بسم الله الرحمن الرحيم

King Abdulaziz University
Engineering College
Department of Production and Mechanical System Design

MENG 366 Automatic Control

First Exam
Closed-book Exam
Wednesday: 3/2/1425 H
Time Allowed: 60 mins

Name:	Sec. No.:	ID No.:

Question 1		25
Question 2		25
Question 3		25
Question 4		25
TOTAL		100

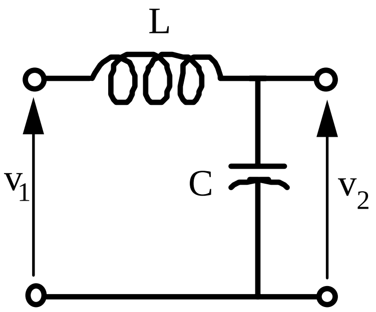
Instructions

2. There are totally 4 problems in this exam.
3. Show all work for partial credit.
4. Assemble your work for each problem in logical order.
5. Justify your conclusion. I cannot read minds.

بسم الله الرحمن الرحيم

Automatic Control	Closed Book Exam
MENG366	Time: 1 Hour
First Exam	Wednesday: 3/2/1425 H

1. Consider the electrical system shown in Figure 1.



   a) Find the transfer function, $V_2(s)/V_1(s)$.
   b) In case of unit step input, find $v_2(t)$.
   c) Draw the system response, $v_2(t)$ for unit step input.

Sol.:

a)

$$V_1 = [Ls]I(s) + [1/Cs]I(s) = [Ls + (1/Cs)]I(s)$$

$$V_2 = [1/Cs]I(s)$$

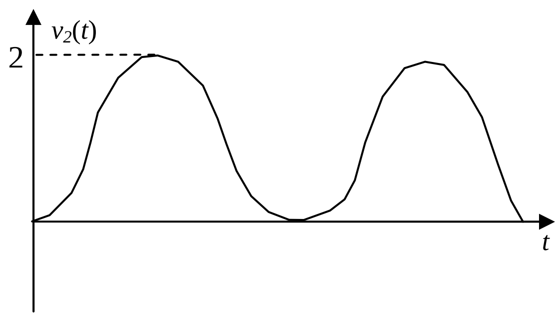
So that,

$$\frac{V_2(s)}{V_1(s)} = \frac{(1/Cs)}{[Ls + (1/Cs)]} = \frac{1}{[LCs^2 + 1]}$$

b) Unit step input: $V_1(s) = \dfrac{1}{s}$

Then,

$$V_2(s) = \frac{1}{s\left(LCs^2+1\right)} = \frac{\frac{1}{LC}}{s\left(s^2+\frac{1}{LC}\right)}$$



From Table 2-1 page 17-18
(raw number 25):

$$v_2(t) = 1 - \cos\left(\frac{1}{\sqrt{LC}}t\right)$$

2. Hamzah is a student in the engineering college in KAAU. He takes his bike to the university every day, and after having studied automatic control, he has started to ponder what transfer function his bike may have. Hamzah decides to use a 1st order model with unknown coefficients to model the transfer function between the angle of the handlebars (input signal $u(t)$) and the position of the tire relative to a reference trajectory (output signal $y(t)$):

$$G(s) = \frac{a}{s+b}$$

Hamzah then performs an experiment where he lets the angle of the handlebars vary as a sinusoid with period of 0.5 s, and measures the output signal. Input and output after the transients have faded is shown in Figure 2 where the input is dashed and the output is a straight line. Determine the constants a and b.

