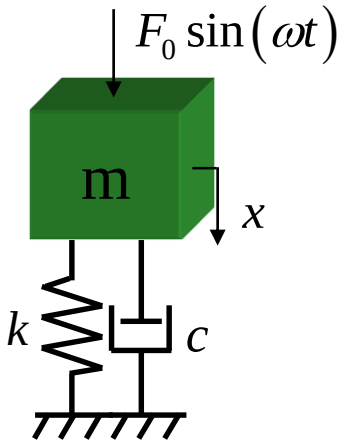
# Vibration Absorbers (Chapter 9, section 10)

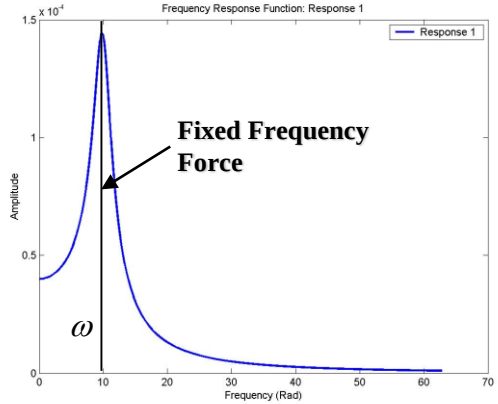
**Excessive vibration can be isolated or controlled**

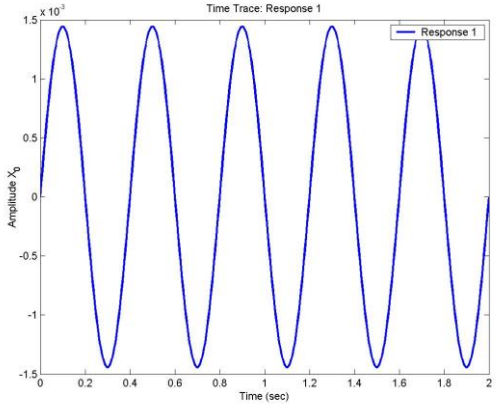
**Solutions:**

1.) Reduce forcing function
2.) Install isolation (Ch. 3, sec. 9.9)
3.) Move resonance (fixed frequency force)
4.) Add damping (moving or broadband frequency force)
5.) Vibration absorber

# Example: Fixed Frequency Force

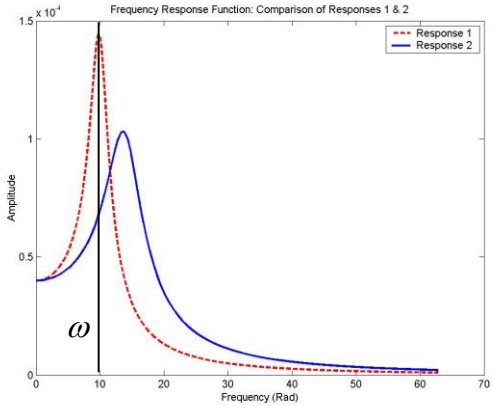


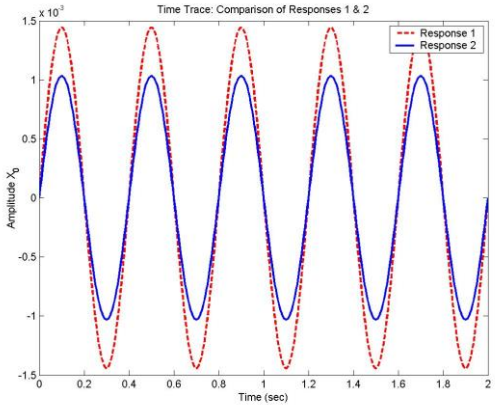




$X_m \sin(\omega t)$ $X_0$ = Amplitude (original)

**Move Resonance**



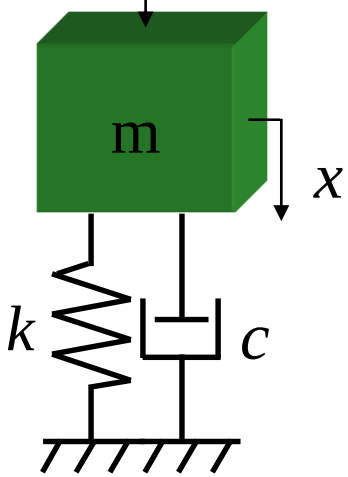

**Amplitude Reduction**

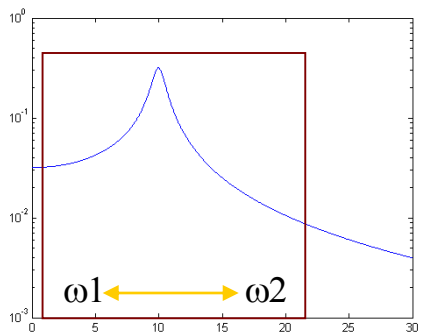


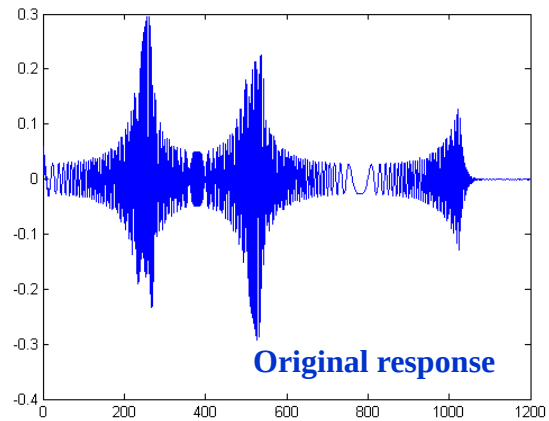

$X_m \sin(\omega t)$ $X_m$ = Amplitude (modified)

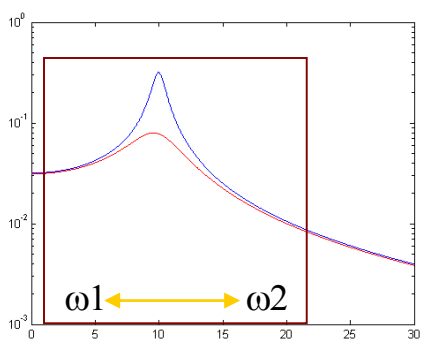
# Example: Moving or Broadband Force
# Effects of Adding Damping

