

The uncontrolled system has the transfer function

$$G(s) = \frac{1}{3s^3 + 2s^2 + 4s + 7}$$

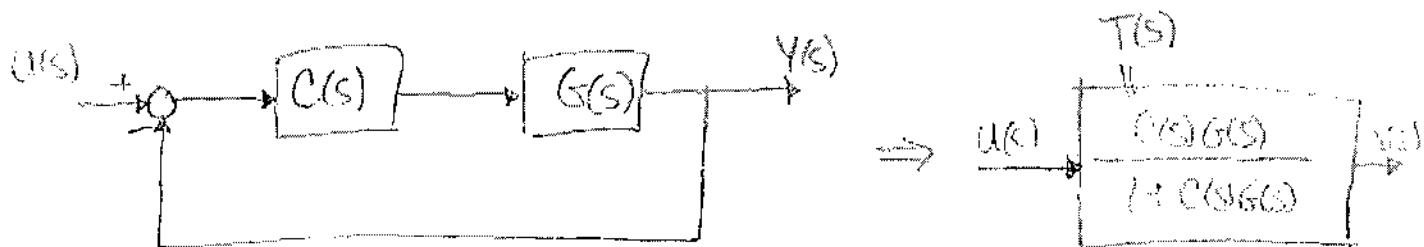
Show $H(s)$ is unstable

$$b_1 = -\frac{\begin{vmatrix} 3 & 4 \\ 2 & 7 \end{vmatrix}}{2} = -\frac{(21-8)}{2} = -6.5$$

s^3	3	4
s^2	2	7
s^1	-6.5	0
s^0	7	

Two sign changes
 \Rightarrow 2 rhp poles
 \Rightarrow Unstable

Now consider the feedback control system



④ Find range of k_p & k_i to stabilize the closed loop system

$$C(s) = \frac{s k_p + k_i}{s}$$

Closed loop:

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} = \frac{\left(s k_p + k_i \right) \left(\frac{1}{3s^3 + 2s^2 + 4s + 7} \right)}{1 + \left(s k_p + k_i \right) \left(\frac{1}{3s^3 + 2s^2 + 4s + 7} \right)}$$

$$T(s) = \frac{s + k_p + k_i}{s(3s^3 + 2s^2 + 4s + 7) + sk_p + k_i}$$

$$= \frac{s k_p + k_i}{3s^4 + 2s^3 + 4s^2 + (7+k_p)s + k_i}$$

$$= \frac{s k_p + k_i}{3s^4 + 2s^3 + 4s^2 + (7+k_p)s + k_i}$$

Use Routh Table to find stable range of k_i , k_p

s^4	3	4	k_i
s^3	2	$7+k_p$	0
s^2	$-6.5 - 1.5k_p$	k_i	0
s^1	$7+k_p$	0	
s^0	k_i	0	

$$D_1 = \frac{1}{2} \begin{vmatrix} 3 & 4 \\ 2 & 7+k_p \end{vmatrix} = \frac{-(3(7+k_p) - 8)}{2} = \frac{-(21 + 3k_p - 8)}{2} = \frac{-(13 + 3k_p)}{2}$$

$$C_1 = \frac{1}{b_1} \begin{vmatrix} 2 & 7+k_p \\ b_1 & k_i \end{vmatrix} = \frac{-(2k_i - b_1(7+k_p))}{b_1} = \frac{-[2k_i - (-6.5 - 1.5k_p)(7+k_p)]}{-6.5 - 1.5k_p}$$

$$C_i = \frac{1}{6.5 + 1.5 k_p}$$

$$= \frac{2k_i + 45.5 + 17k_p + 1.5k_p^2}{6.5 + 1.5 k_p}$$

For stability:

$$k_i > 0$$

$$C_i > 0$$

$$-6.5 - 1.5 k_p > 0$$

$$2k_i + 45.5 + 17k_p + 1.5k_p^2 > 0$$

$$-6.5 > 1.5 k_p$$

pick

$$0 > -\frac{13}{3} > k_p$$

$$45.5 + 17k_p + 1.5k_p^2 > -2k_i$$

$$22.75 + 8.5k_p + 0.75k_p^2 > -k_i$$

condition

$$\boxed{k_i > 0 \quad -\frac{13}{3} > k_p}$$

✗

Steady state error $e(t) = u(t) - y(t)$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} sE(s)$$

$$E(s) = U(s) - Y(s)$$

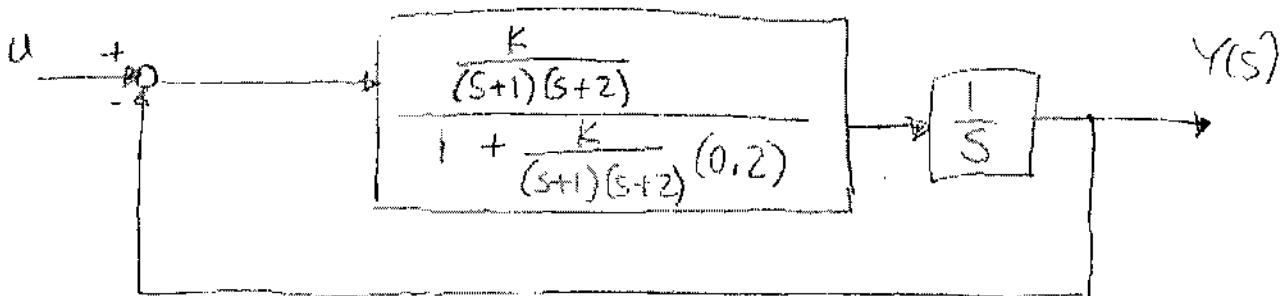
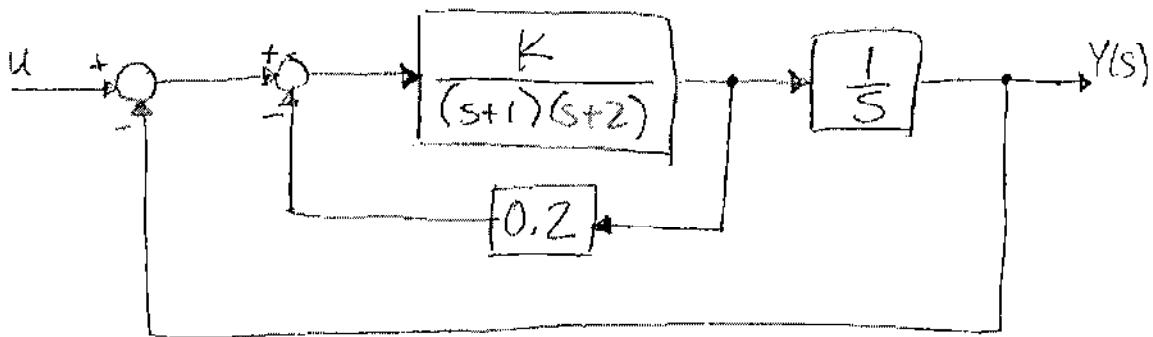
$$Y(s) = U(s)T(s)$$