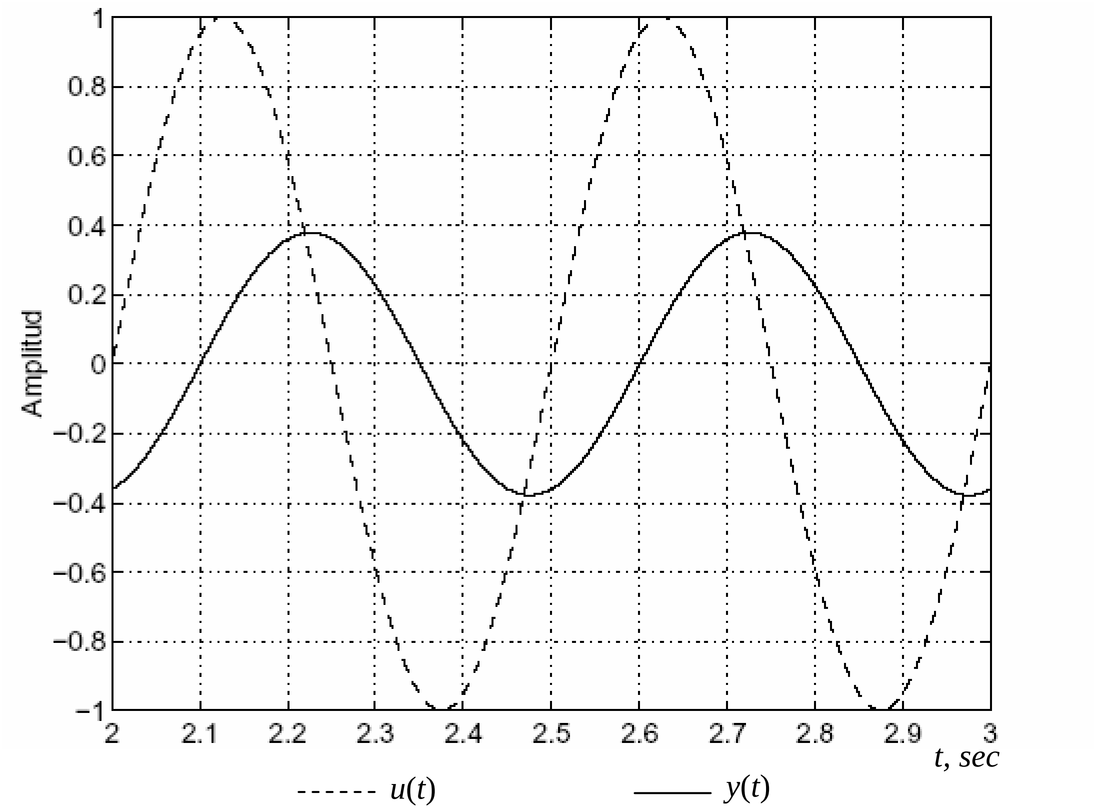


Figure 2

Sol.:

$\omega = \frac{2\pi}{T} = 4\pi$

$$u(t) = \sin(4\pi t) \Rightarrow \ y(t) = |G(4\pi i)| \sin\left(4\pi t + \arg(G(4\pi i))\right)$$

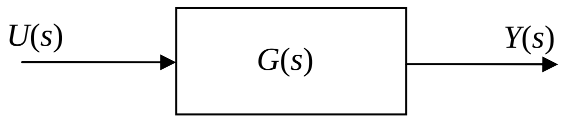
	Model	Figure
$\arg(G(4\pi i))$	$-\arctan(\frac{4\pi}{b})$	$-0.1\text{s} \cdot 4\pi \text{ rad/s} = -0.4\pi \text{ rad}$
$\lvert G(4\pi i)\rvert$	$\frac{a}{\sqrt{16\pi^2+b^2}}$	0.39

$$\frac{4\pi}{b} = \tan(0.4\pi) \Rightarrow \ b = 4.083$$

$$\frac{a}{\sqrt{16\pi^2 + b^2}} = \frac{a}{13.2} = 0.39 \Rightarrow \ a = 5.15$$

$$\boxed{G(s) = \frac{5.51}{s+4.08}}$$

3. Consider the following system:



$$G(s) = \frac{s+4}{s^2+4s+3}$$

a) Find all the zeros and the poles of the system.
b) Find the response of the system if the input is a unit impulse.
c) Write the state space representation of the system.
d) Is the system stable? Why?

Sol.:
a) One zero at s = -4 and two poles at s = -1 and s = -3.
b)

$$G(s) = \frac{s+4}{s^2+4s+3} = \frac{s+4}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3} = \frac{As+3A+Bs+B}{(s+1)(s+3)} = \frac{(A+B)s+(3A+B)}{(s+1)(s+3)}$$

$A+B=1$ and $3A+B=4$

Hence, $A = \frac{3}{2}$ and $B = -\frac{1}{2}$.

$\frac{Y(s)}{U(s)} = \frac{3}{2}\frac{1}{s+1} - \frac{1}{2}\frac{1}{s+3}$. However, $R(s)=1$

Then, $Y(s) = \frac{3}{2}\frac{1}{s+1} - \frac{1}{2}\frac{1}{s+3}$.

$y(t) = \frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t}$

c) As $\frac{Y(s)}{U(s)} = \frac{s+4}{s^2+4s+3}$ then the differential equation is:

$$\ddot{y} + 4\dot{y} + 3y = \dot{u} + 4u$$

Then,

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \text{and} \quad D = 0$$

d) The system is *stable* because all the poles lie on the left side of s-plane.

4. A system can be described with the following block diagram:

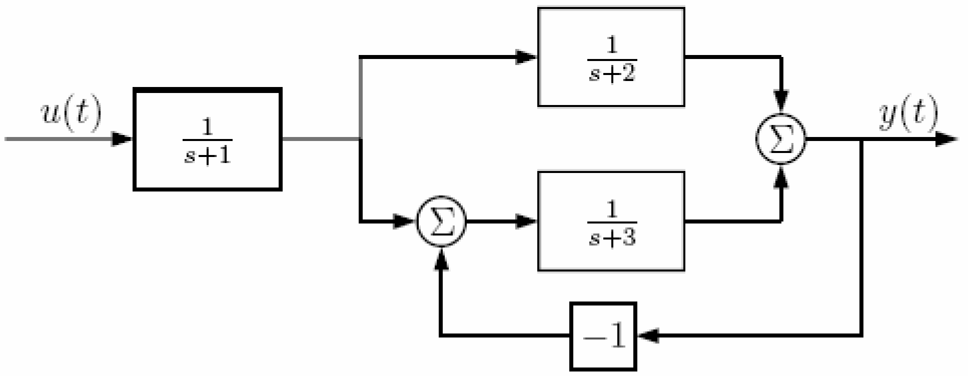


Figure 1

Determine the transfer function from *u*(*t*) to *y*(*t*)?

