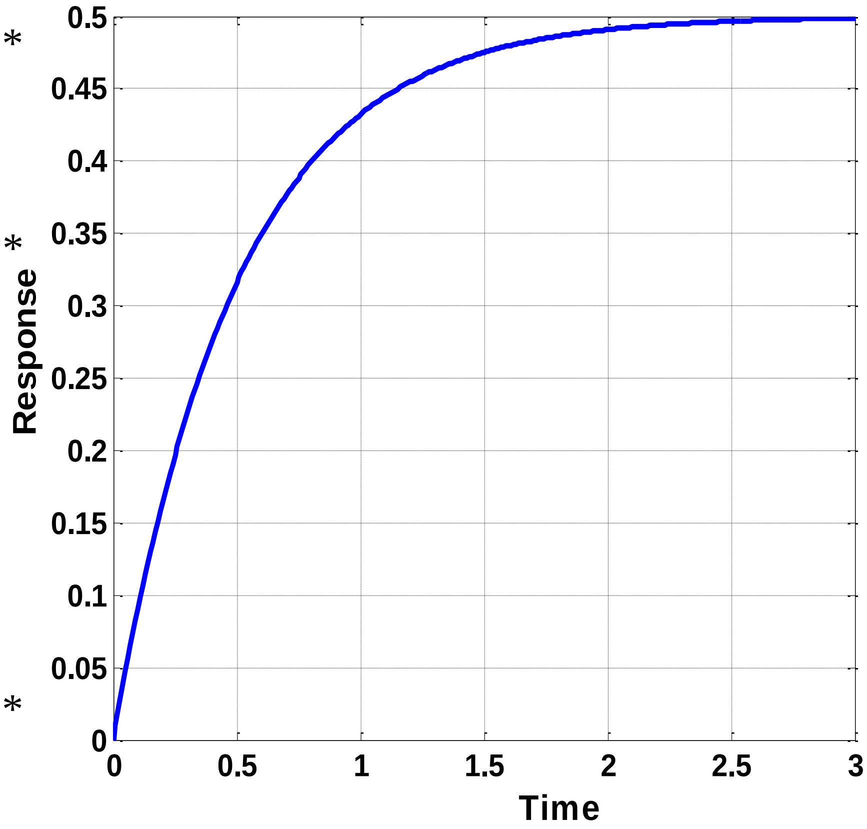


Response from T. f^n

$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

MATLAB Time Response

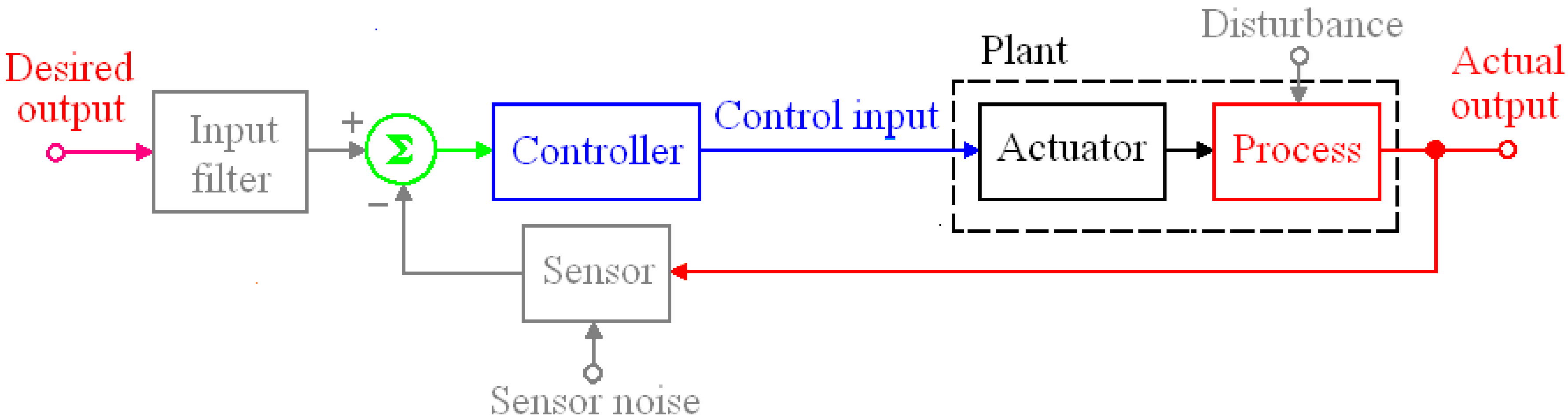
```
%*********************************************
********
%********Step
Input******************************
%*********************************************
********
syms s t c  C ct

eqn=sym('D(c)(t)+2*c(t)=1');
lteqn=laplace(eqn,t,s)
neweqn=subs(lteqn,{'laplace(c(t),t,s)','c(0)'},{C,0})
cs=solve(neweqn,C)
ct=ilaplace(cs,s,t)
%*********************************************
*********
ezplot(ct,[0 3])
grid
xlabel('Time - s')
ylabel('Response - c(t)')
%*********************************************
```



Block Diagram

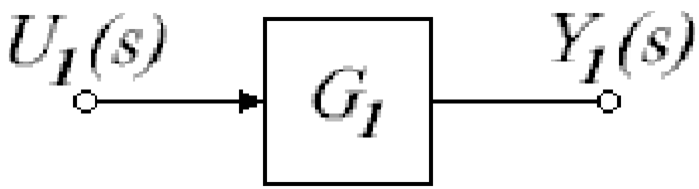
- *A graphical tool* can help us to ***visualize the model*** of a system and ***evaluate the mathematical relationships between their elements***, using their transfer functions.
- In many control systems, the system of equations can be written so that their components do not interact ***except by having the input of one part be the output of another part***.
- In these cases, it is very easy to draw a block diagram that represents the mathematical relationships in similar manner to that used for the component block diagram.

Reminder: Component Block Diagram



Block Diagram

- It represents the ***mathematical relationships*** between the elements of the system.



$$U_1(s)\,G_1(s) = Y_1(s)$$

- The ***transfer function*** of each component is placed ***in box***, and the ***input-output relationships*** between components are indicated by ***lines and arrows***.

Block Diagram Algebra

- Using block diagram, we can ***solve the equations by graphical simplification***, which is often easier and more informative than algebraic manipulation, even though the methods are in every way equivalent.

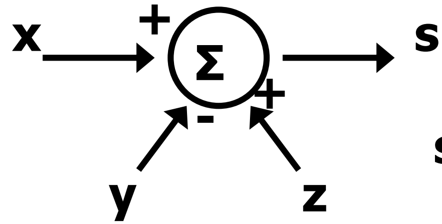
- It is convenient to think of ***each block as representing an electronic amplifier*** with the transfer function printed inside.

- The interconnections of blocks include summing points, where any number of signals may be added together.

Block Diagrams

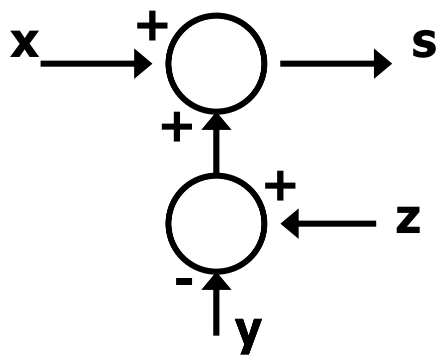
- A line is a signal
- A block is a gain
- A circle is a sum
- Due to h.f. noise,
  use <u>proper blocks</u>: num deg ≤ den deg
- Try to use just horizontal or vertical lines
  - Use additional "Σ" to help

$y = Gx$



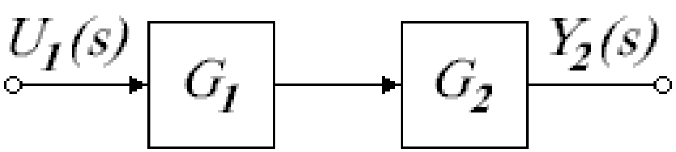
$s = x + z - y$

e.g.



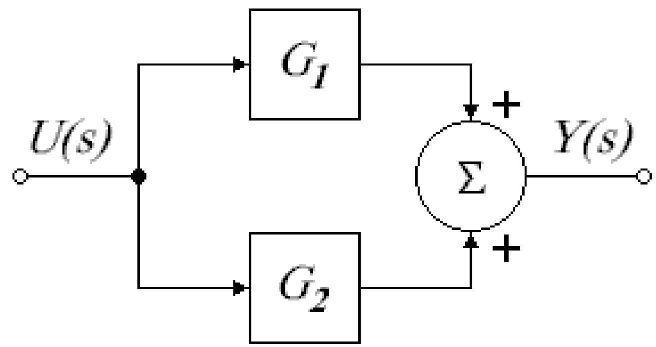
1st & 2nd Elementary Block Diagrams

- Block in series:



$$\frac{Y_2(s)}{U_1(s)} = G_1 G_2$$

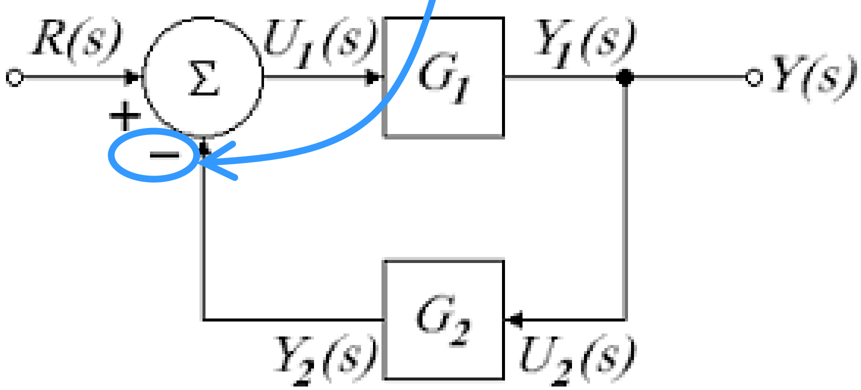
- Blocks in parallel with their outputs added:



$$\frac{Y_2(s)}{U_1(s)} = G_1 + G_2$$

3rd Elementary Block Diagram

- ***Single-loop negative feedback***

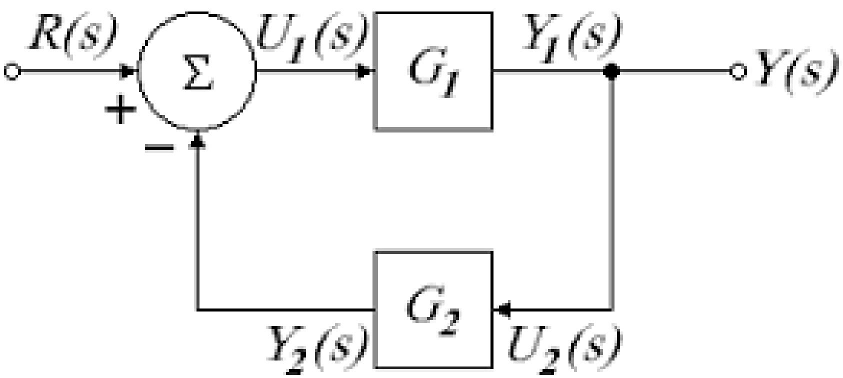


Two blocks are connected in a feedback arrangement so that each feeds into the other:

- The *overall transfer function* is given by:

$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1 G_2}$$

Feedback Rule

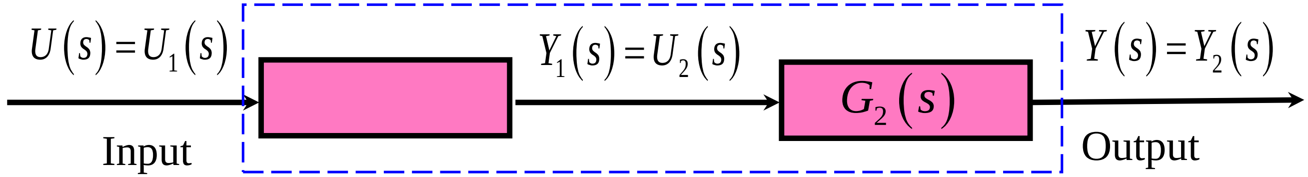


$$\frac{Y(s)}{R(s)} = \frac{G_1}{1+G_1G_2}$$

The gain of a single-loop negative feedback system is given by the forward gain divided by the sum of 1 plus the loop gain

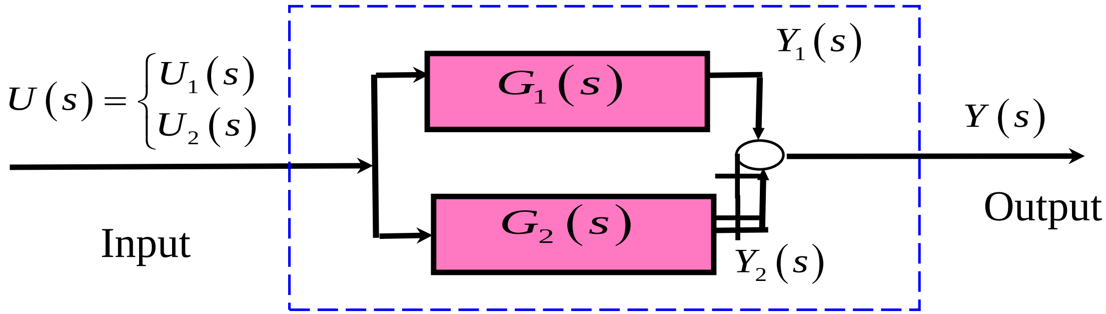
System Connections

Cascaded System



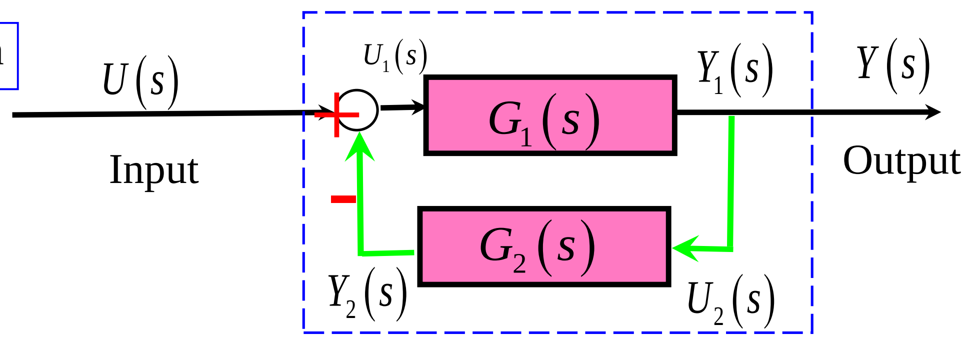
$$G(s)=G_2(s)G_1(s)$$

Parallel System



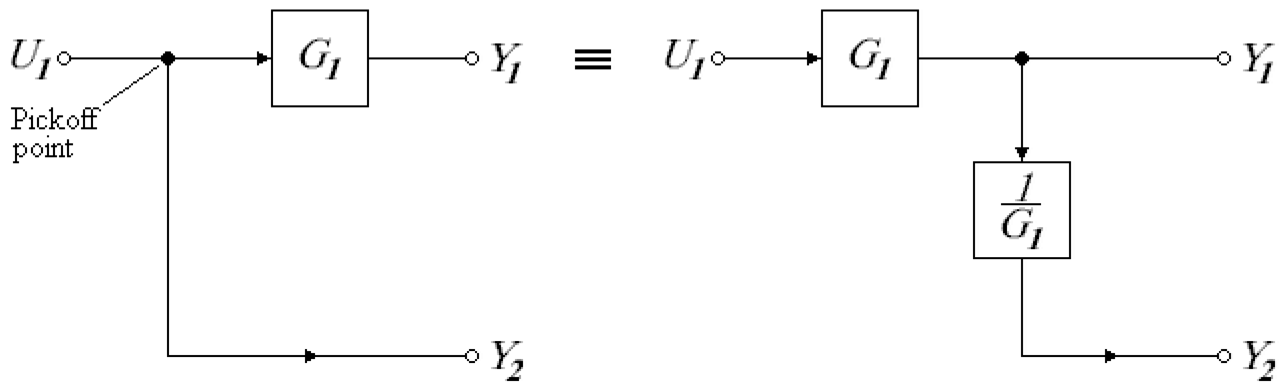
$$G(s)=G_1(s)+G_2(s)$$

Feedback System

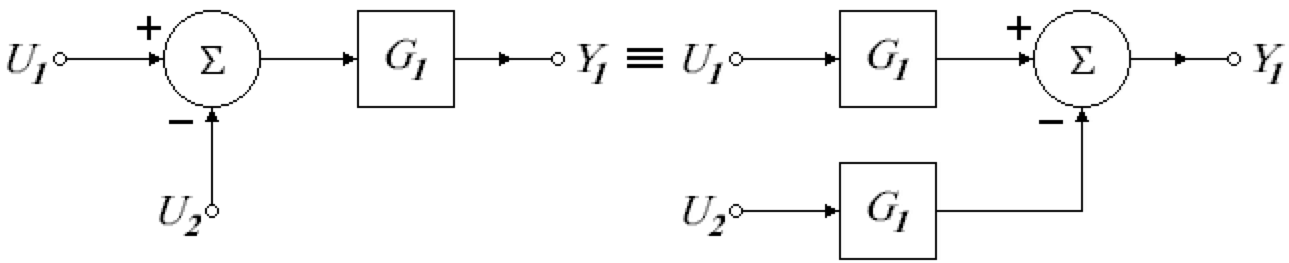


$$G(s)=\frac{G_1(s)}{1+G_1(s)G_2(s)}$$

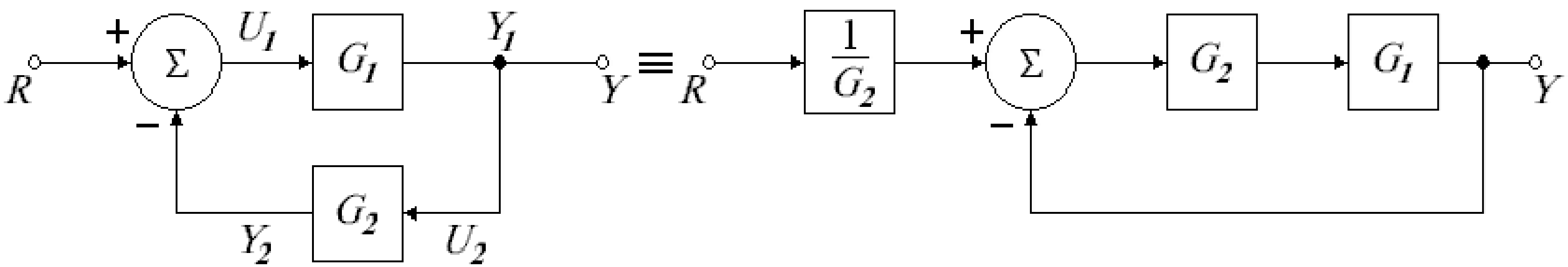
1st Elementary Principle of Block Diagram Algebra