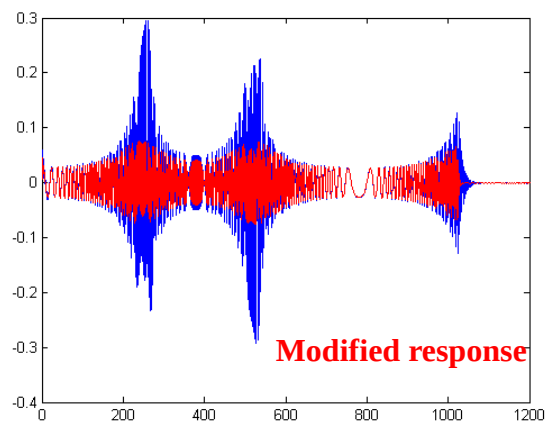
$$F_0 \sin\left(\left(\omega_1 \rightarrow \omega_2\right)t\right)$$



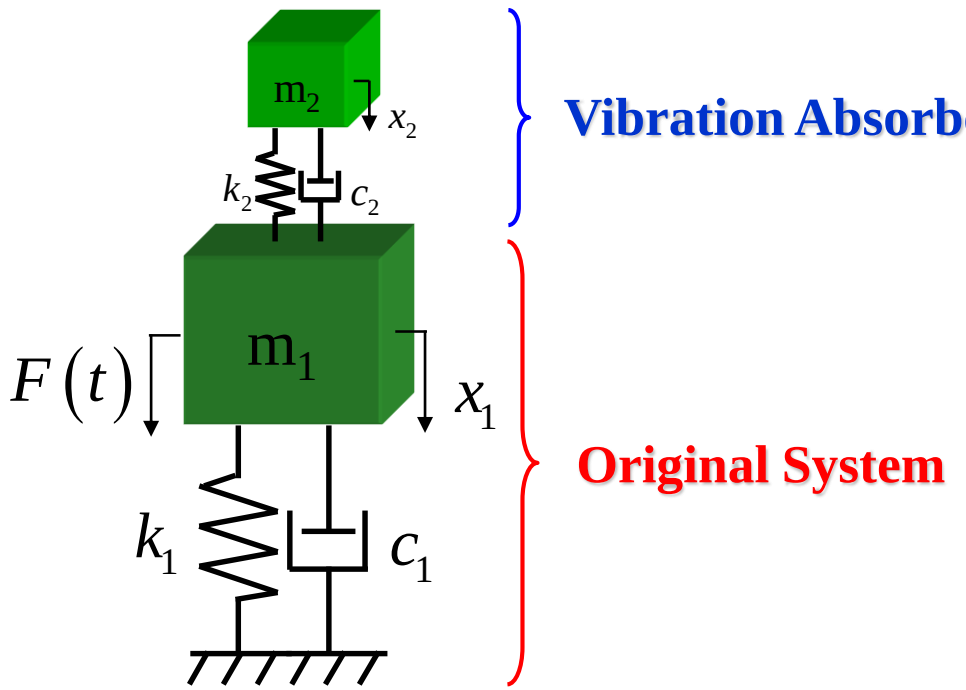



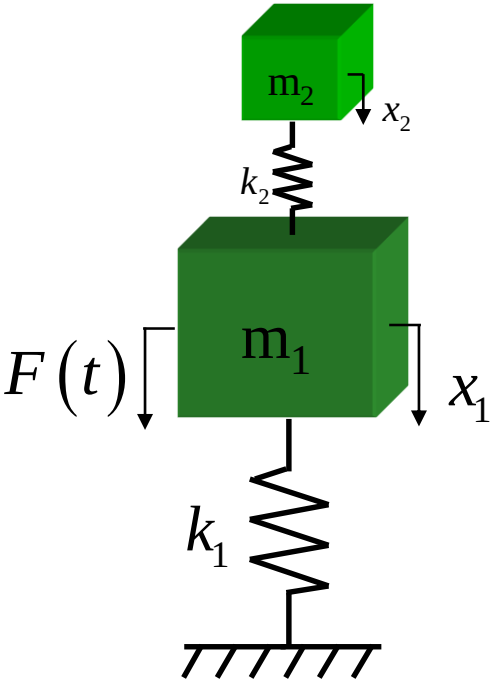
# Vibration Absorber (a.k.a Tuned Mass Damper)

1.) Reduces amplitude response at or near a resonance.

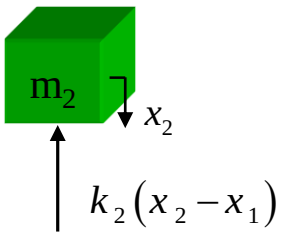
2.) Reduces amplitude response at a specific frequency.



# Vibration Absorber (undamped configuration)



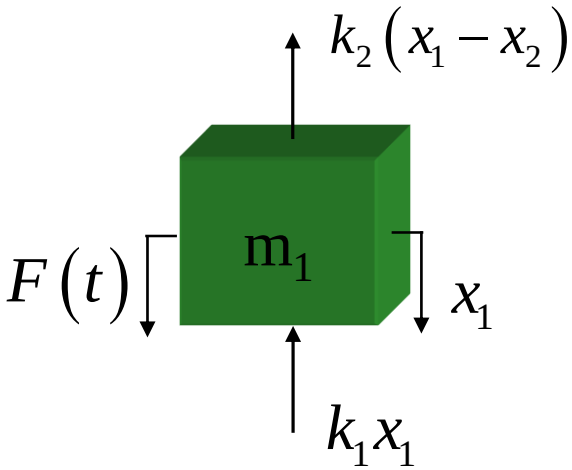

$$F(t) = F_0 \sin(\omega t)$$




$$\sum F_{x_2} = m_2 \ddot{x}_2$$

$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0$$

**FBD's**




$$\sum F_{x_1} = m_1 \ddot{x}_1$$

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = F_0 \sin(\omega t)$$

# Vibration Absorber (undamped configuration)

## Equations of Motion

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \sin(\omega t) \\ 0 \end{Bmatrix}$$

## Solution

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \sin(\omega t) \quad \Rightarrow$$

$$X_1 = \frac{\left(k_2 - m_2\omega^2\right) F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2}$$

$$X_2 = \frac{k_2 F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2}$$

# Vibration Absorber

$$X_1 = \frac{\left(k_2 - m_2\omega^2\right)F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2}$$