$$E(s) = U(s) - U(s)T(s) = (1 - T(s))U(s)$$

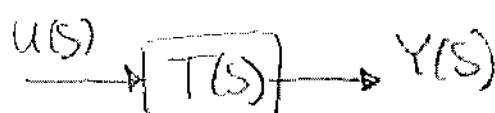
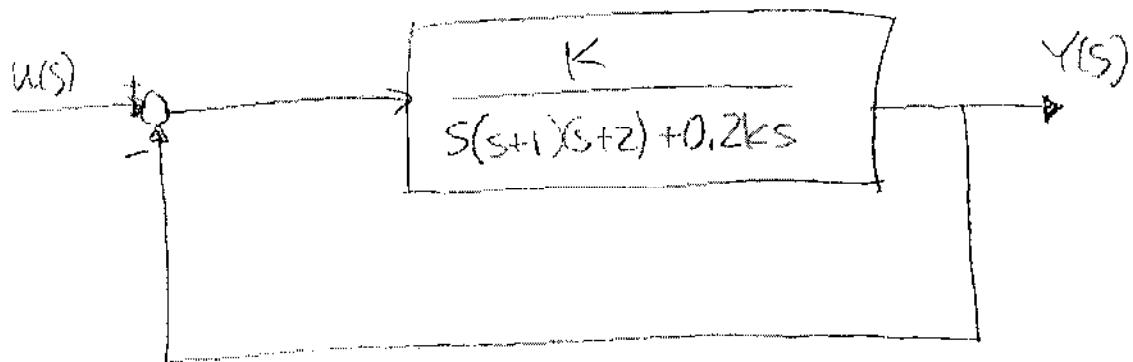
$$E(s) = \left(1 - \frac{s k_p + k_i}{3s^4 + 2s^3 + 4s^2 + 9s + 5k_p k_i}\right) \cdot \frac{1}{s}$$

$$\lim_{s \rightarrow 0} sE(s) = \left(1 - \frac{k_i}{k_i}\right) = (1 - 1) = \boxed{0 = e_\infty}$$

Plot root locus of figure 3 by hand & find range of K for stability.



$$\frac{\frac{K}{(s+1)(s+2)}}{s(s+1)(s+2) + 0.2ks} = \frac{K}{s(s+1)(s+2) + 0.2ks}$$



Final closed loop calculation

$$\frac{\frac{K}{S(S+1)(S+2)+0.2KS}}{1 + \frac{K}{S(S+1)(S+2)+0.2KS}} = \frac{K}{\frac{S(S+1)(S+2)+0.2KS}{S(S+1)(S+2)+0.2KS} + K} = \frac{K}{\frac{S(S+1)(S+2)+K(0.2S+1)}{S(S+1)(S+2)}} = \frac{K}{1 + \frac{K(0.2S+1)}{S(S+1)(S+2)}} = T(s)$$

Closed loop characteristic equation

① $1 + \frac{K(0.2S+1)}{S(S+1)(S+2)} = 1 + G_{OL}(s)$

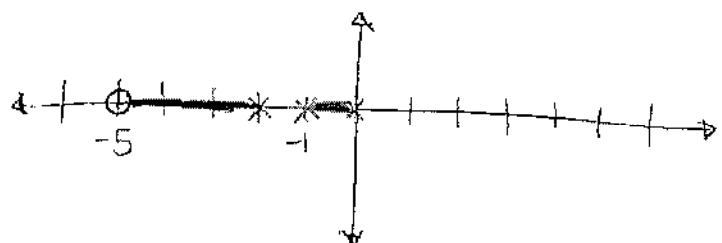
② open loop zeros and poles, $G_{OL}(s)$ is open loop TF

$$z_1 = -5 \quad p_1 = 0 \quad p_2 = -1 \quad p_3 = -2$$

$m=1$ zeros

$n=3$ poles

③ find the root locus on the real axis



center of asymptotes

$$C_0 = \frac{P_1 + P_2 + P_3 - Z_1}{n-m} = \frac{0 + (-1) + (-2) - (-5)}{3-1} = 1$$

angle of asymptotes

$$\theta = \pm \frac{(2k+1)180^\circ}{n-m} = \pm \frac{(2k+1)180^\circ}{2} = \pm (2k+1)90^\circ$$

