بسم الله الرحمن الرحيم

Automatic Control MENG366 First Exam	Closed Book Exam Time: 1 ½ hrs Mon: 19/8/1424 H

Name:	Number: ....
Q1: ...., Q2: ...., Q3: ...., Q4: ....,	Total: ..../40

1. Pick an example of a closed-loop system that you are familiar with (e.g. temperature control system in an office, liquid-level control system). Draw a simple block diagram of that system, similar to the one shown in Figure 1. For the system you choose, specify the physical input, the physical output, and a possible source of disturbance and noise. Also write down the physical devices that constitute the "controller", the "plant", and the "sensor", but don't worry about writing any transfer functions! Discuss briefly (in one paragraph) how effective or ineffective the control system would be without feedback.

Answer:

For example, the cruise control system in a car: the input is the desired speed and the output is the actual speed of the car. The controller is the cruise control computer, which compares the actual speed with the desired speed, and sets the throttle accordingly. The plant is the car driving on the road. The sensor is the tachometer. A source of disturbance could be a gear change, and a source of noise could be voltage fluctuations on the power bus. (Of course, real cruise control systems are much more complicated and incorporate more than one sensor, but the basic idea is the same.)

Without feedback, the controller would have to "guess" at a throttle setting, based only on the desired speed. If the model of the plant was absolutely perfect, then it could work, but the instant something about the car changed–e.g. it started going up a hill, or the engine began to wear out and run less efficiently, the car's speed would also change and the controller would have no idea how to correct for it.

2. The following block diagram represents a generic closed-loop linear system consisting of a reference input $R(s)$, a disturbance $D(s)$, and sensor noise $N(s)$ and output $C(s)$.

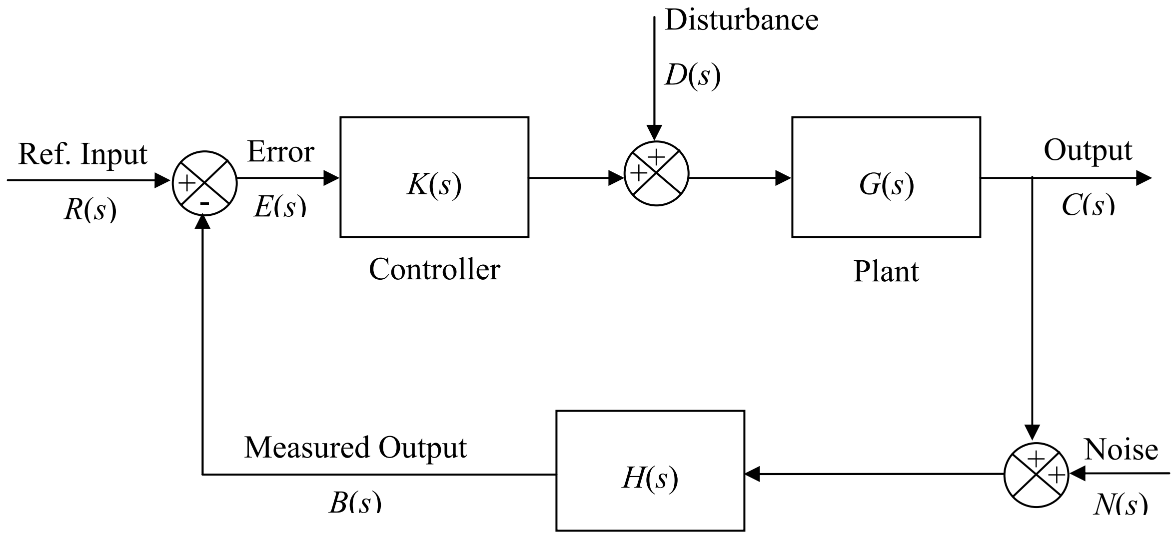


Figure 1: Feedback Loop

a) Evaluate the transfer functions relating the output $C(s)$ to each of the inputs $R(s)$, $D(s)$, and $N(s)$.
Hint: since the system is linear use the property of superposition. For example, to calculate the transfer function C(s)/R(s), set D(s) and N(s) to zero.

b) Assume that $H(s) = 1$ (i.e. unity feedback) and $K(s) = K$ (i.e. the controller is a simple amplifier).
Assume also that there is no noise or disturbance (i.e. N(s) = 0 and D(s) = 0). Derive a simple expression for the error $E(s) = R(s)\text{-}B(s)$, in terms of $R(s)$, $G(s)$, and K.

c) Consider the following plant: $G(s) = \dfrac{1}{s^2 + 2}$ and take $R(s)$ as a unit step input. Assume $H(s) = 1$, $K(s) = K$, $N(s) = 0$, and $D(s) = 0$. Use the Final Value Theorem to find the steady state error, (i.e. $e(t)$ as t goes to infinity).

Using the superposition property of linear systems we can compute the transfer functions from each input the the output individually and then obtain the overall transfer function by adding the individual contributions.