**Goal: Reduce amplitude $X_1$**

$$\therefore \quad \left(k_2 - m_2\omega^2\right) = 0 \quad \Longrightarrow \quad \omega^2 = \frac{k_2}{m_2}$$

**The natural frequency of the vibration absorber equals the frequency of the forcing function:**

$$F_0 \sin\left(\omega t\right)$$

# Vibration Absorber: Design for Resonance

$$\omega_2 = \sqrt{\frac{k_2}{m_2}} = \omega_1 = \sqrt{\frac{k_1}{m_1}}$$

$$\frac{X_1}{\delta_{st}} = \frac{1-\left(\frac{\omega}{\omega_2}\right)^2}{\left[1+\frac{k_2}{k_1}-\left(\frac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_2}\right)^2\right]-\frac{k_2}{k_1}}$$

**Define:** $\delta_{st} = \frac{F_0}{k_1}$

$$\frac{X_2}{\delta_{st}} = \frac{1}{\left[1+\frac{k_2}{k_1}-\left(\frac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_2}\right)^2\right]-\frac{k_2}{k_1}}$$

# Vibration Absorber: Design for Resonance

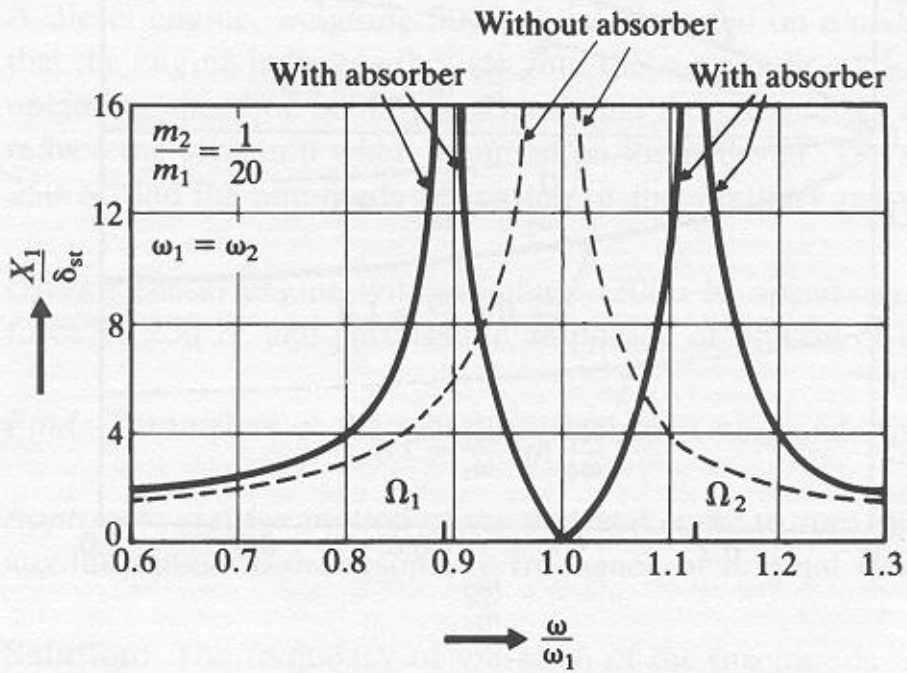



**FIGURE 9.25** Effect of undamped vibration absorber on the response of machine.

**$\Omega_1$ , $\Omega_2$ = natural frequency of the "new" combined system**

# Vibration Absorber: Amplitude of the Absorber

$$X_2 = \frac{k_2 F_0}{\left(k_1 + k_2 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2^2}$$