$$k=0 \quad \theta = 90^\circ$$

$$k=1 \quad \theta = 270^\circ$$

$$k=2 \quad \theta = 450^\circ = 90^\circ \quad \text{repeat}$$

⑤ Break-in, Break-away pts

$$R(s) = D(s)N(s) - D(s)N'(s) = 0$$

$$D(s) = s(s+1)(s+2) = s(s^2+3s+2) = s^3+3s^2+2s$$

$$N(s) = 0.2s+1$$

$$R(s) = (s^3+3s^2+2s)(0.2s+1) - (s^3+3s^2+2s)(0.2) = 0$$

$$0.6s^2 + 1.2s^2 + 0.4s + 3s^2 + 6s + 2 - 0.2s^3 - 0.6s^2 - 0.4s = 0$$

$$0.4s^3 + 3.6s^2 + 6s + 2 = 0$$

$$s_1 = -0.448$$

on root locus

$$s_2 = -1.61$$

not on root locus

$$s_3 = -6.94$$

locus

b.c. there are no complex poles or zeros

⑦ Imaginary Axis Crossings

use closed loop characteristic equation in the form of a polynomial

$$P(s) = s(s+1)(s+2) + K(0.2s+1)$$

$$P(s) = (s^3 + 3s^2 + 2s) + 0.2Ks + K$$

substitute $s=j\omega$, & set polynomial to 0

$$\begin{aligned} P(j\omega) &= (j\omega)^3 + 3(j\omega)^2 + 2j\omega + 0.2Kj\omega + K = 0 \\ &= -j\omega^3 - 3\omega^2 + 2j\omega + 0.2Kj\omega + K = 0 \\ &= -3\omega^2 + K + j\omega(-\omega^2 + 2 + 0.2K) = 0 \end{aligned}$$

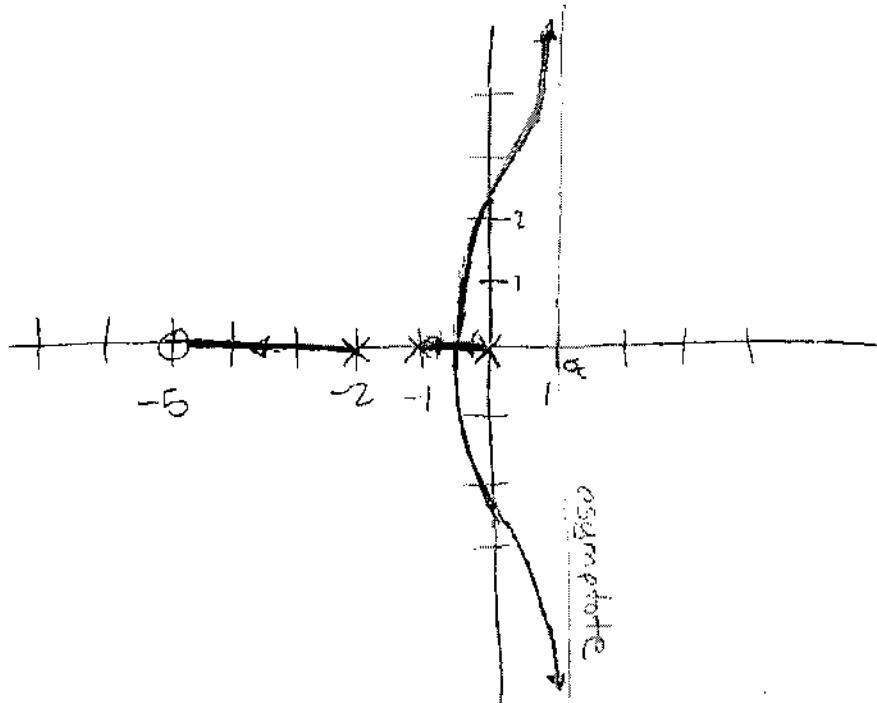
Solve for ω & K

$$-3\omega^2 + K = 0 \quad -\omega^2 + 2 + 0.2K = 0$$

$$\omega^2 = \frac{K}{3} \quad -\frac{K}{3} + 2 + 0.2K = 0$$

$$K = 0, \omega = 0 \quad K = 15 \quad \omega = \pm 2.24$$

Final plot



Range of K for stability comes from step ⑦ and looking at the direction of the root locus

$$[0 < K < 15]$$

for stability

Problem 3 SOLUTION - Root Locus

