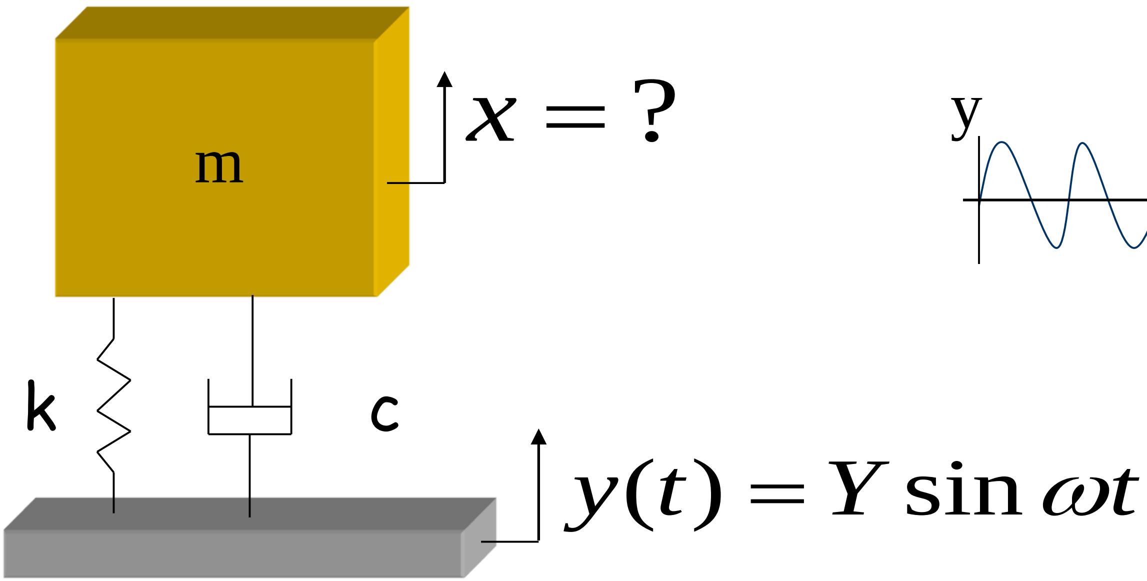
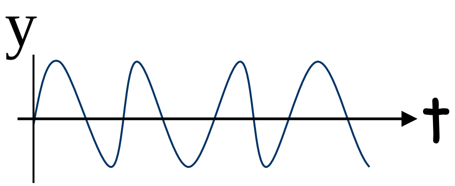
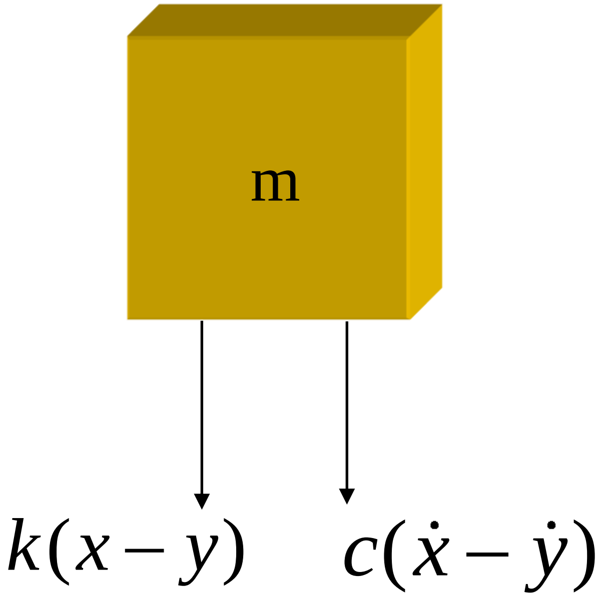
Base Excitation (Displacement Transmissibility) Section 3.6 in text





1. What is the absolute motion of the mass (x)?
2. What is the relative motion between the mass and the base?
3. How much force is transmitted to the mass?



absolute motion X

$$\sum F_x = m\ddot{x}$$

$$-k(x-y)-c(\dot{x}-\dot{y}) = m\ddot{x}$$

$$m\ddot{x}+c\dot{x}+kx = c\dot{y}+ky$$

Base Excitation

$$m\ddot{x}+c\dot{x}+kx=c\dot{y}+ky \qquad y=Y\sin(\omega t)$$

$$m\ddot{x}+c\dot{x}+kx=c\omega Y\cos(\omega t)+kY\sin(\omega t)$$

$$m\ddot{x}+c\dot{x}+kx=A\sin(\omega t-\alpha)$$

where $A=Y\sqrt{k^2+(c\omega)^2}$ $\qquad \alpha=\tan^{-1}\left(-\frac{c\omega}{k}\right)$

Base excitation is equivalent to applying a force of magnitude A to the mass.