Let's start with the transfer function from r to c. Disregard the other two inputs, i.e. set $D(s) = 0$ and $N(s) = 0$.

$$C(s) = K(s)G(s)E(s) = K(s)G(s)(R(s) - B(s)) = K(s)G(s)\,R(s) - K(s)G(s)H(s)\,C(s) \tag{1}$$

rearranging:

$$C(s) = \frac{K(s)G(s)}{1 + K(s)G(s)H(s)}\,R(s) \tag{2}$$

In a similar fashion, considering one input at the time, for disturbance we get:

$$C(s) = \frac{G(s)}{1 + K(s)G(s)H(s)}\,D(s) \tag{3}$$

and for noise:

$$C(s) = \frac{-K(s)G(s)H(s)}{1 + K(s)G(s)H(s)}\,N(s) \tag{4}$$

As a final result, the overall transfer function is:

$$C(s) = \frac{1}{1 + K(s)G(s)H(s)}\,[G(s)D(s) + K(s)G(s)R(s) - K(s)G(s)H(s)N(s)] \tag{5}$$

If there is no noise and no disturbance, and $H(s) = 1$, the error signal is given by:

$$E(s) = R(s) - B(s) = R(s) - C(s) = R(s) - \frac{1}{1 + KG(s)}\,[KG(s)R(s)] = \frac{1}{1 + KG(s)}R(s) \tag{6}$$

3. Find the closed-loop transfer function $C(s)/R(s)$ for each one of the block diagrams shown in Figure 1 and Figure 2.

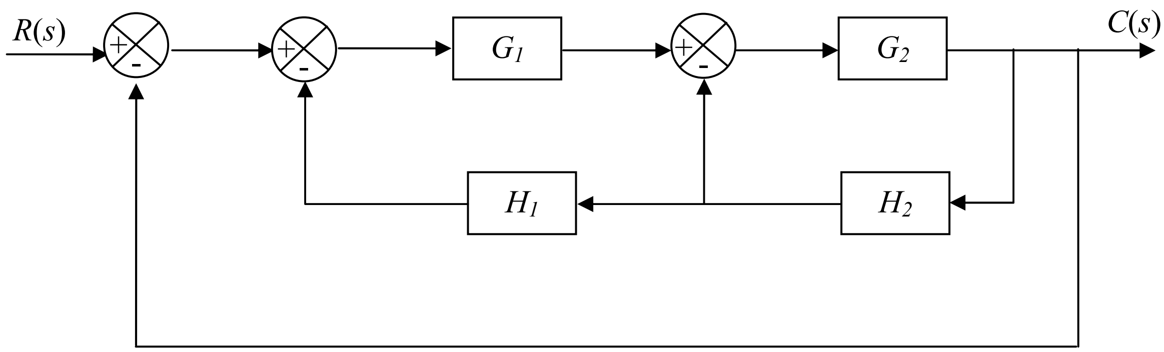
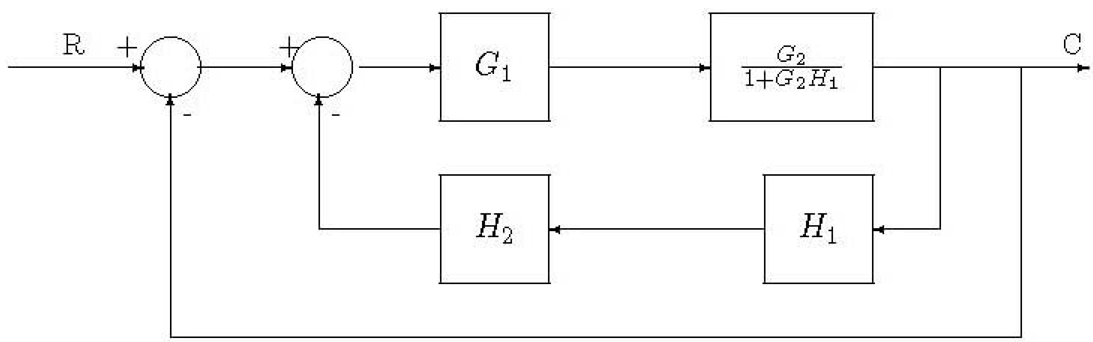
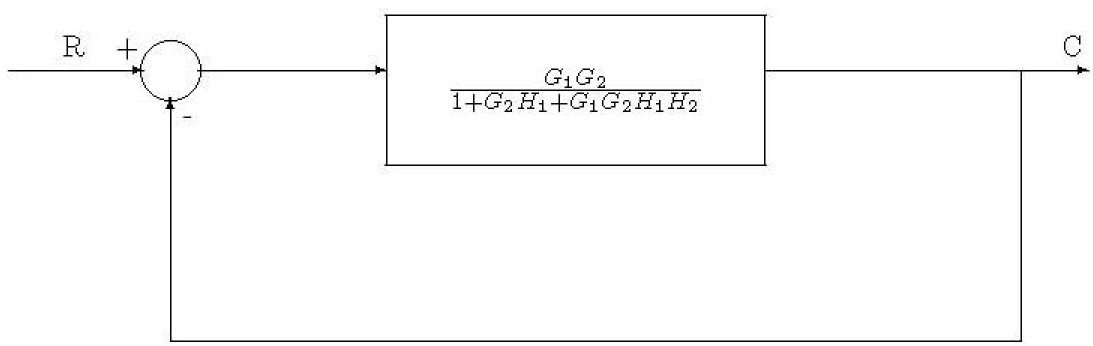