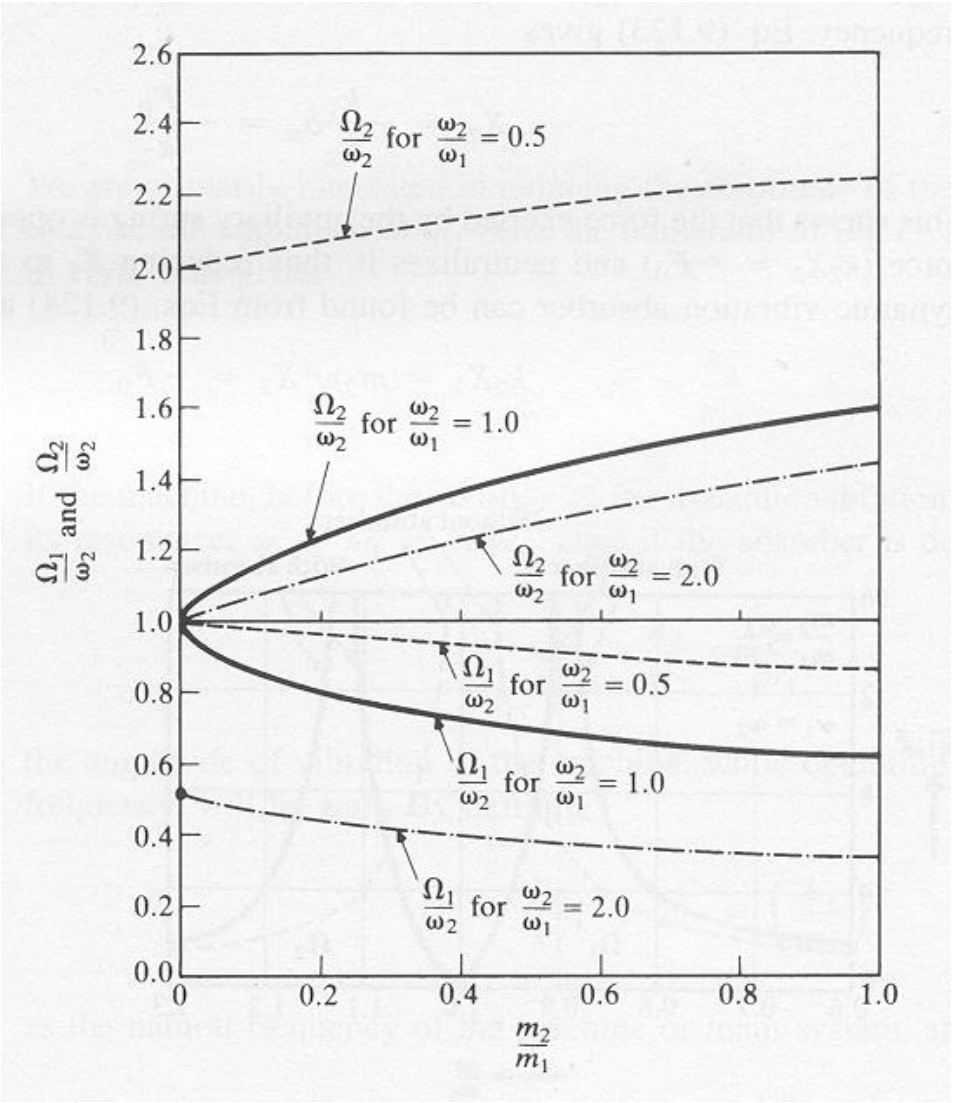
$$X_2 = -\frac{F_0}{K_2}$$

**Absorber design must accommodate this.**

# Vibration Absorber:

**Effect of** $\dfrac{\omega_2}{\omega_1}$

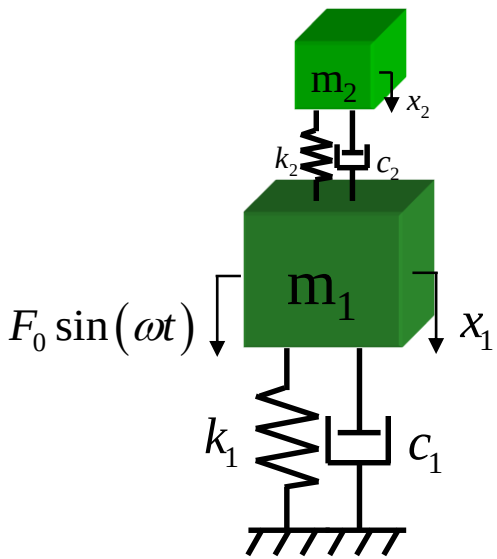
**Effect of the size of $m_2$**



# Vibration Absorber: Damped System

## Equations of Motion

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}+\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} F_0\sin(\omega t) \\ 0 \end{Bmatrix}$$




**Solution:**

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}e^{j\omega t}$$

# Vibration Absorber: Damped System

$$X_1 = \frac{F_0\left(k_2 - m_2\omega^2 + jc_2\omega\right)}{\left[\left(k_1 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - m_2k_2\omega^2\right] + j\omega c_2\left(k_1 - m_1\omega^2 - m_2\omega^2\right)}$$

$$X_2 = \frac{X_1\left(k_2 + j\omega c_2\right)}{\left(k_2 - m_2\omega^2 + j\omega c_2\right)}$$

# Vibration Absorber: Damped System

**By defining:**

$$\mu = \frac{m_2}{m_1} = \text{ Mass ratio } = \text{ Absorber mass / main mass}$$

$$\delta_{st} = \frac{F_0}{k_1} = \text{ Static deflection of the system}$$

$$\omega_a^2 = \frac{k_2}{m_2} = \text{ Square of the natural frequency of the absorber}$$

$$\omega_n^2 = \frac{k_1}{m_1} = \text{ Square of the natural frequency of the main mass}$$

$$f = \frac{\omega_a}{\omega_n} = \text{ Ratio of natural frequencies}$$

$$g = \frac{\omega}{\omega_n} = \text{ Forced frequency ratio}$$

$$c_c = 2m_2\omega_a = \text{ Critical damping constant}$$

$$\zeta = \frac{c_2}{c_c} = \text{ Damping ratio}$$

# Vibration Absorber: Damped System

**Solution:**

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} e^{j\omega t}$$

**The magnitudes, $X_1$ and $X_2$, can be expressed as:**

$$\frac{X_1}{\delta_{st}} = \left[ \frac{(2\zeta g)^2 + \left(g^2 - f^2\right)^2}{(2\zeta g)^2 \left(g^2 - 1 + \mu g^2\right)^2 + \left\{\mu f^2 g^2 - \left(g^2 - 1\right)\left(g^2 - f^2\right)\right\}^2} \right]^{1/2}$$

$$\frac{X_2}{\delta_{st}} = \left[ \frac{(2\zeta g)^2 + f^4}{(2\zeta g)^2 \left(g^2 - 1 + \mu g^2\right)^2 + \left\{\mu f^2 g^2 - \left(g^2 - 1\right)\left(g^2 - f^2\right)\right\}^2} \right]^{1/2}$$

# Vibration Absorber: Damped System

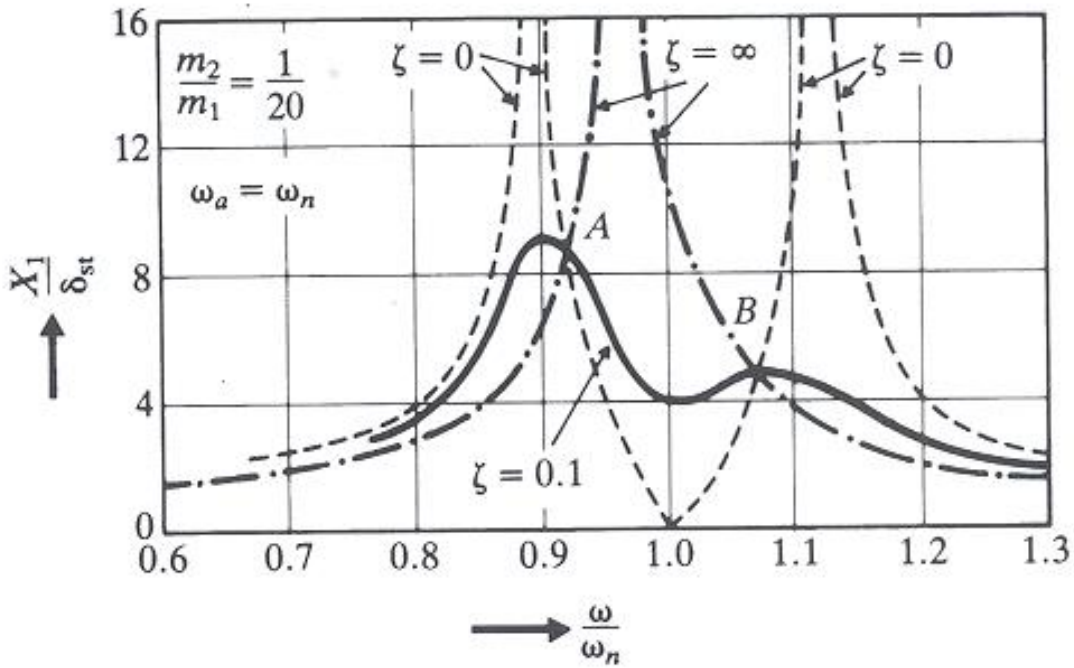
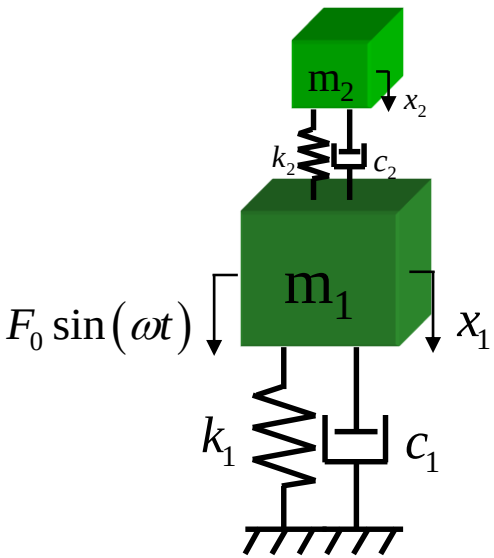