2nd Elementary Principle of Block Diagram Algebra



3rd Elementary Principle of Block Diagram Algebra



Example 1: Transfer function from a Simple Block Diagram

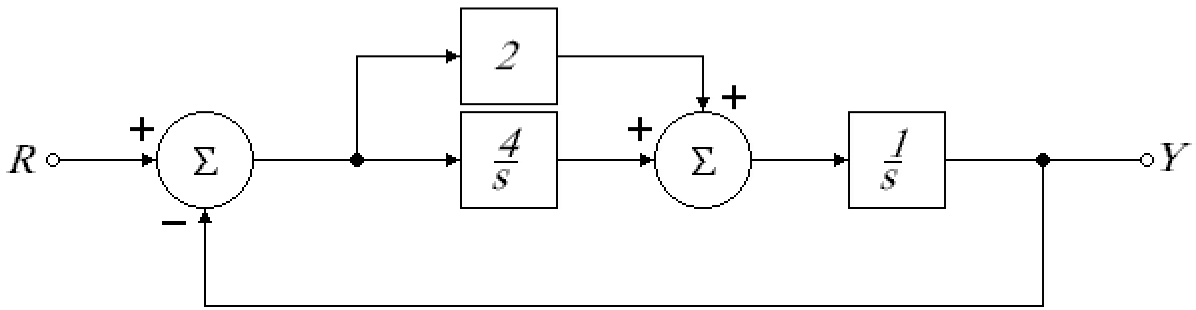


fig. (a)

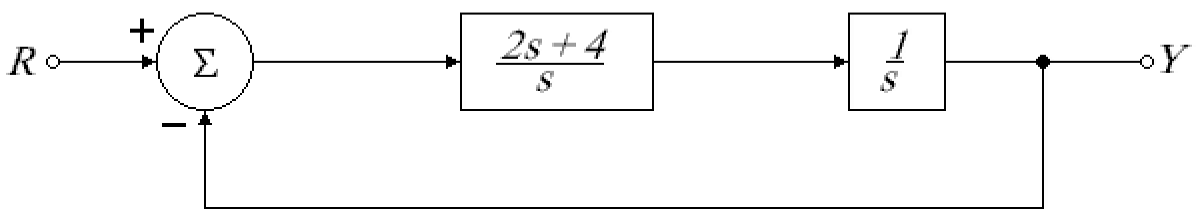


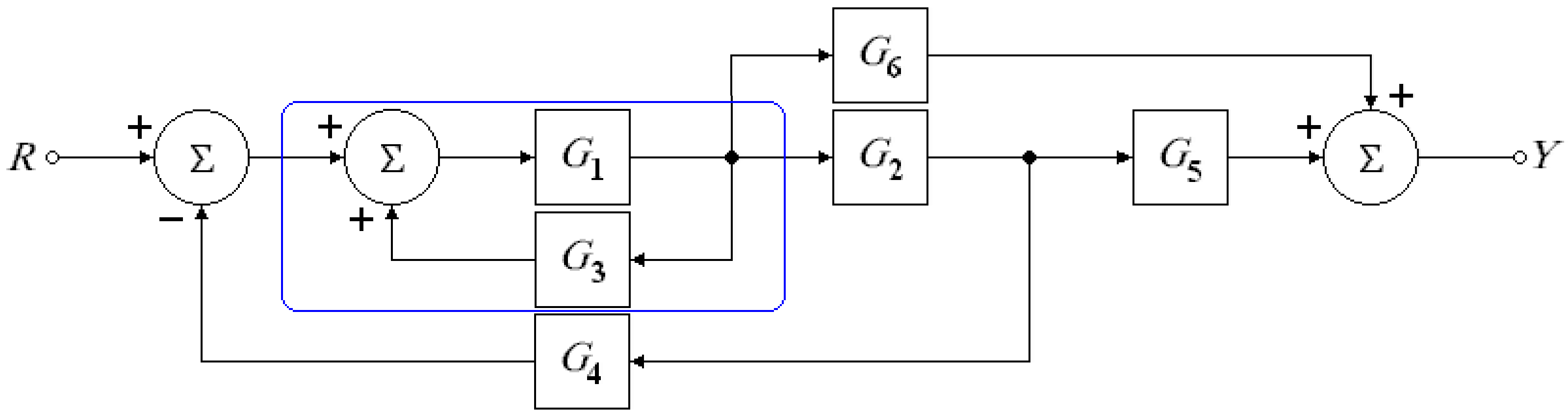
fig. (b)

$$T(s) = \frac{Y(s)}{R(s)}$$

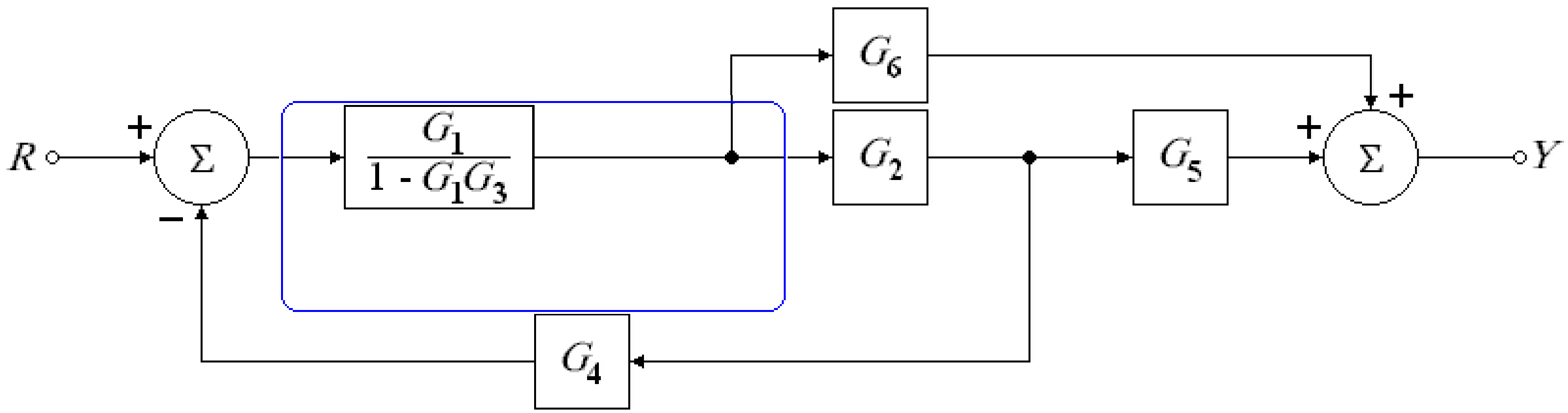
$$T(s) = \frac{\dfrac{2s+4}{s^2}}{1 + \dfrac{2s+4}{s^2}}$$