Figure 2: Block Diagram

Answer:





$$\Rightarrow T(s) = \frac{G_1 G_2}{1 + G_1 G_2 + G_2 H_1 + G_1 G_2 H_1 H_2}$$

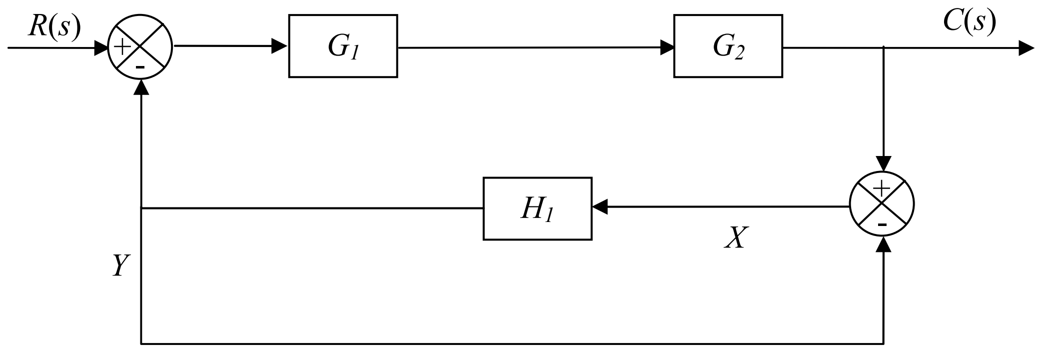
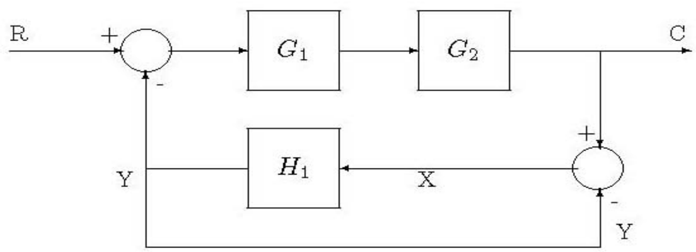


Figure 3: Block Diagram

Answer:



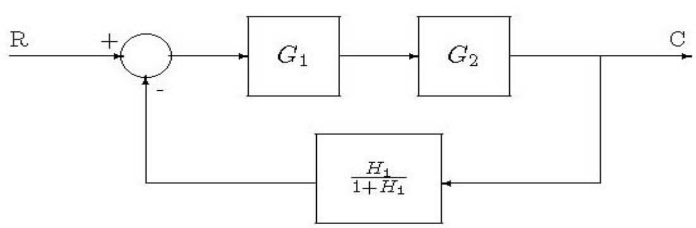
This one is a little trickier. Label the input to H_1 as X, and the output of H_1 as Y, as shown. Therefore, we have:

$$\begin{aligned} X &= C - Y \\ Y &= H_1 X \end{aligned}$$

Solve for X and Y to get:

$$\begin{aligned} X &= \frac{C}{1 + H_1} \\ Y &= \frac{H_1}{1 + H_1} C \end{aligned}$$

So the block diagram can be rewritten as:



And now we can write down the closed-loop transfer function:

$$T(s) = \frac{G_1 G_2 (1 + H_1)}{1 + H_1 (1 + G_1 G_2)}$$

4. Solve the following differential equations using Laplace transforms (again, $u(t)$ is the unit step function):

a) $2\dot{y}(t) + 3y(t) = e^{-t}$, with $y(0) = 1$;
b) $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = u(t)$, with $\dot{y}(0) = 1/2$ and $y(0) = -1$

Answer:

a) Taking the Laplace transform (remember the initial conditions), we get:

$$2sY(s) - 2y(0) + 3Y(s) = \frac{1}{s+1}$$

Solving for $Y(s)$ we get:

$$Y(s) = \frac{1}{2s+3}\left[\frac{1}{s+1} + 2\right] = \frac{1}{s+1}$$

Finally, going back to the time domain:

$$y(t) = e^{-t}$$

b) Proceed in the same way for the second problem:

$$(s^2Y(s) - sy(0) - \dot{y}(0)) + 5(sY(s) - y(0)) + 6Y(s) = \frac{1}{s}$$

$$(s^2Y(s) + s - \frac{1}{2}) + 5(sY(s) + 1) + 6Y(s) = \frac{1}{s}$$

Solving for $Y(s)$:

$$Y(s) = \frac{1}{2}\left[\frac{-2s^2 - 9s + 2}{(s+3)(s+2)s}\right]$$

Expanding into partial fractions:

$$Y(s) = \frac{11/6}{s+3} - \frac{3}{s+2} + \frac{1/6}{s}$$

Going back to the time domain:

$$y(t) = \frac{11}{6}e^{-3t} - 3e^{-2t} + \frac{1}{6}$$