**FIGURE 9.29** Effect of damped vibration absorber on the response of the machine.

**Example:**

$$\omega_a = \omega_n = \sqrt{\frac{k_1}{m_1}}$$




$$m_1 = 10 \qquad m_2 = 2$$

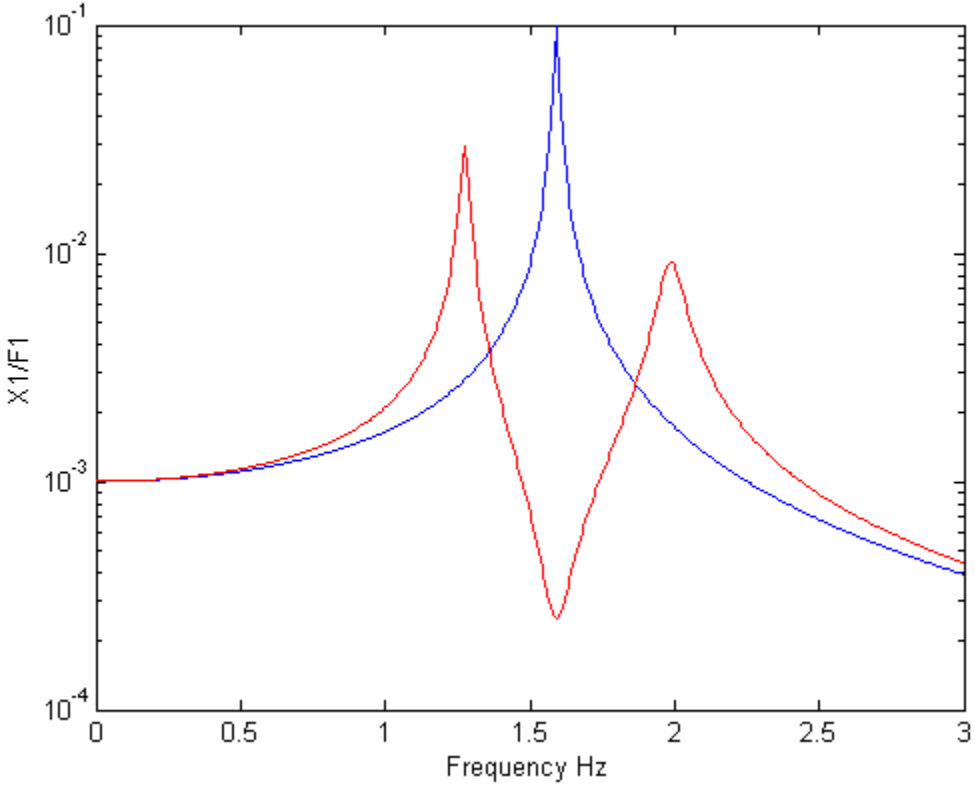
$$k_1 = 1000 \qquad k_2 = 200$$

$$c_1 = 1 \qquad c_2 = 1$$

$$\begin{bmatrix} 10 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 1200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \sin(\omega t) \\ 0 \end{Bmatrix}$$

