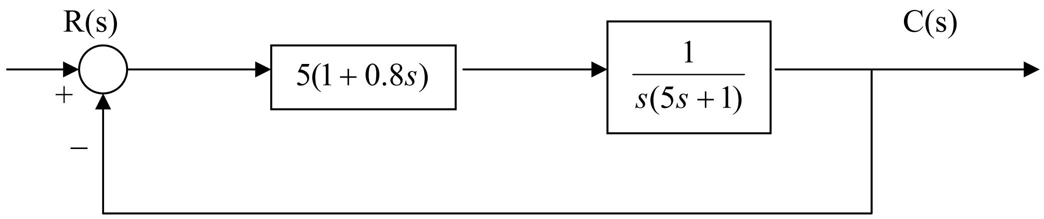
بسم الله الرحمن الرحيم

Automatic Control	Closed Book Exam
MENG366	Time: 1 ½ hrs
Second Exam	Wednesday: 19/8/1424 H

Name:				No.:
Q1:	Q2:	Q3:	Q4:	Total: /40

1. The unity feedback positional servo system with PD-Control is given. What is the type of the system? Find positional error, velocity error and acceleration error constants and corresponding steady state errors of the system.



Solution;

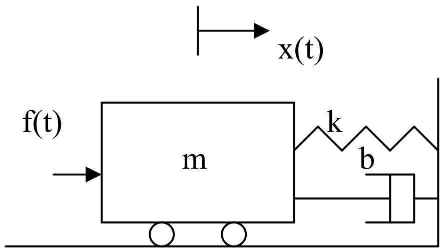
$$G(s) = \frac{5(1+0.8s)}{s(5s+1)}$$ Type 1 system

$$K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{5(1+0.8s)}{s(5s+1)} = \infty \qquad e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\frac{5(1+0.8s)}{s(5s+1)} = 5 \qquad e_{ss} = \frac{1}{K_v} = \frac{1}{5} = 0.2$$

$$K_a = \lim_{s\to 0} s^2G(s) = \lim_{s\to 0} s^2\frac{5(1+0.8s)}{s(5s+1)} = 0 \qquad e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

2. The mechanical system has a mass of 1kg, stiffness of 40 N/m and damping coefficient of 4 Ns/m. If the external force is $f(t) = 12\delta(t)$. Find the response x(t).



solution;

$m\ddot{x} + b\dot{x} + k = f(t)$ equation of motion

$\ddot{x} + 4\dot{x} + 40 = 12\delta(t)$

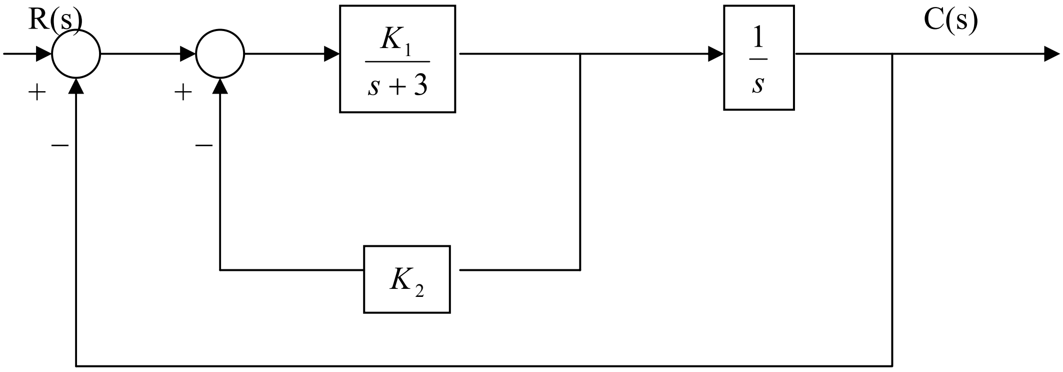
Transfer function between X(s) and F(s) is

$$X(s) = \frac{1}{s^2 + 4s + 40} F(s)$$

$$X(s) = \frac{12}{s^2 + 4s + 40} = \frac{12}{6} \frac{6}{(s+2)^2 + 36}$$ from Laplace table

$$x(t) = 2e^{-2t} \sin 6t$$

3. For the following closed loop control system; overshoot should not exceed 10% and peak time should be less than or equal 0.4s. Find K_1 and K_2 gain values that will satisfy the conditions. What is the settling time of the system if 2% error is assumed.