$$T(s) = \frac{2s+4}{s^2 + 2s + 4}$$

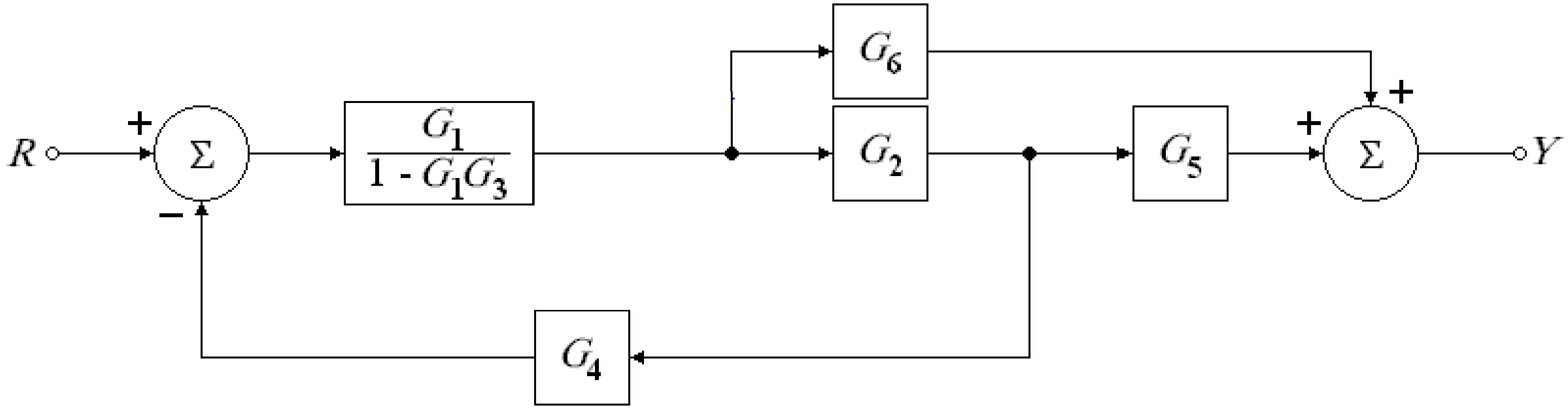
Example 2: TF from the Block Diagram



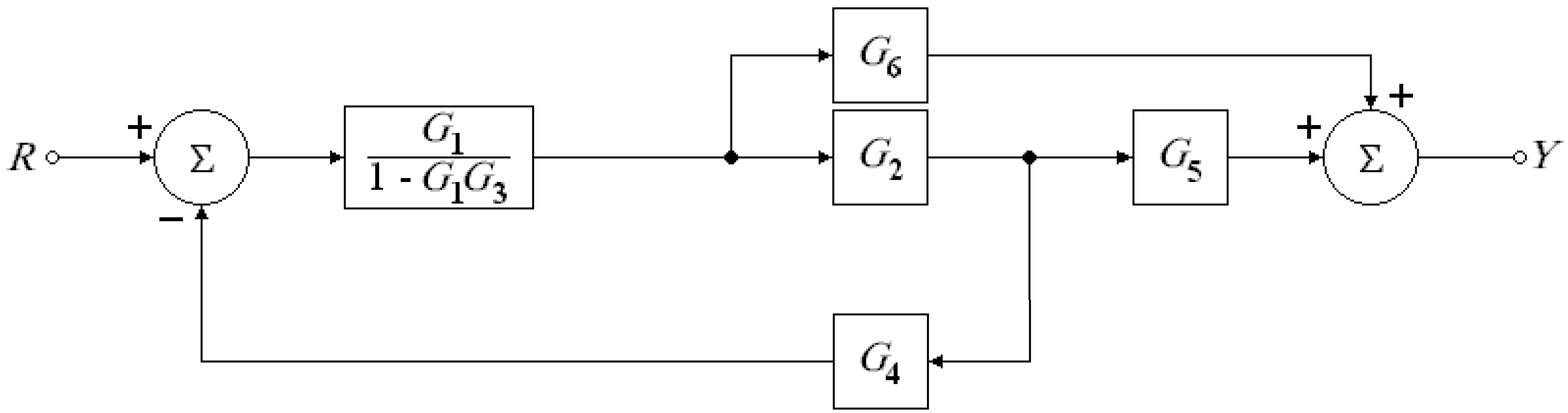
Example 2: TF from the Block Diagram



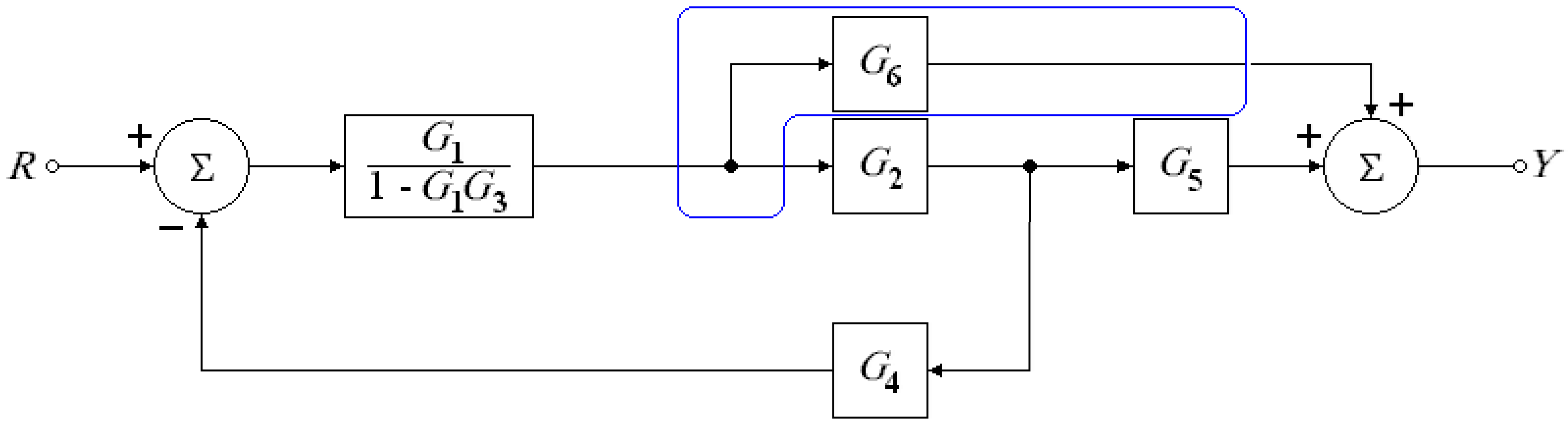
Example 2: TF from the Block Diagram



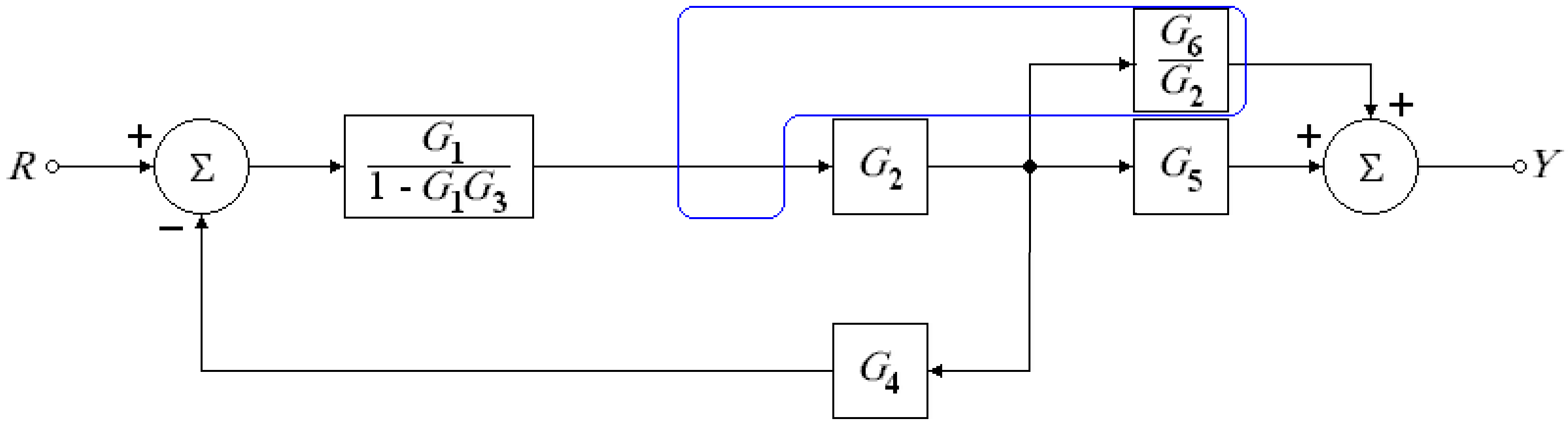
Example 2: TF from the Block Diagram



Example 2: TF from the Block Diagram



Example 2: TF from the Block Diagram



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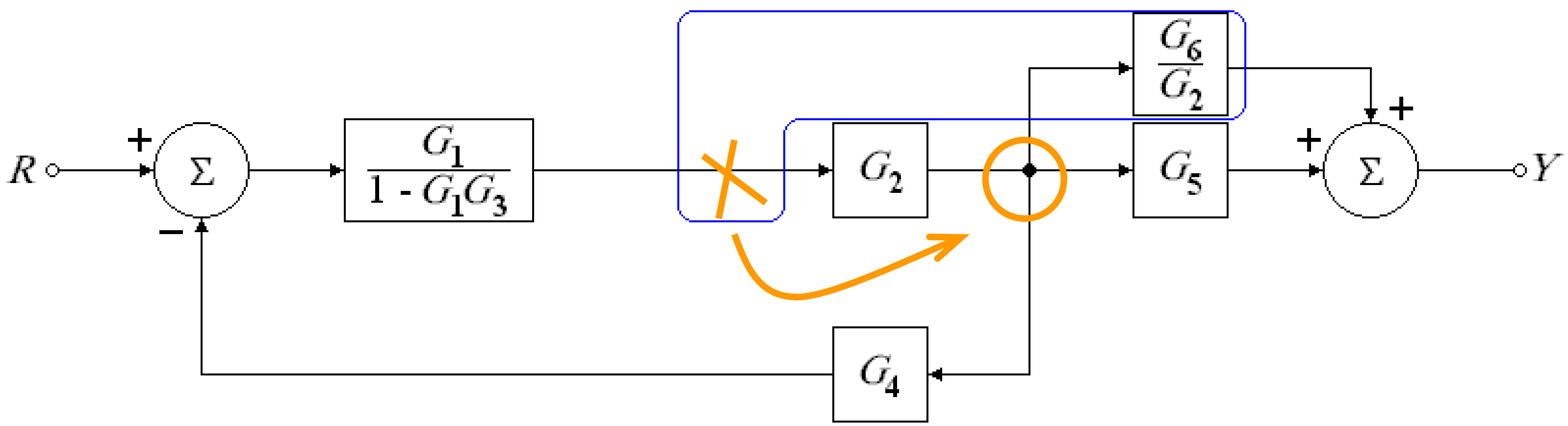
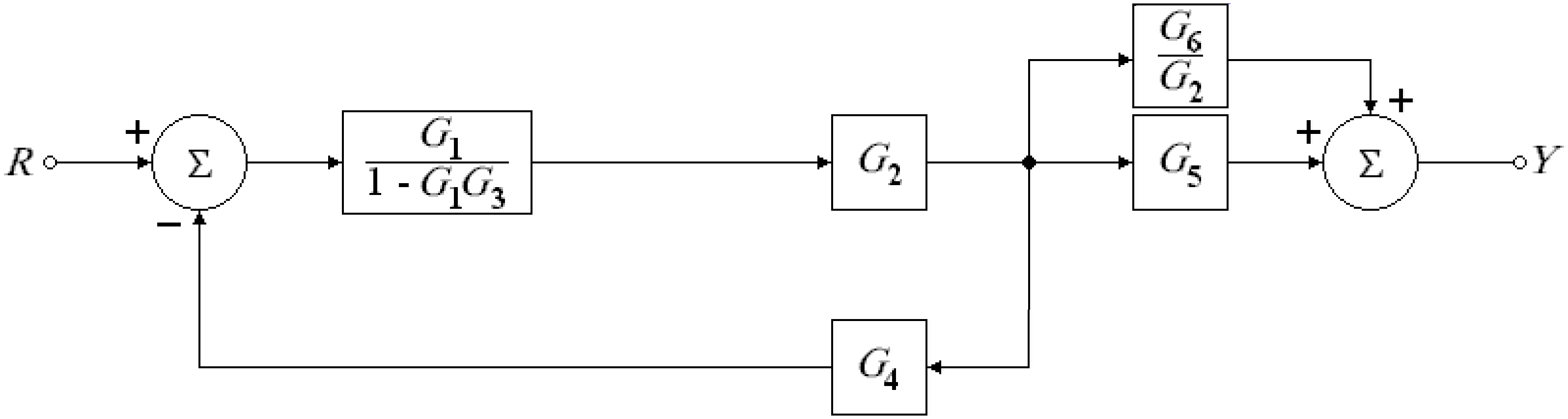


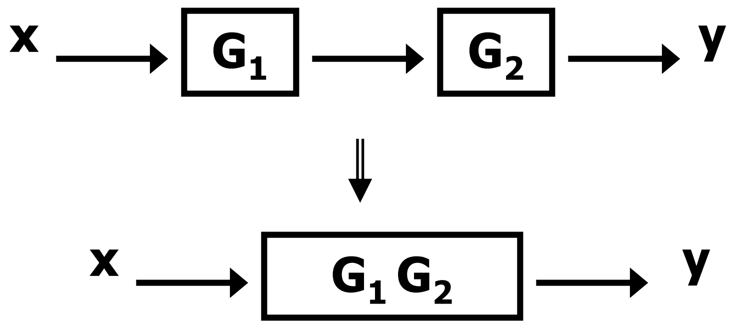
Example 2: TF from the Block Diagram



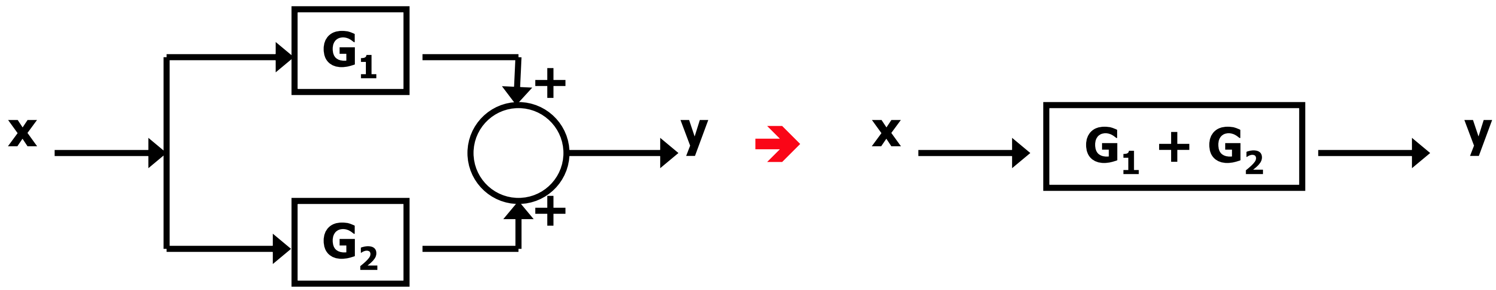
$$T(s) = \frac{G_1G_2G_5 + G_1G_6}{1 - G_1G_3 + G_1G_2G_4}$$

Block Diagram Reduction

- Series:

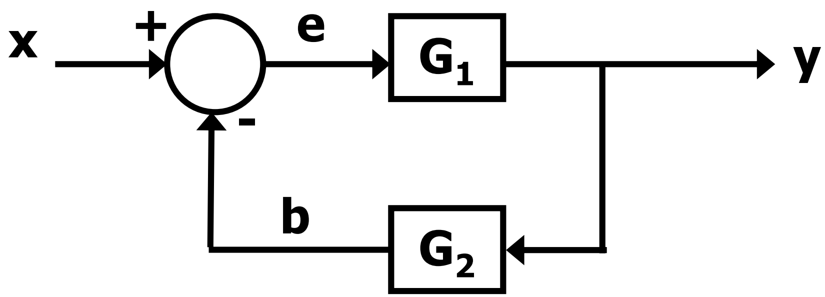


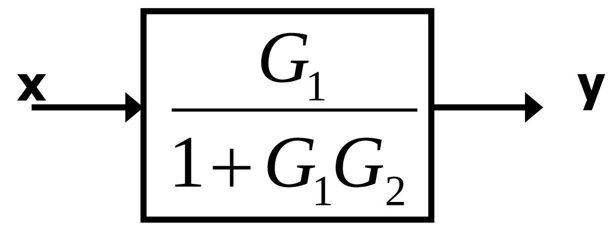
- Parallel:



Block Diagram Reduction

- Feedback:





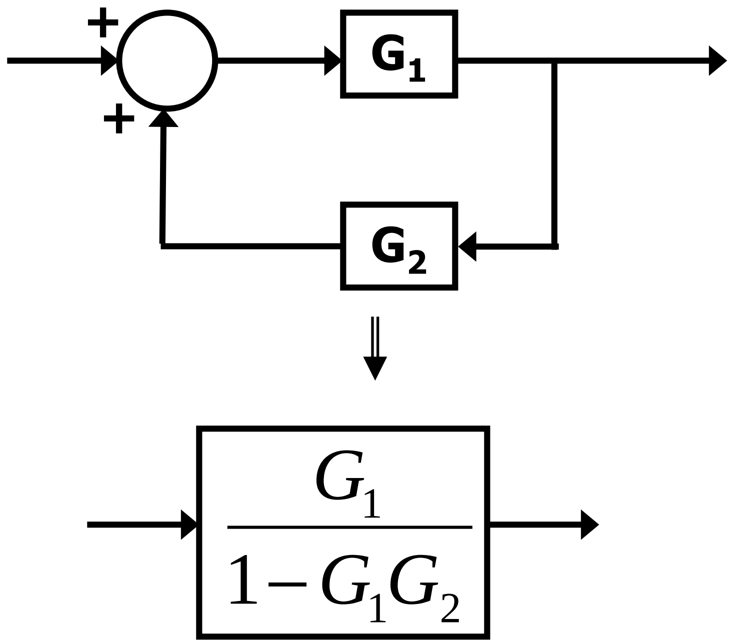
$$y = \frac{G_1}{1+G_1G_2}x$$

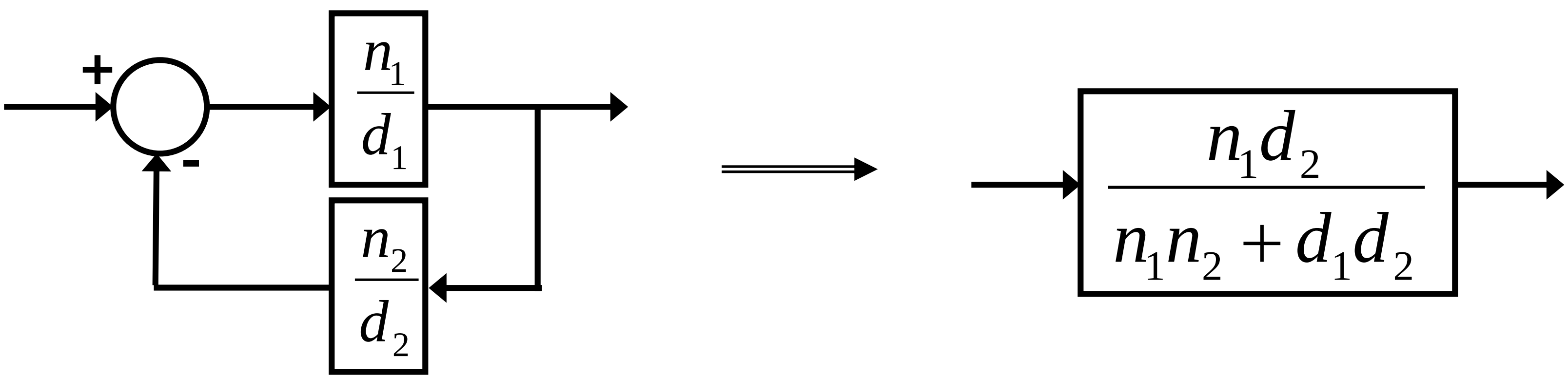
- Proof: $e = x - b, \quad b = G_2 y, \quad y = G_1 e \quad \Rightarrow \quad y = \dfrac{G_1}{1+G_1G_2}x$

$$e = x - G_2 G_1 e$$

$$(1+G_1G_2)e = x \;\Rightarrow\; e = \frac{1}{1+G_1G_2}x$$

Block Diagram Reduction





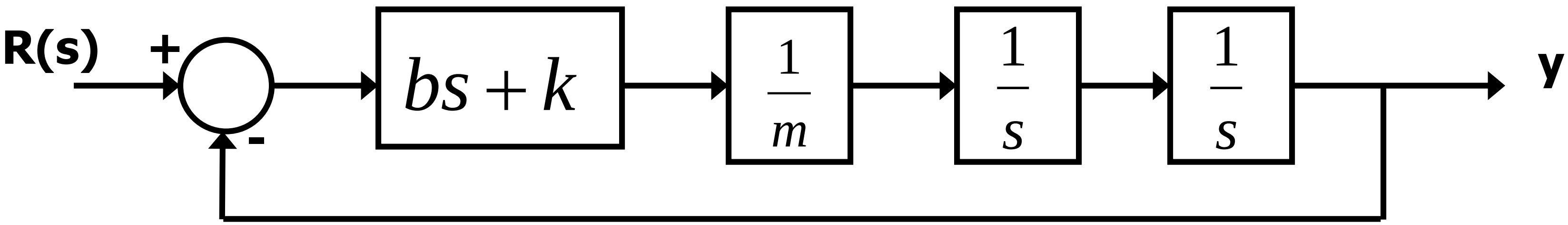
Block Diagram Reduction

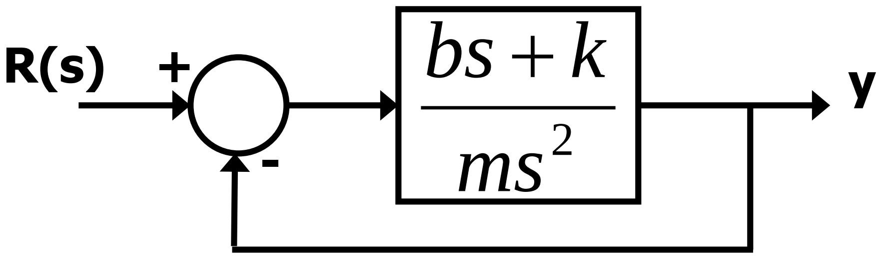
```
>> s=tf('s')
Transfer function:
s

>> G1=(s+1)/(s+2)
Transfer function:
s + 1
-----
s + 2

>> G2=5/(s+5)
Transfer function:
  5
-----
s + 5
```

```
>> G=G1*G2
Transfer function:
   5 s + 5
--------------
s^2 + 7 s + 10

>> H=G1+G2
Transfer function:
s^2 + 11 s + 15
---------------
s^2 + 7 s + 10

>> HF=feedback(G1, G2)
Transfer function:
 s^2 + 6 s + 5
---------------
s^2 + 12 s + 15
```