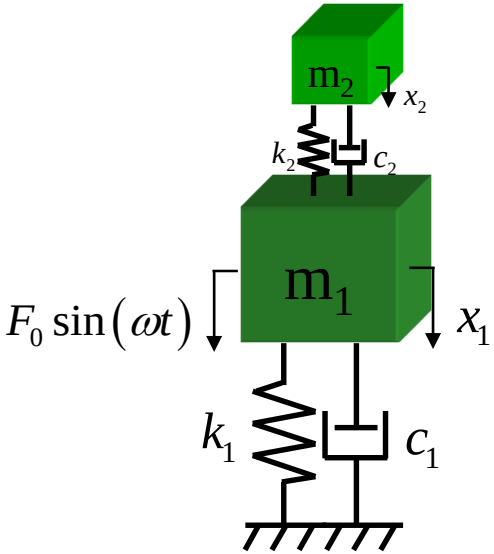
**Example:**

$$\omega_a = \omega_n = \sqrt{\frac{k_1}{m_1}}$$



**Example:**

$$\omega_a = 1.75Hz$$




$$m_1 = 10 \qquad m_2 = 2$$

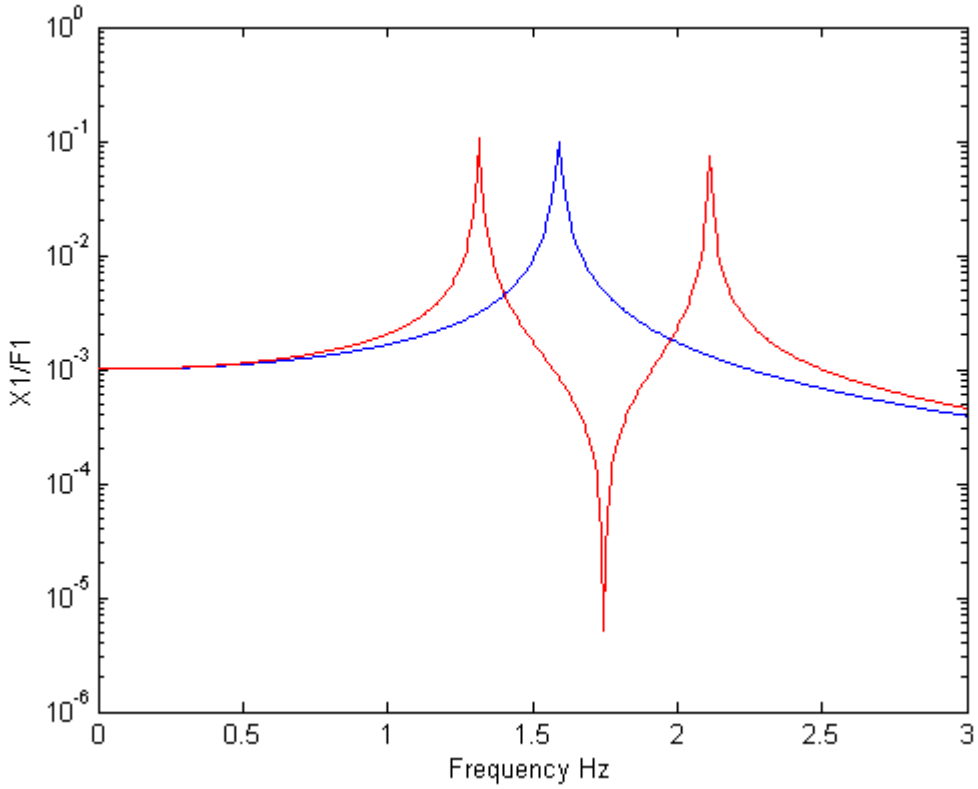
$$k_1 = 1000 \qquad k_2 = 241$$

$$c_1 = 1 \qquad c_2 = 0$$

$$\begin{bmatrix} 10 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 1241 & -241 \\ -241 & 241 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \sin(\omega t) \\ 0 \end{Bmatrix}$$

**Example:**

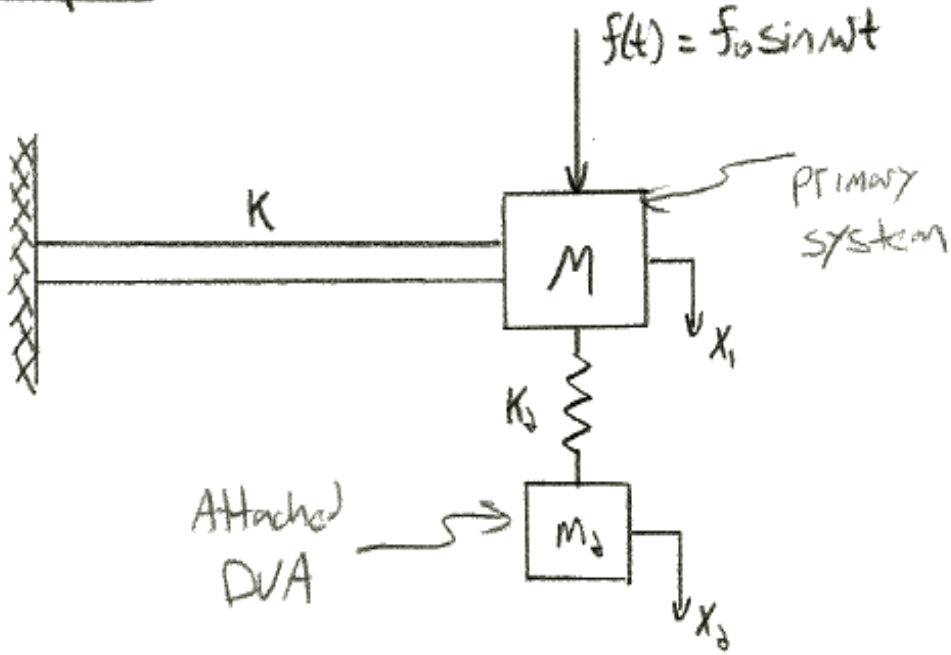
$$\omega_a = 1.75Hz$$



# Applications of Tuned Mass Damper

- **Automotive** (Exhaust, Steering & Suspension Systems, Rear view mirror, airbag, battery, etc.)
- **Appliances** (Washers, Dryers, Dishwasher, Refrigerators, etc.)
- **Machine Tools** (Lathes, Milling Machines, Grinders, etc.)
- **Buildings** (John Hancock Tower, Citicorp Center, Canadian National Tower)
- **Aircraft & Spacecraft** (Turbines, Inlet & Exhaust, accessory component motors, etc.)

## Example (Dynamic Vibration Absorber)




Data: $M = 250\ Kg$

$K = 2.5 \times 10^6\ N/m$

$f_0 = 250\ N$

Choose $m_2 = 25\ Kg$

↳ $K_2$ automatically prescribed.

DVA tuning: $\omega_0^2 = \frac{K_2}{m_2} \Rightarrow K_2 = m_2 \omega_0^2$

$$K_2 = (25\,kg)(105\,rad/s)^2 = 2.756 \times 10^5\ kg/s^2$$

If $K_2 = m_2 \omega_0^2$ then $X_2 = -\frac{f_0}{K_2}$

$$\therefore |X_2| = \frac{(250\,N)}{(2.756 \times 10^5\ kg/s^2)} \left(\frac{1000\,mm}{1\,m}\right) = 0.907\ mm$$

$$(\omega_n)_{primary} = \sqrt{\frac{K}{M}} = 100\ rad/s$$

Excitation frequency:

$\omega$ = 105 rad/s

Natural frequency of original system:

$\omega_{10}$ = 100. rad/s

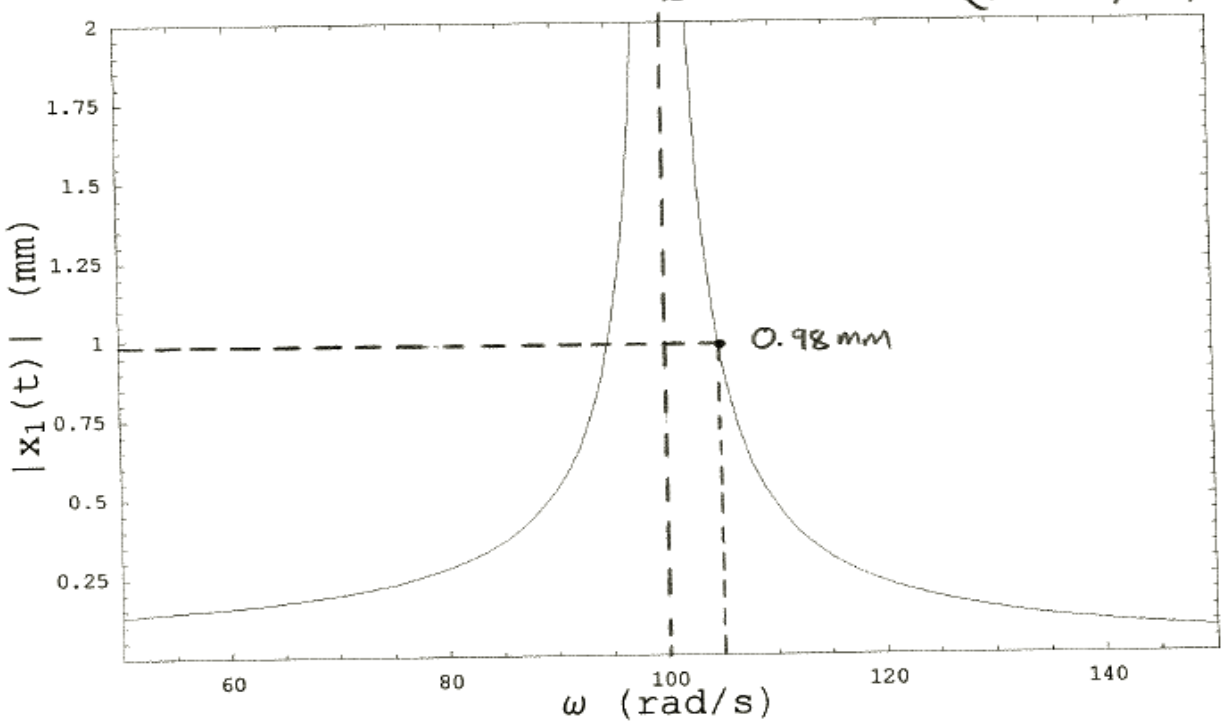
Natural frequencies of composite system:

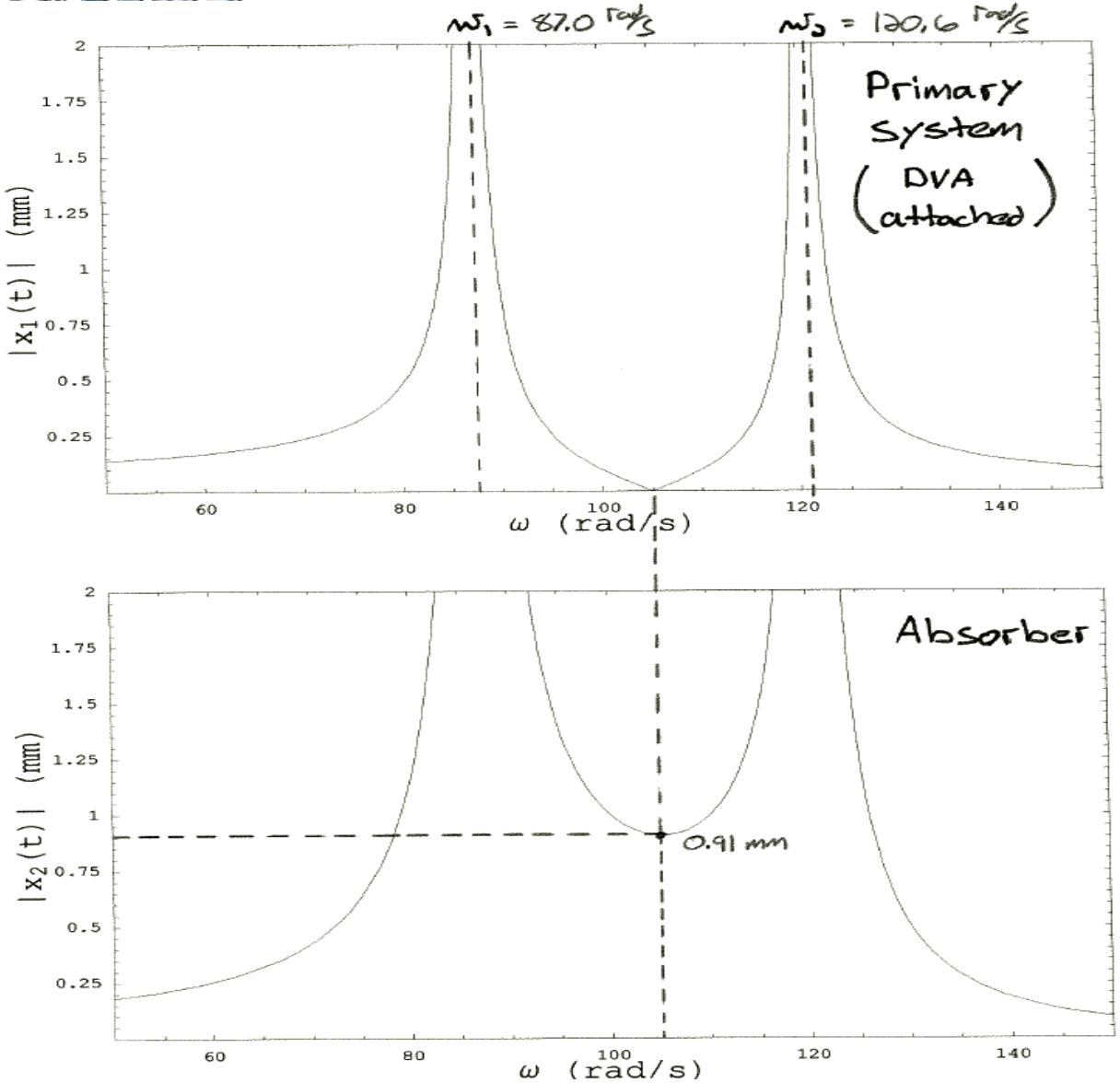
$\omega_1$ = 87.0467 rad/s

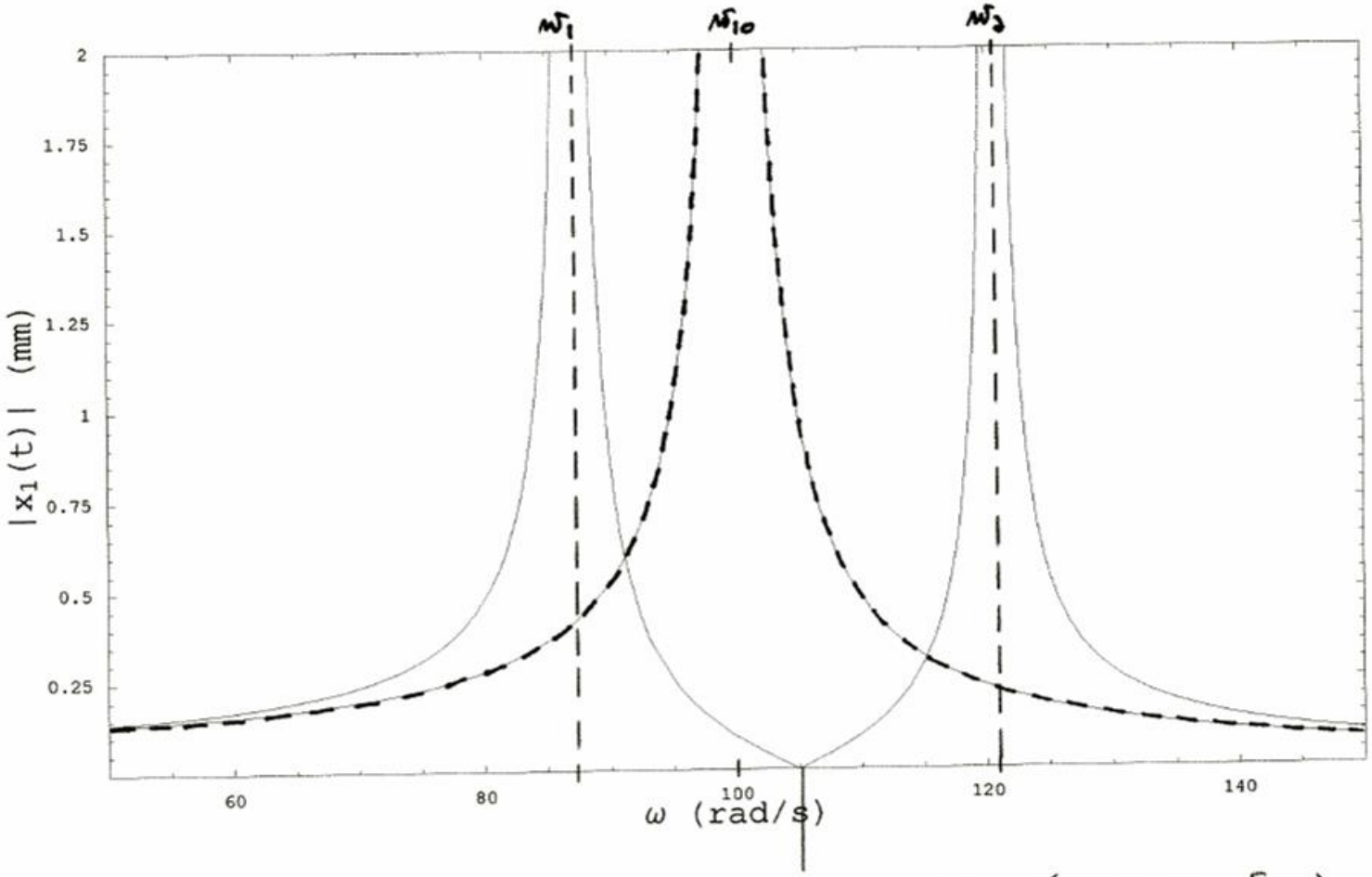
$\omega_2$ = 120.625 rad/s

Primary System before absorber is attached.

$\omega_{10}$ = 100 rad/s (Natural frequency of Primary system)








$\omega = 105$ rad/s (excitation freq)

$\omega_{10} = 100$ rad/s

$\omega_1 = 87.0$ rad/s

$\omega_2 = 120.6$ rad/s

# Summary

## 2 DOF Systems
## Forced Harmonic Vibration of Undamped Systems

**STEP 1.** Derive EOM. Cast into the matrix-vector form[1]

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \begin{cases} \mathbf{f}\sin\omega t \\ \mathbf{f}\cos\omega t \end{cases},$$

where $\mathbf{x}(t) = (x_1(t), x_2(t))^\top$, $\mathbf{f} = (f_{1o}, f_{2o})^\top$, and $\omega$ is the excitation, or driving frequ

**STEP 2.** Try the steady-state (particular) solution

$$\mathbf{x}_{ss}(t) = \begin{cases} \mathbf{u}\sin\omega t \\ \mathbf{u}\cos\omega t \end{cases},$$

where $\mathbf{u} = (X_1, X_2)^\top$. Map trial solution into the EOM. This yields

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{u} = \mathbf{f},$$

or

$$\mathbf{Z}\mathbf{u} = \mathbf{f},$$

where $\mathbf{Z} = (\mathbf{K} - \omega^2\mathbf{M})$ is the *impedance matrix*.

**STEP 3.** Invert the impedance matrix. The unknown vector $\mathbf{u}$ is then given by

$$\mathbf{u} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \mathbf{Z}^{-1}\mathbf{f} = \frac{1}{\det(\mathbf{Z})}\mathrm{adj}(\mathbf{Z})\mathbf{f}, \tag{5}$$

where $\mathbf{Z}^{-1} = \frac{\mathrm{adj}(\mathbf{Z})}{\det(\mathbf{Z})}$ is the inverse of $\mathbf{Z}$.

**STEP 4.** Write the steady-state response. It is given by Equation (2), where $\mathbf{u}$ is given by Equation (5). The result is

$$\mathbf{x}_{ss}(t) = \frac{1}{\det(\mathbf{Z})}\mathrm{adj}(\mathbf{Z})\mathbf{f}\sin\omega t = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}\sin\omega t,$$

or

$$\mathbf{x}_{ss}(t) = \frac{1}{\det(\mathbf{Z})}\mathrm{adj}(\mathbf{Z})\mathbf{f}\cos\omega t = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}\cos\omega t.$$