Solution:

Closed loop transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{K_1}{s^2 + (3 + K_1 K_2)s + K_1}$$

$$e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.10 \quad \text{solution of this equation gives} \quad \zeta = 0.59$$

peak time is

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

natural frequency is found as

$$\omega_n = \frac{\pi}{t_p \sqrt{1-\zeta^2}} = \frac{\pi}{0.4\sqrt{1-0.59^2}} = 9.73 \text{ rad/s}$$

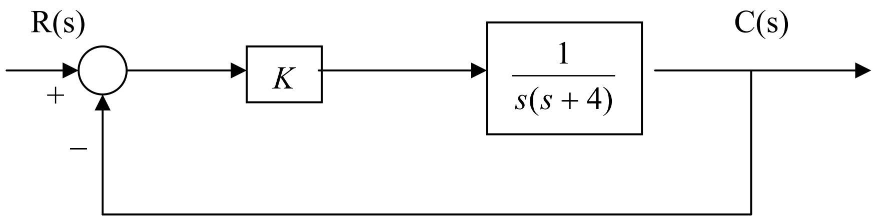
From the characteristic equation

$$2\zeta\omega_n = 3 + K_1 K_2 = 2(0.59)(9.73) = 11.48$$

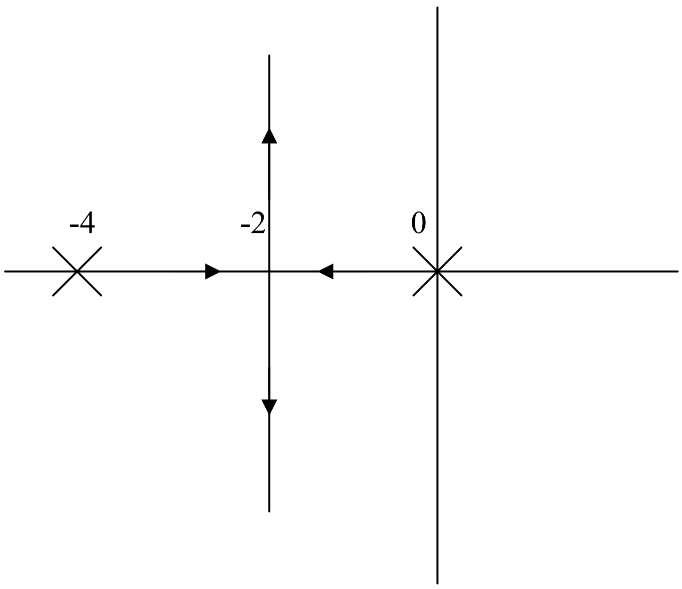
$$\omega_n^2 = K_1 = 9.73^2 = 94.67 \qquad K_2 = 0.0896$$

settling time $t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.59(9.73)} = 0.7s$

4. Plot the root loci for the system given. What is the value of K, if roots are equal to $s_{1,2} = -2 \mp 2j$.



Solution



$$1 + G(s)H(s) = 1 + \frac{K}{s(s+4)} = 0$$

$$\left|\frac{K}{s(s+4)}\right| = 1 \qquad K = |s||s+4| = |-2+2j||-2+2j+4| = 8$$

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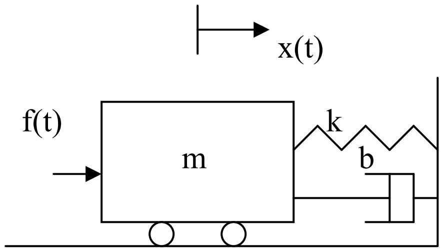

solution;

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