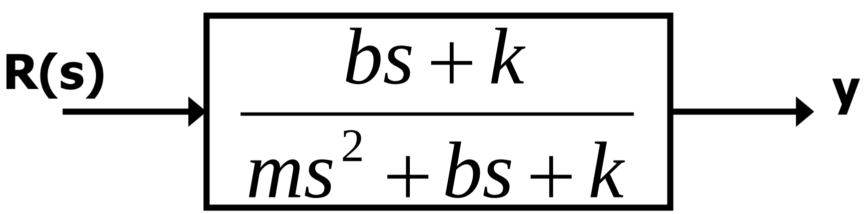
Quarter car suspension

Series





Feedback



$$TF = H(s) = \frac{bs + k}{ms^2 + bs + k}$$

Block Diagram Reduction

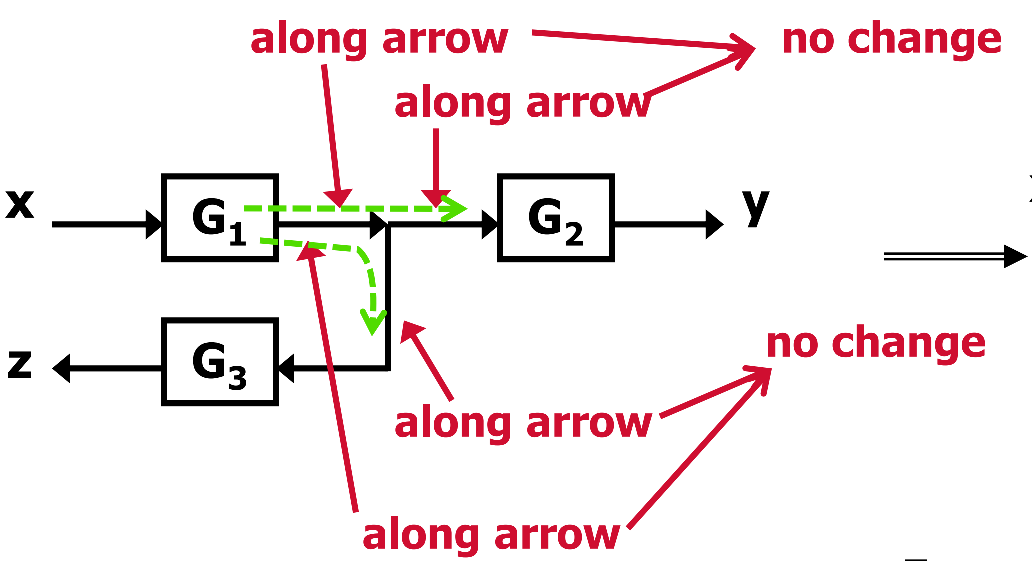
```
>> b=sym('b');
>> m=sym('m');
>> k=sym('k');
>> s=sym('s');
>> G1=b*s+k
G1 =
b*s+k

>> G2=1/m*1/s*1/s
G2 =
1/m/s^2

>> G=G1*G2
G =
(b*s+k)/m/s^2
```

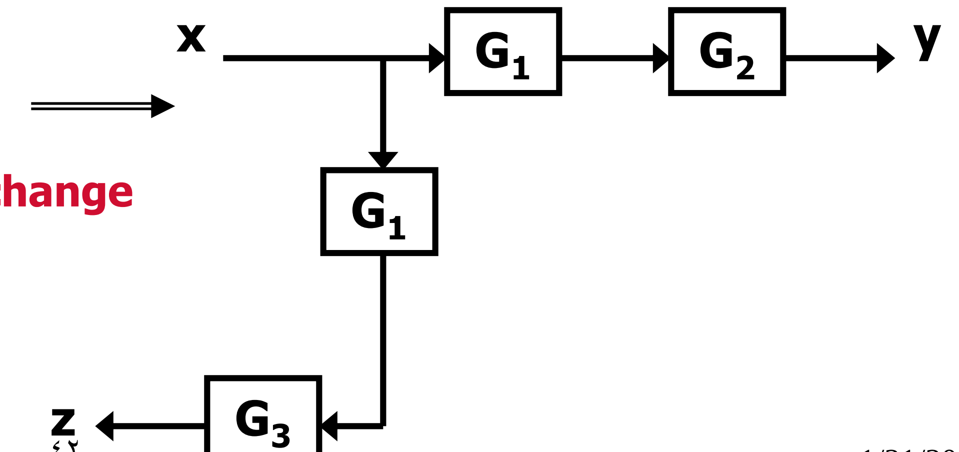
```
>> Gcl=G/(1+G)

Gcl =

(b*s+k)/m/s^2/(1+(b*s+k)/m/s^2)

>> simplify(Gcl)

ans =

(b*s+k)/(m*s^2+b*s+k)
```

Block Diagram Reduction

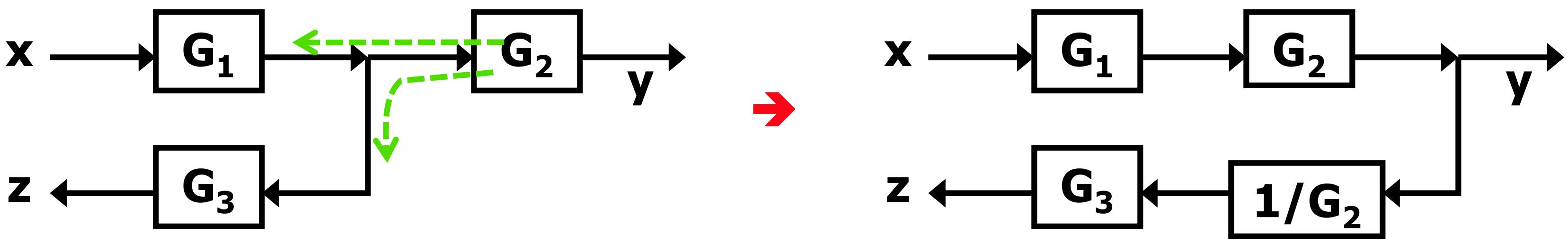
- Move a block (G1) across a summation / pick-up point into all touching lines:

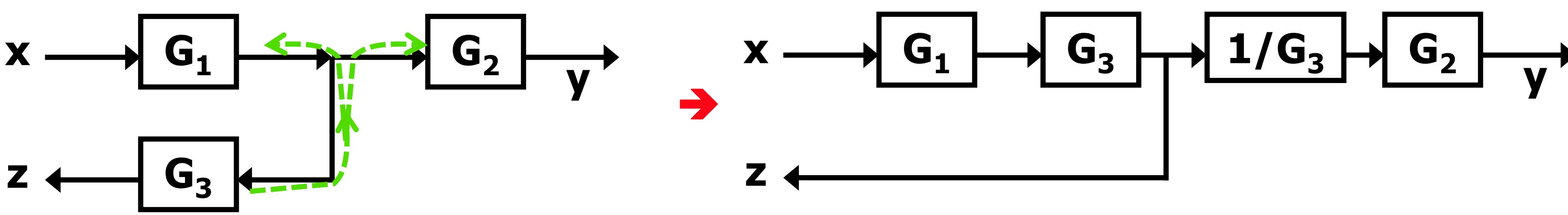
  - e.g.