Sol.:

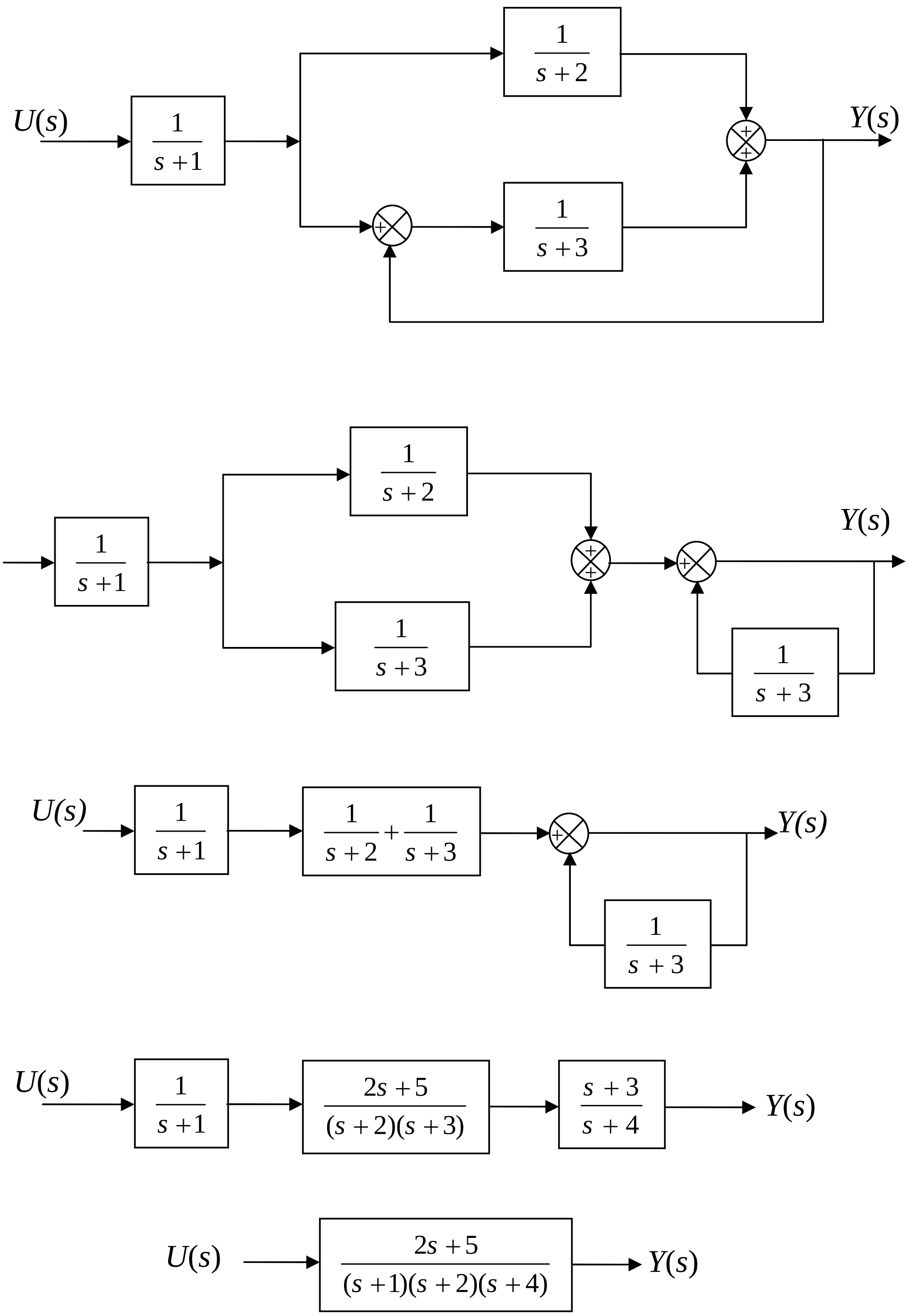
Method I:

$$Y(s) = \frac{1}{s+1}\frac{1}{s+2}U(s) + \frac{1}{s+1}\frac{1}{s+3}U(s) - \frac{1}{s+3}Y(s) \Rightarrow$$

$$\frac{s+4}{s+3}Y(s) = \frac{1}{s+1}\frac{2s+5}{(s+2)(s+3)}U(s)$$

$$Y(s) = \frac{2s+5}{(s+1)(s+2)(s+4)}$$

Method II:



can check your answers online right after the exam.
go to the course website at:
http://www.asiri.net

مع دعواتي لكم بالتوفيق والنجاح

د. سعيد عسيري