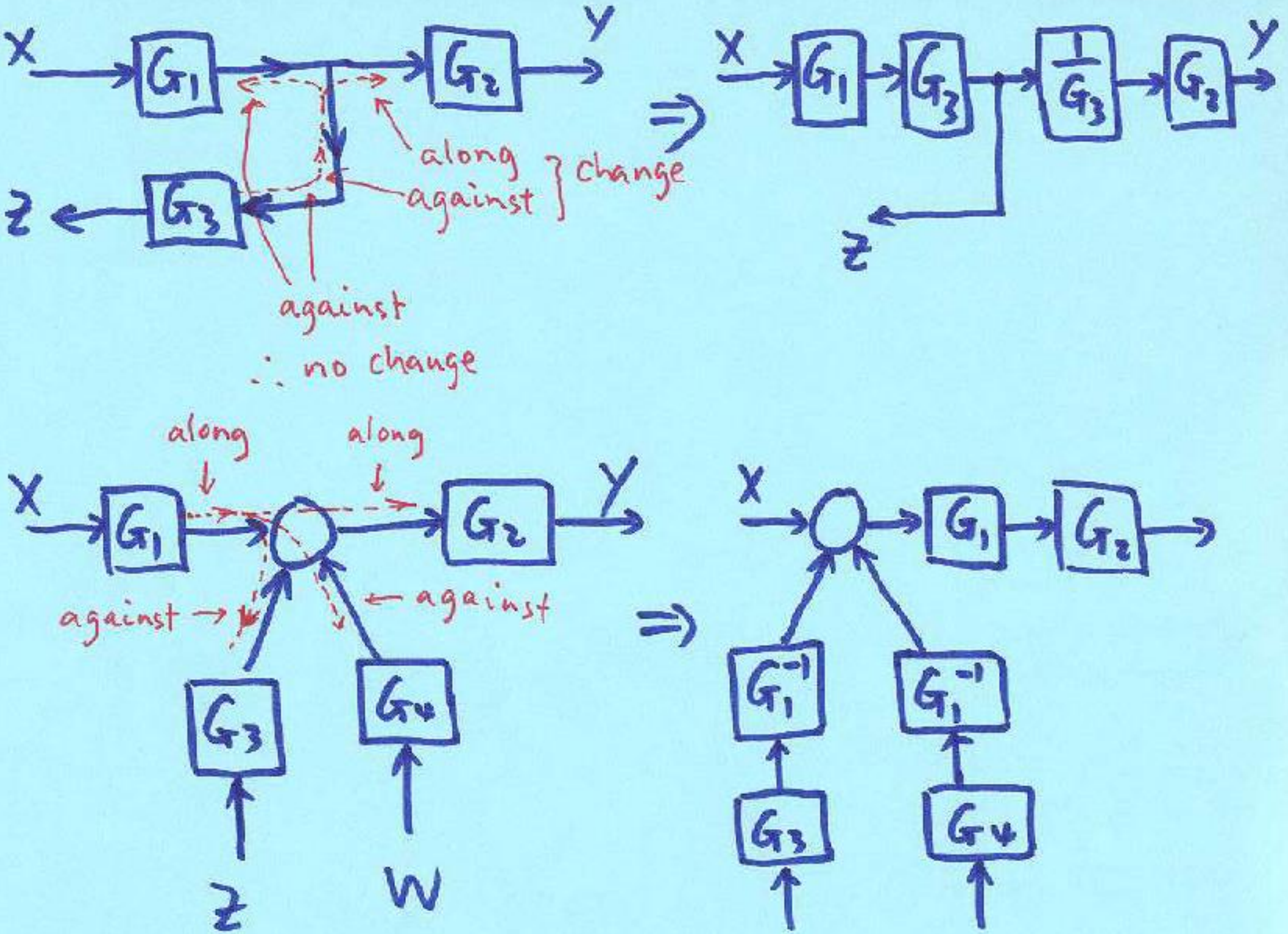


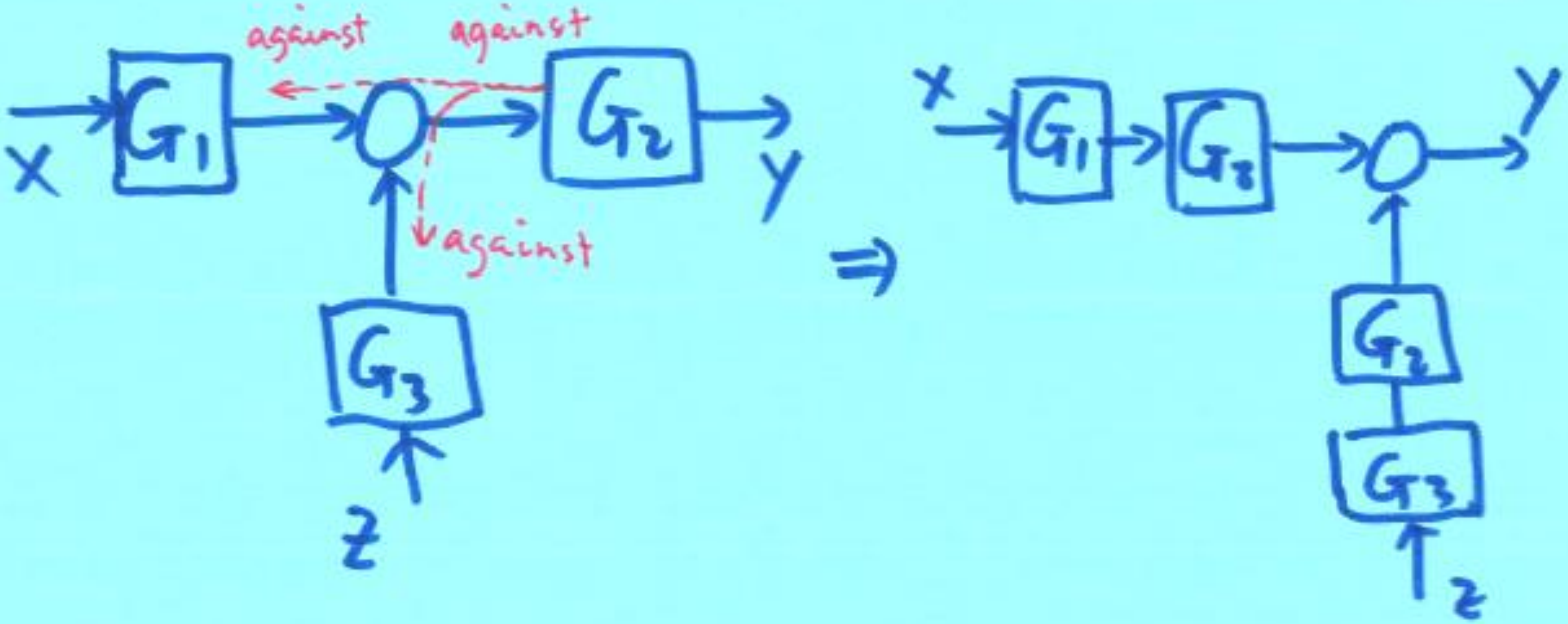


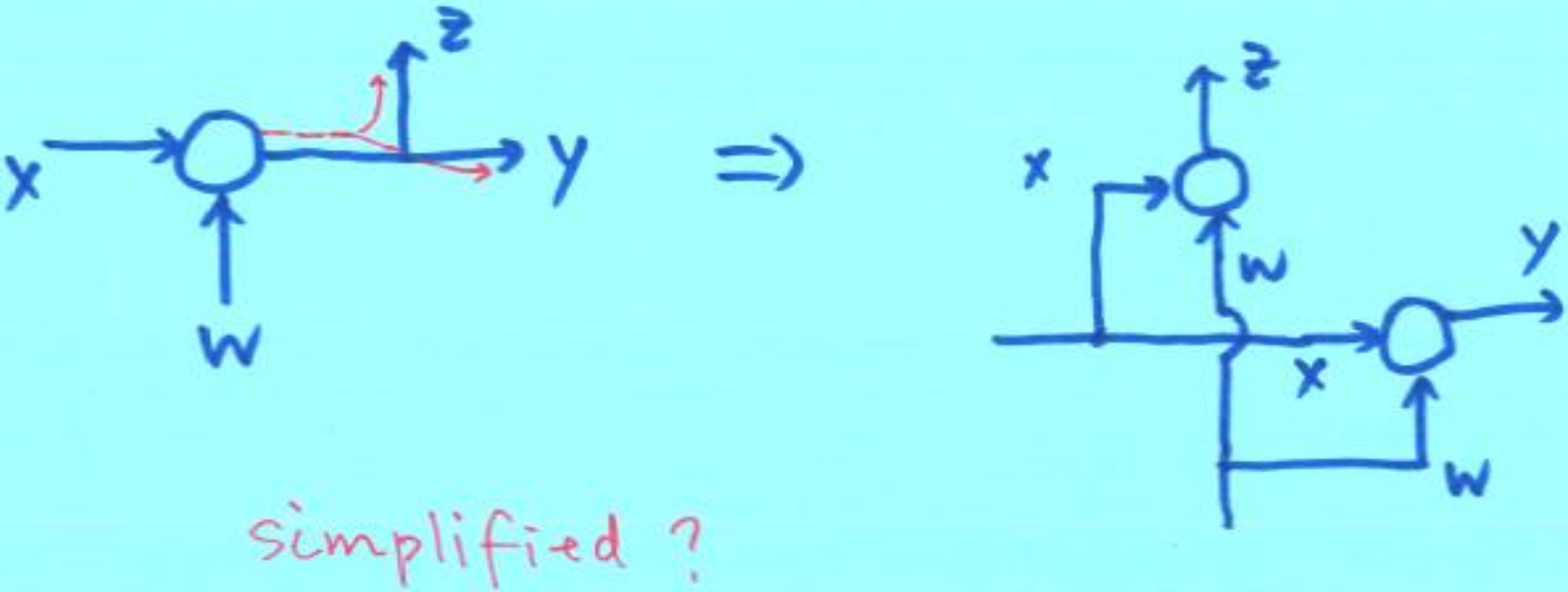
Block Diagram Reduction



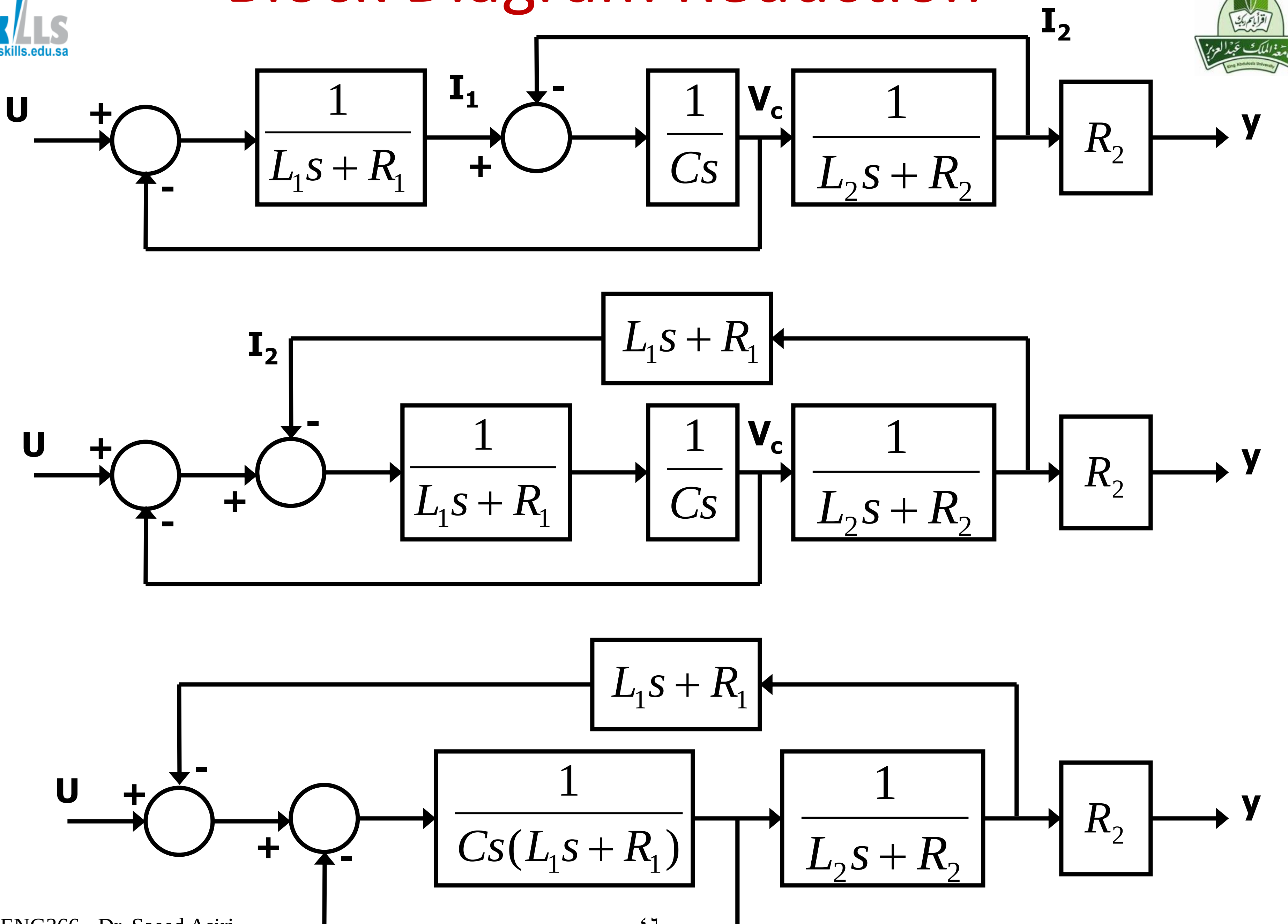



X
G1
G2
Y
G3
Z
along
against
change
against
∴ no change
1/G3
along
along
against
against
W
G1^-1
G4

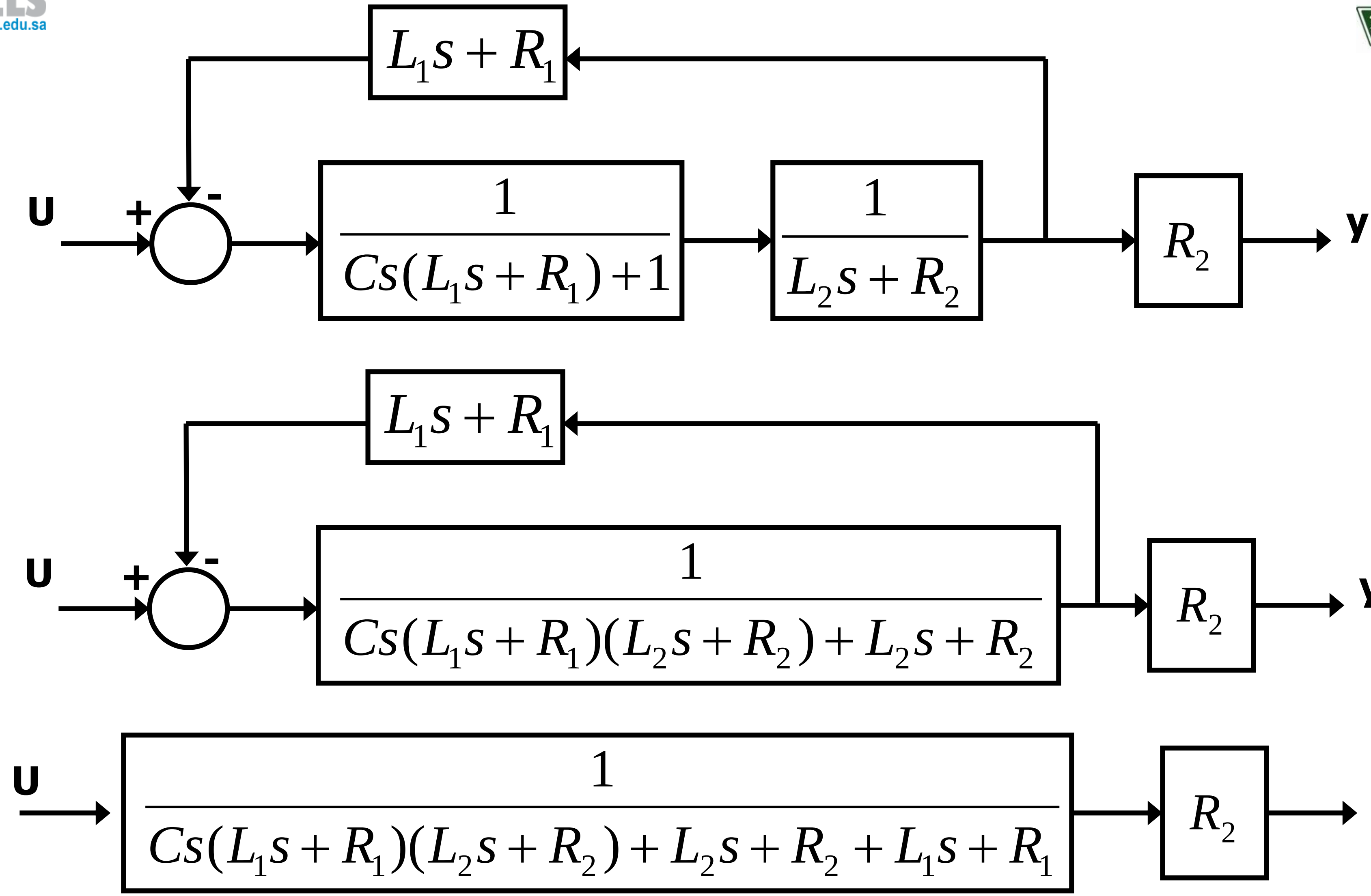

against
against
against
X
G1
G2
G3
Y
Z
⇒
X
G1
G2
Y
G2
G3
Z


X
W
Y
Z
⇒
X
Z
W
Y
X
W
simplified ?

Block Diagram Reduction



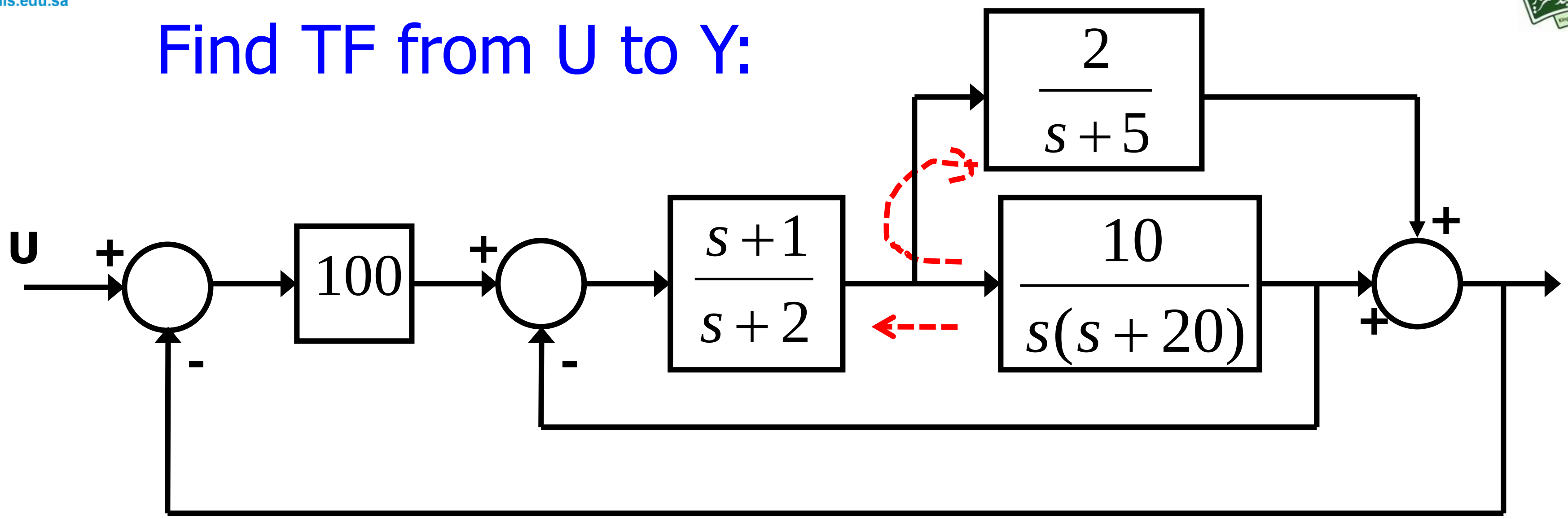
Block Diagram Reduction



$$T.F. = \frac{R_2}{Cs(L_1s+R_1)(L_2s+R_2)+L_2s+R_2+L_1s+R_1}$$

Block Diagram Reduction

Find TF from U to Y:

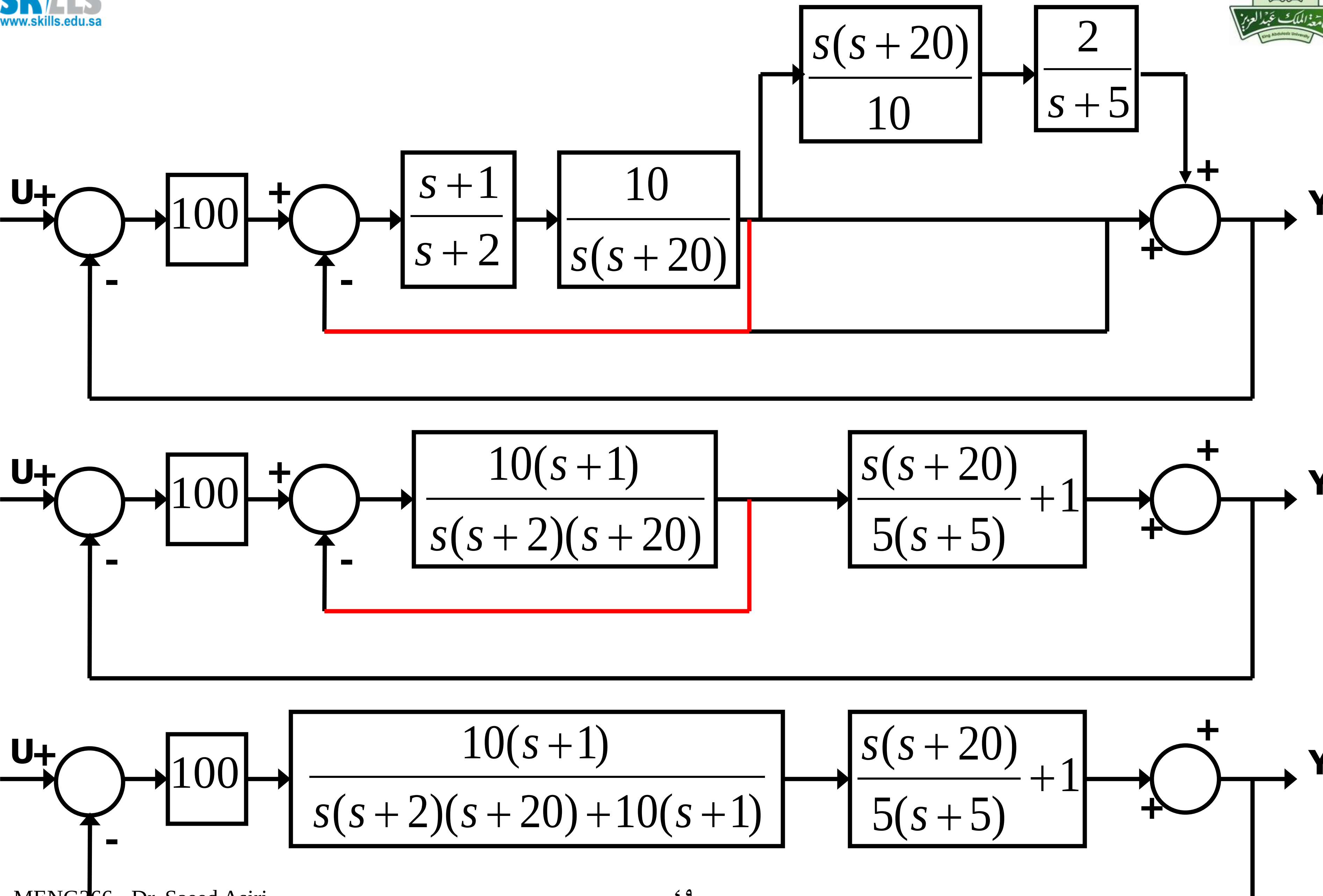


- No pure series/parallel/feedback
- Needs to move a block, but which one?

Key: move one block to create pure series or parallel or feedback!

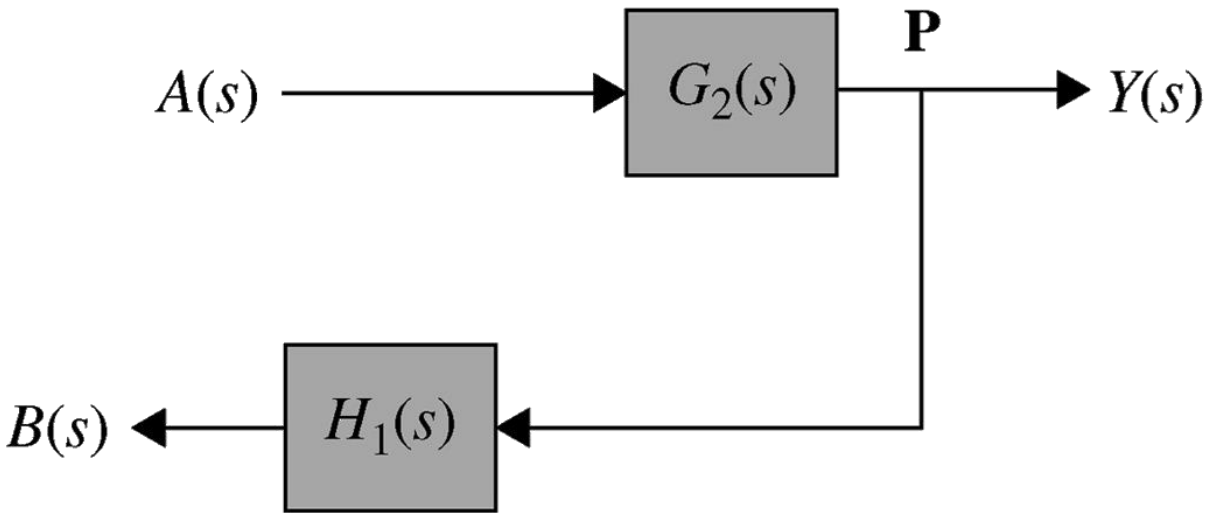
So move $\frac{10}{s(s+20)}$ either left or right.

Block Diagram Reduction

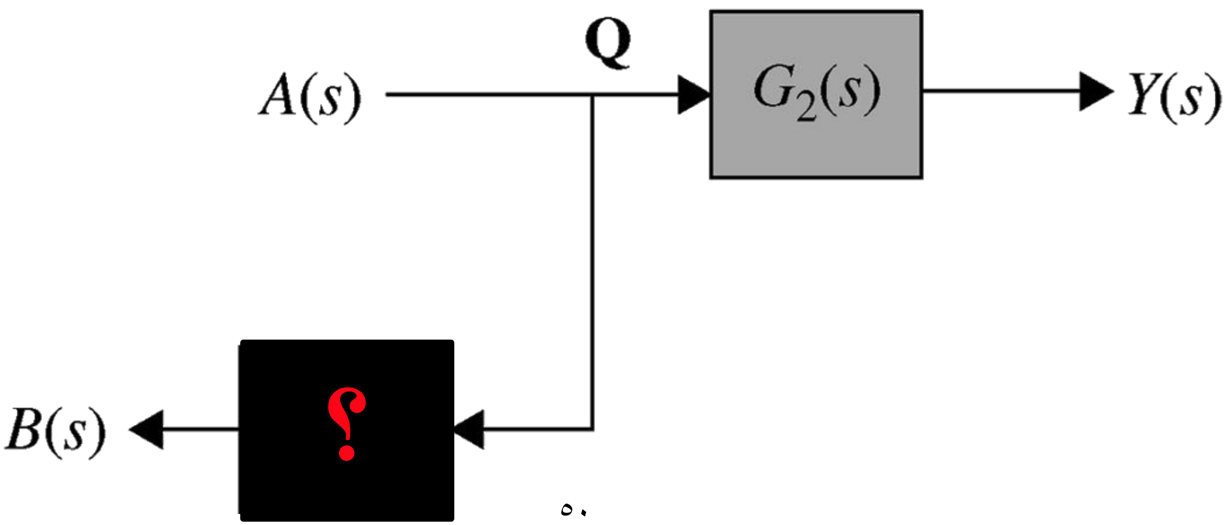


Block Diagram Reduction

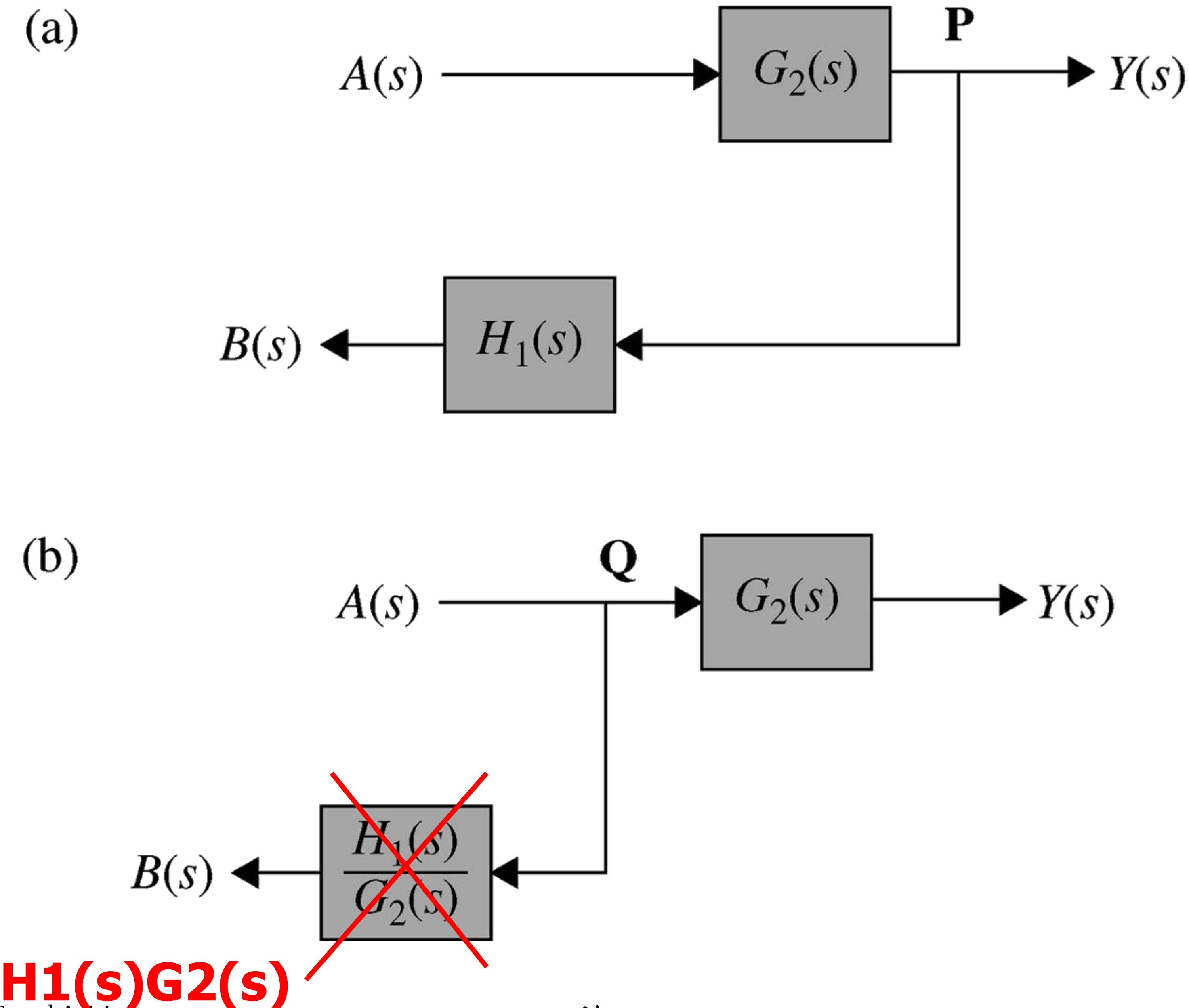
(a)



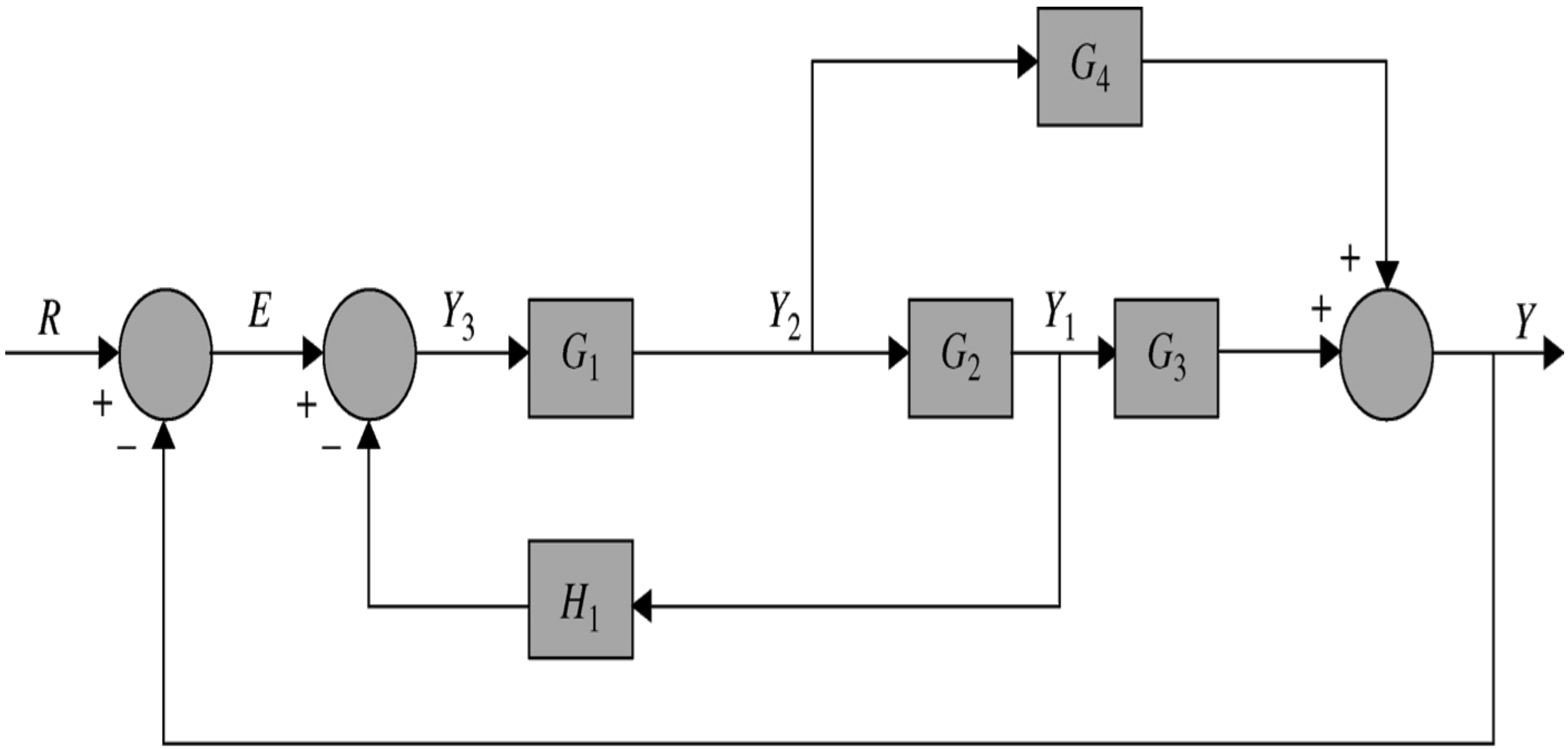
(b)



Block Diagram Reduction

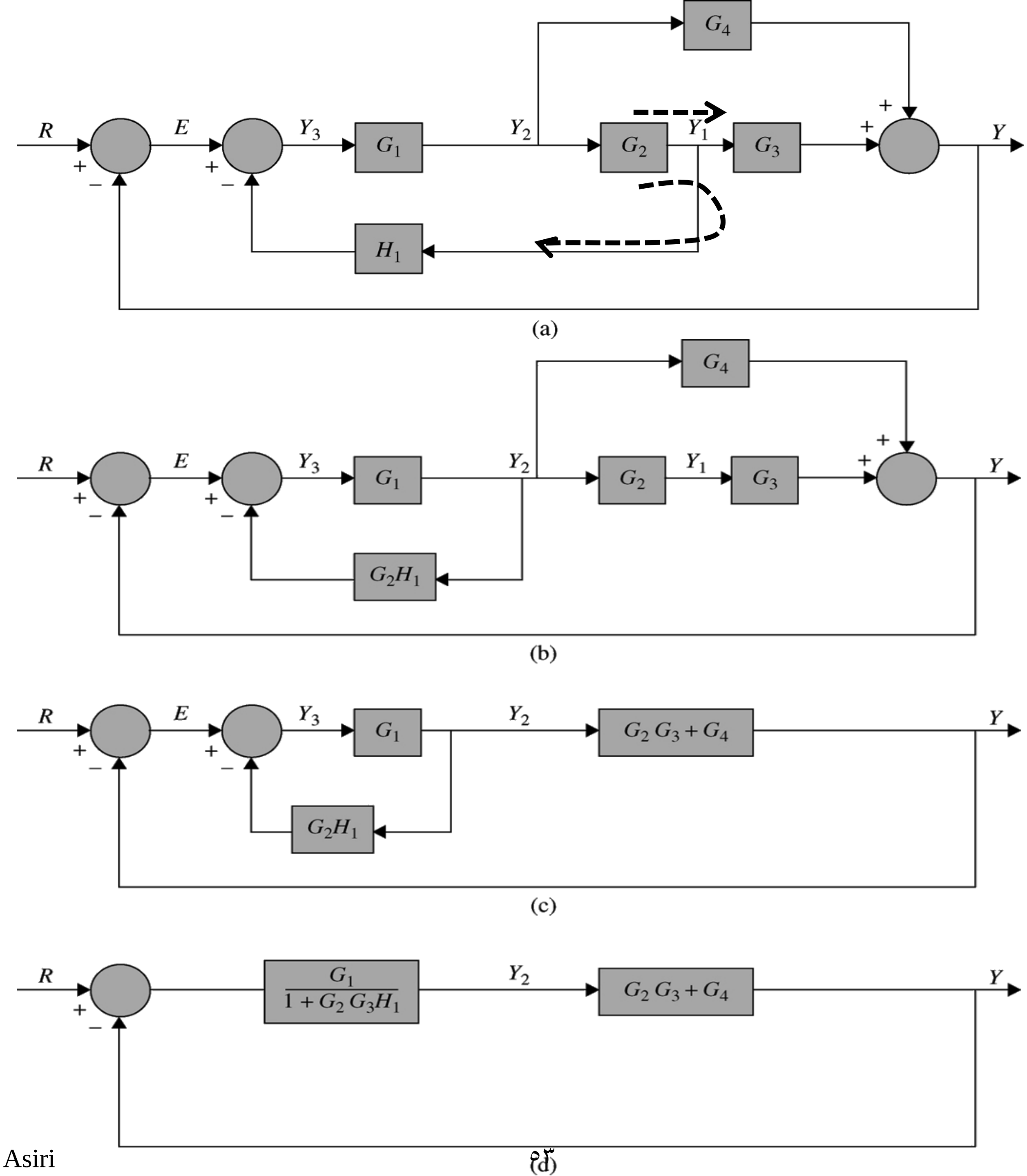


Block Diagram Reduction

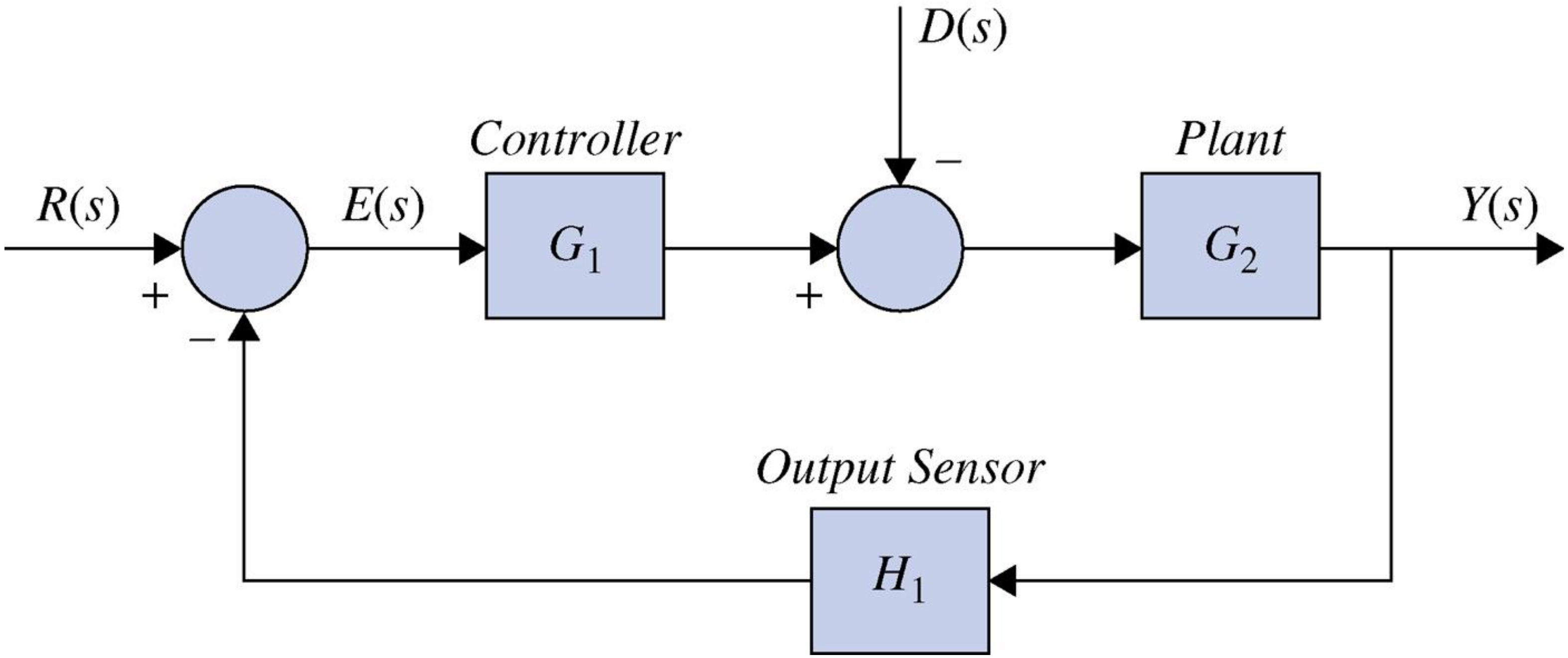


(a)

Block Diagram Reduction



Block Diagram Reduction



Can use superposition:

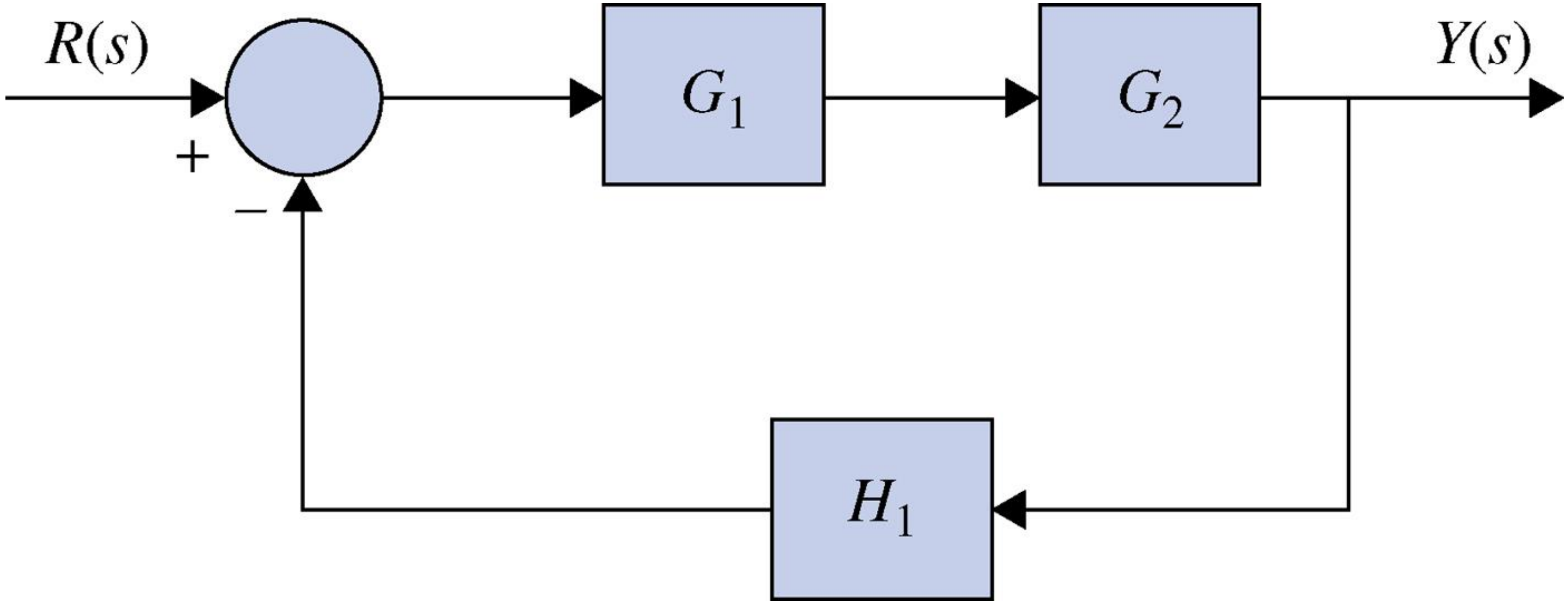
First set D=0, find Y due to R

Then set R=0, find Y due to D

Finally, add the two component to get the overall Y

Block Diagram Reduction

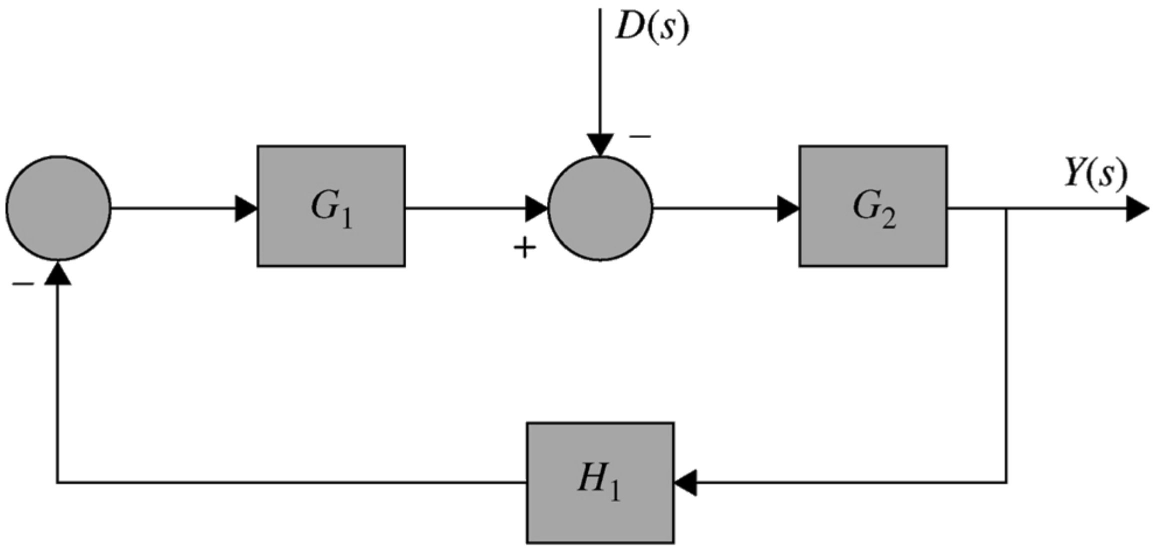
First set D=0, find Y due to R



$$Y_1(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_1} R(s)$$

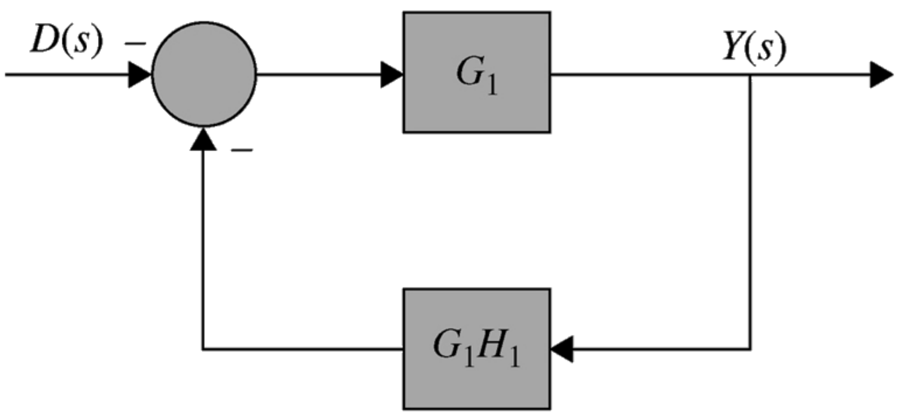
Block Diagram Reduction

Then set R=0, find Y due to D



(a)

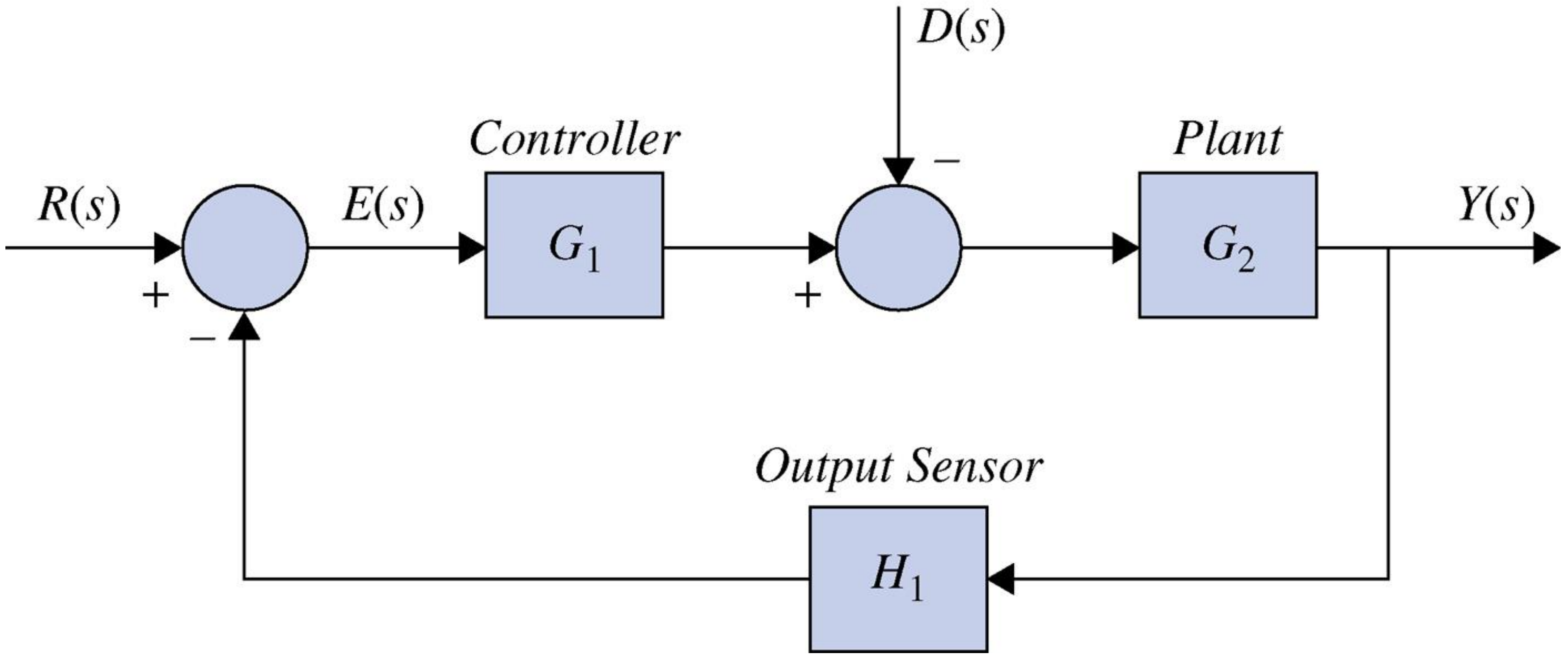
$$Y_2(s) = \frac{G_2}{1 + G_1 G_2 H_1}\left(-D(s)\right)$$



(b)

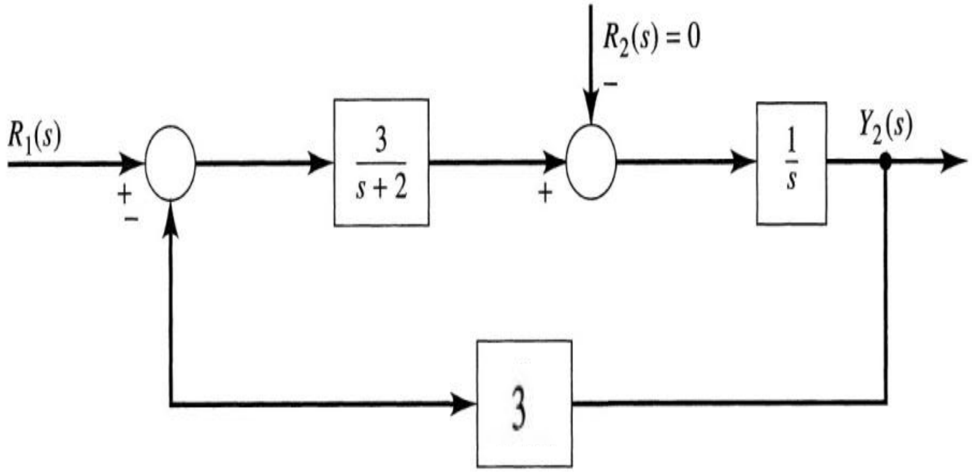
Block Diagram Reduction

Finally, add the two component to get the overall Y

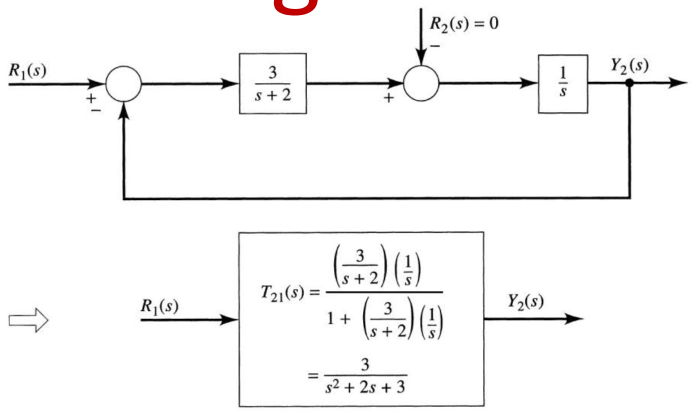


$$Y(s) = \frac{G_1 G_2}{1 + G_1 G_2 H_1} R(s) - \frac{G_2}{1 + G_1 G_2 H_1} D(s)$$

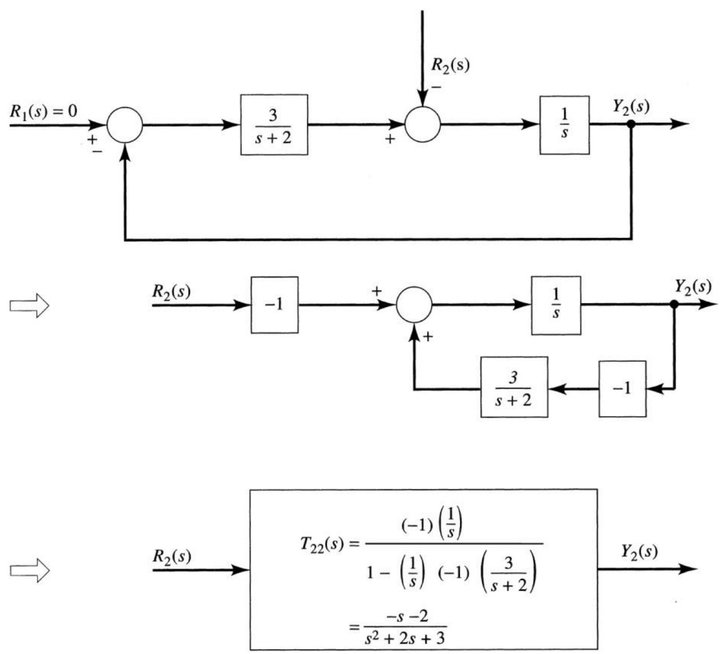
Block Diagram Reduction



Block Diagram Reduction

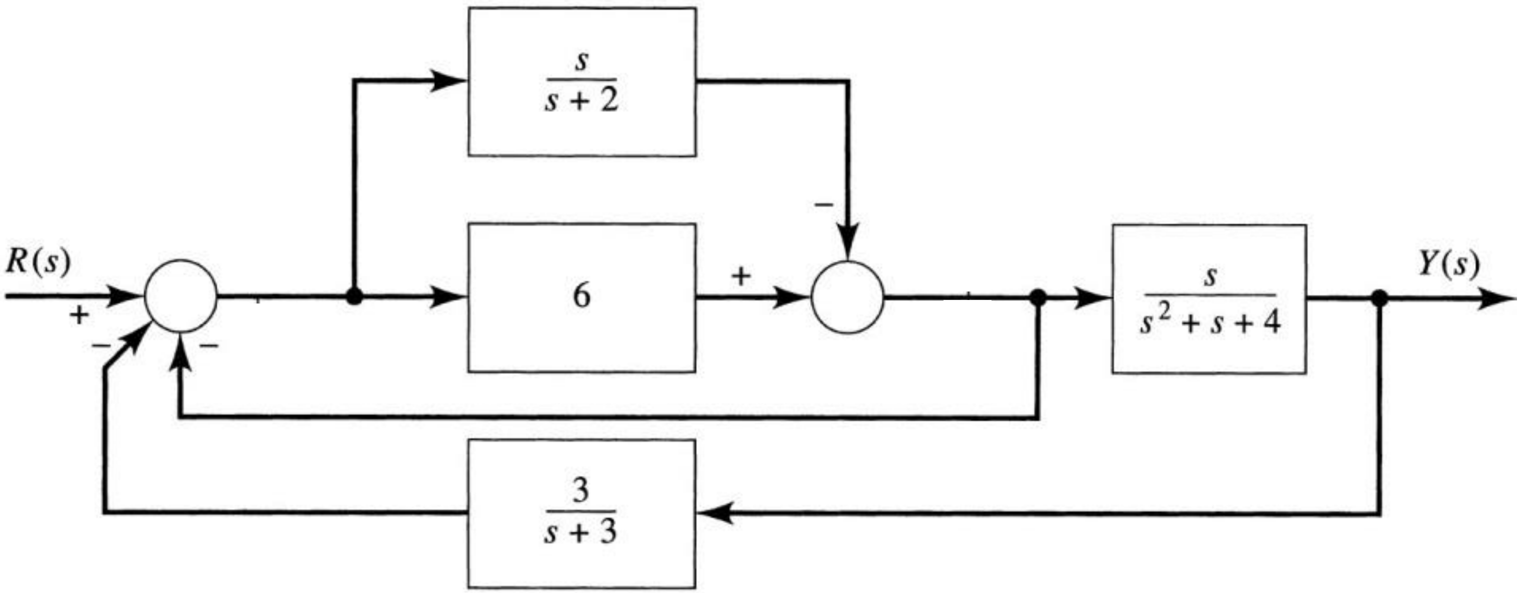


(a)

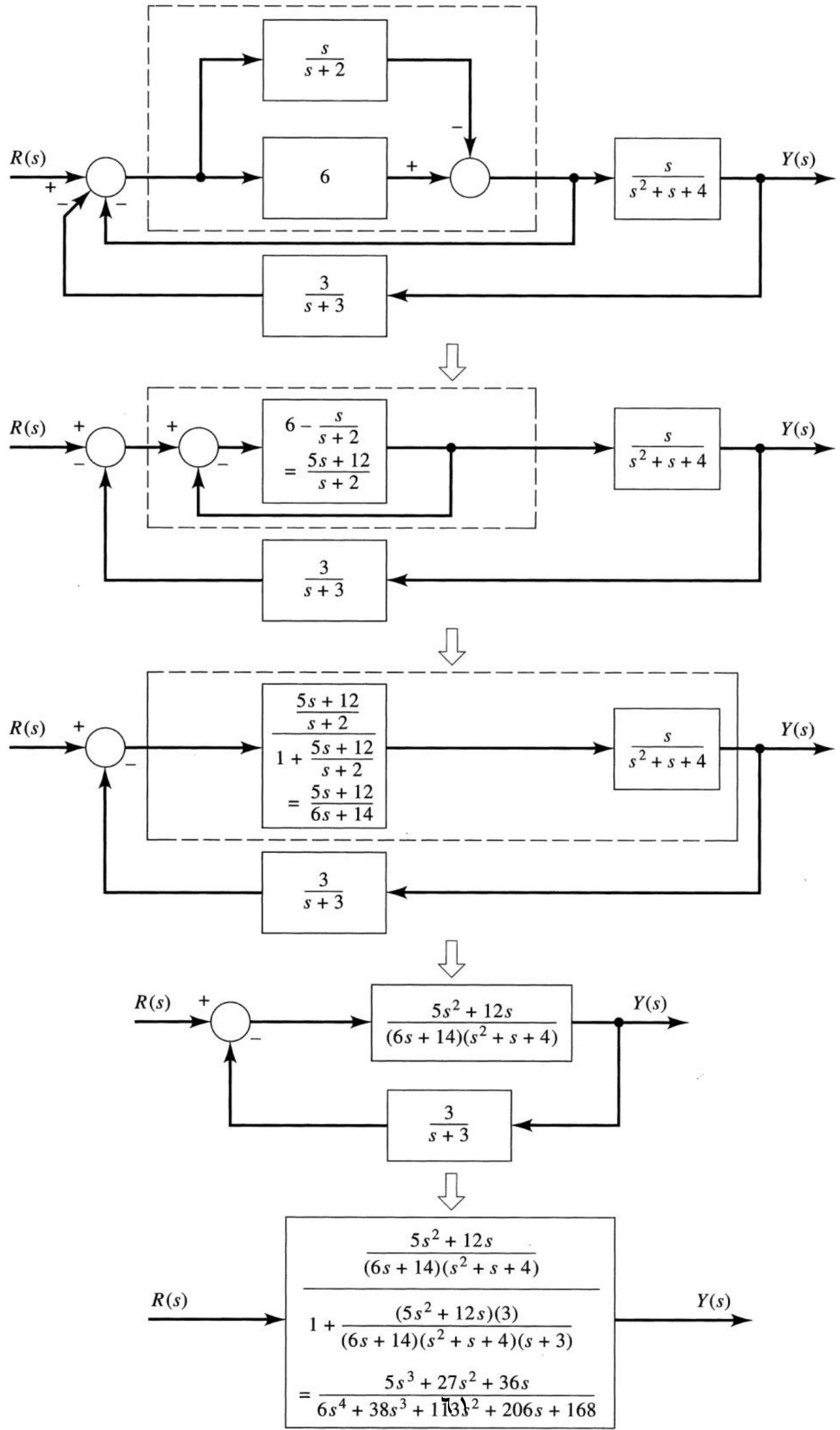


(b)

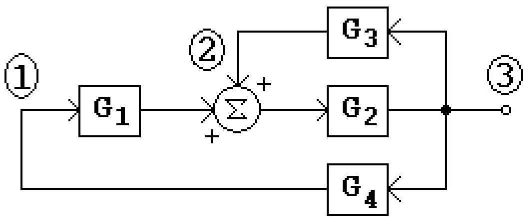
POP. Quiz. 3

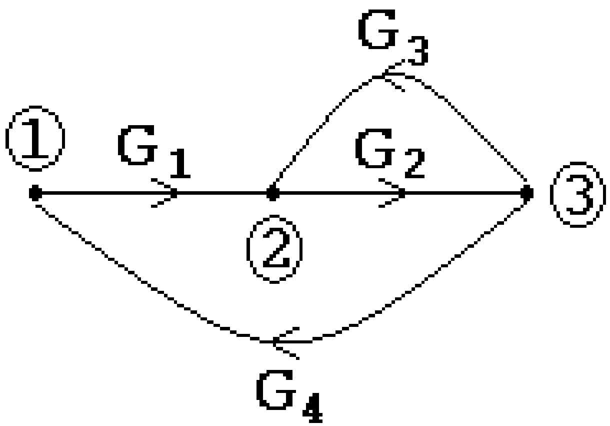


Block Diagram Reduction



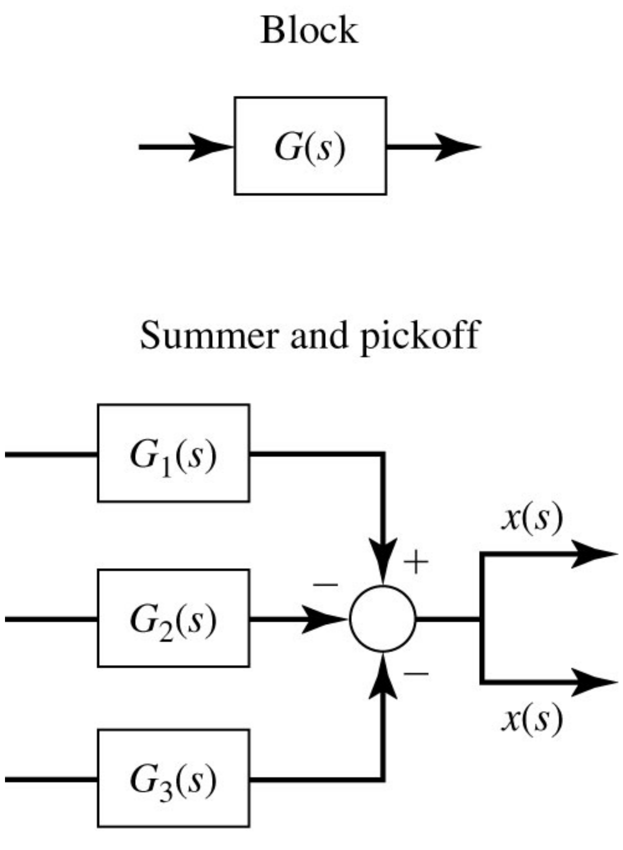
Signal Flow Graph



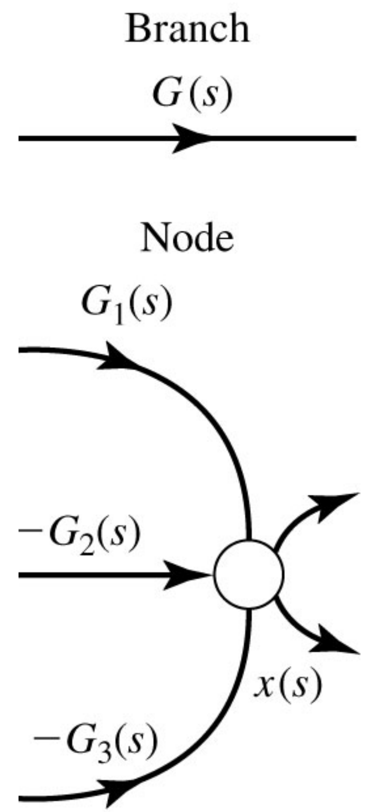


- Compact alternative ***notation to the block diagram***.
- It characterizes the system by a network of directed branches and associated transfer functions.
- The two ways of depicting signal are equivalent.

Signal Flow Graph

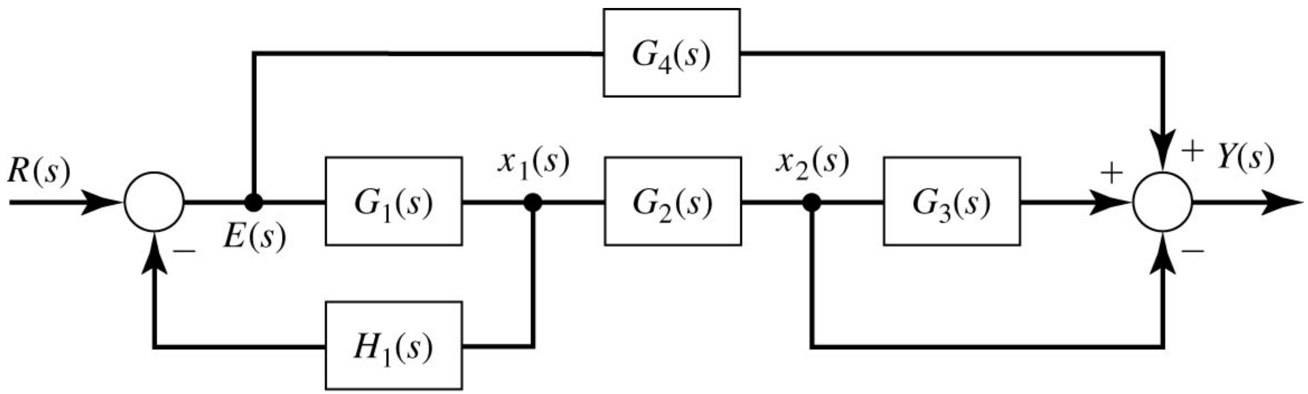


(a)

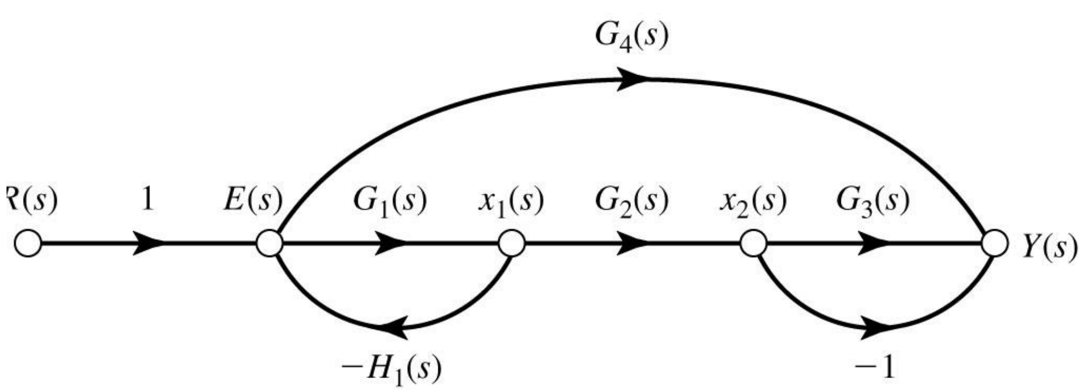


(b)

Signal Flow Graph



(a)



(b)

Signal Flow Graph

Mason's Rule

Mason's gain rule is as follows: the transfer function of a system with signal-input, signal-output flow graphs is

$$T(s) = \frac{p_1\Delta_1 + p_2\Delta_2 + p_3\Delta_3 + \cdots}{\Delta}$$

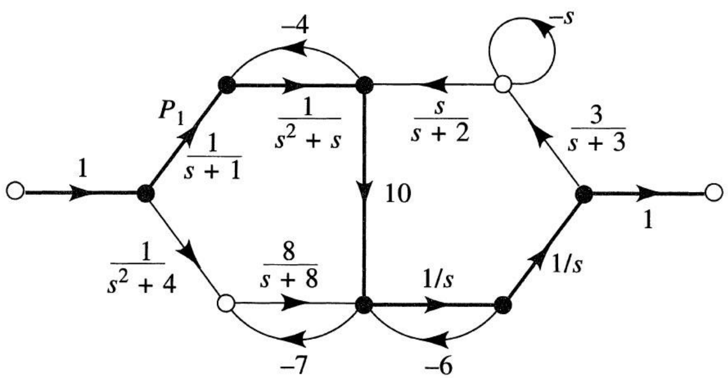
Δ=1-(sum of all loop gains)+(sum of products of gains of all combinations if 2 nontouching loops)- (sum of products of gains of all combinations if 3 nontouching loops)+…

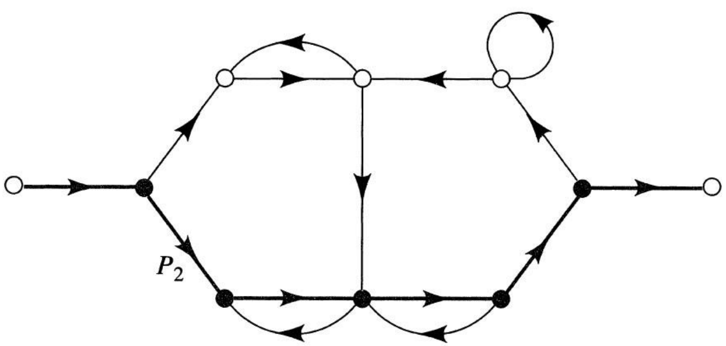
*A **path** is any succession of branches, from input to output, in the direction of the arrows, that does not pass any node more than once.*

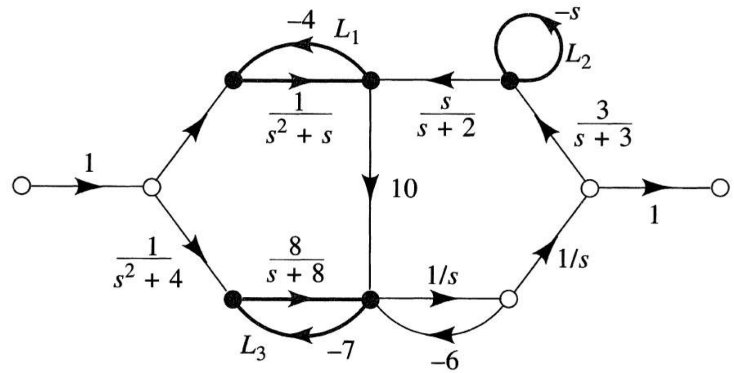
*A **loop** is any closed succession of branches in the direction of the arrows that does not pass any node more than once.*

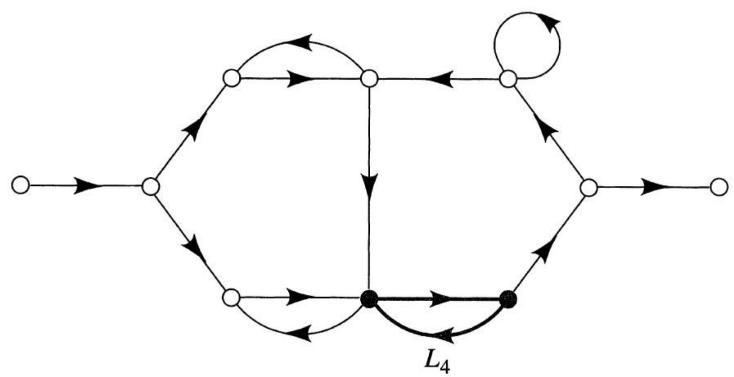
Block Diagram Reduction

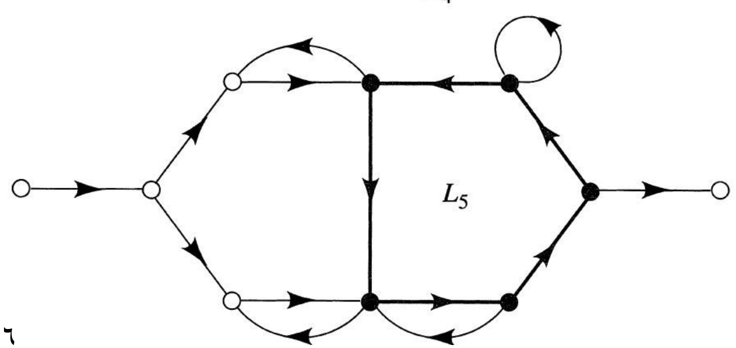
Example 3:











Example 4:

Find: $\dfrac{y_5}{y_3}$