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Base Excitation

$$m\ddot{x} + c\dot{x} + kx = A\sin(\omega t - \alpha)$$

The steady state solution $x_{ss}(t)$ $\left(x_p(t) \text{ in textbook}\right)$

Absolute motion of mass → $x_{ss}(t) = X\sin(\omega t - \alpha - \phi_1)$

$$X = \frac{A}{\sqrt{\left(k - m\omega^2\right)^2 + (c\omega)^2}} \qquad \phi_1 = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)$$

Base Excitation

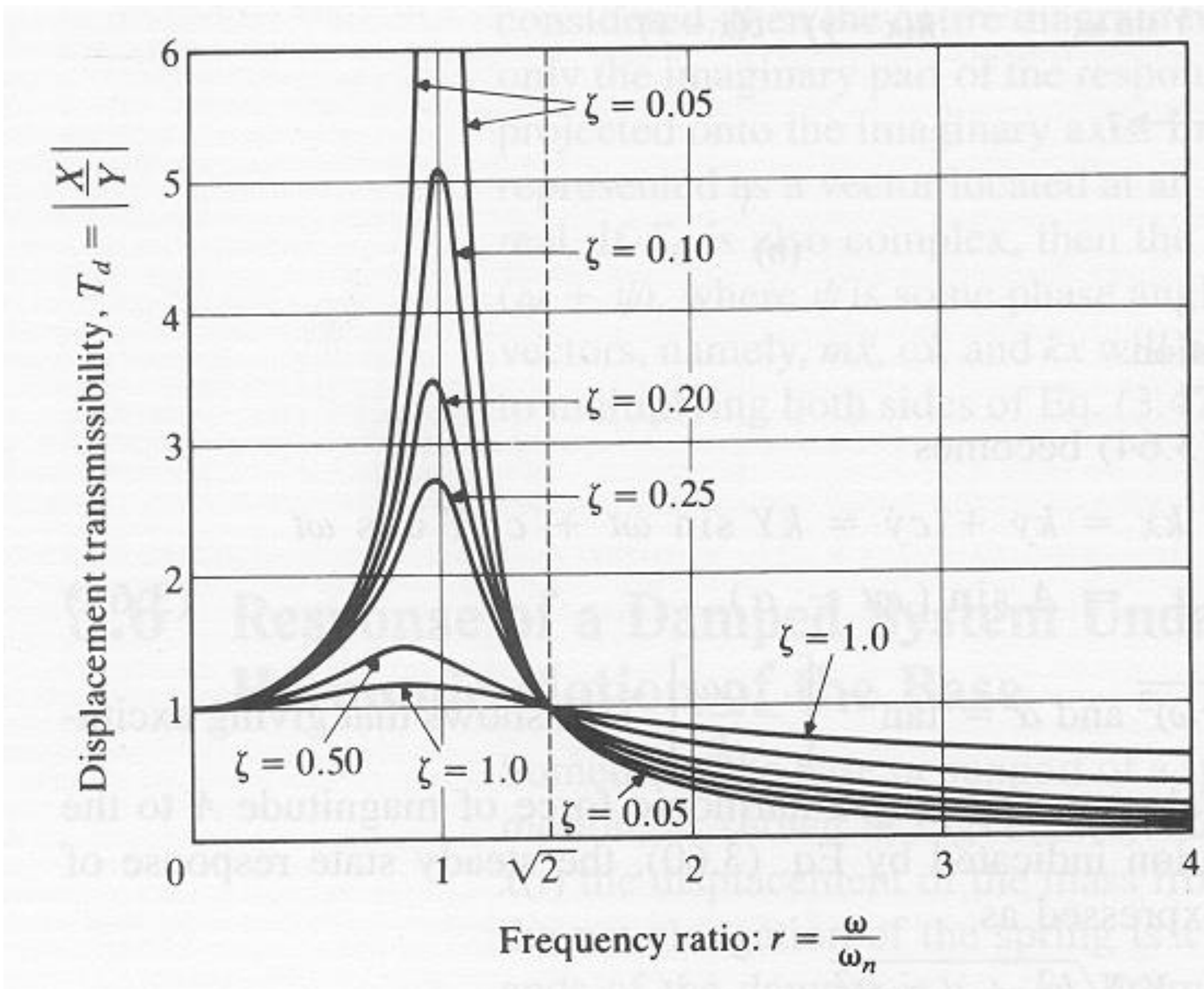
$$x_{ss}(t) = X\sin(\omega t - \alpha - \phi_1) = X\sin(\omega t - \phi)$$

$$\frac{X}{Y} \equiv \text{ Displacement Transmissibility}$$

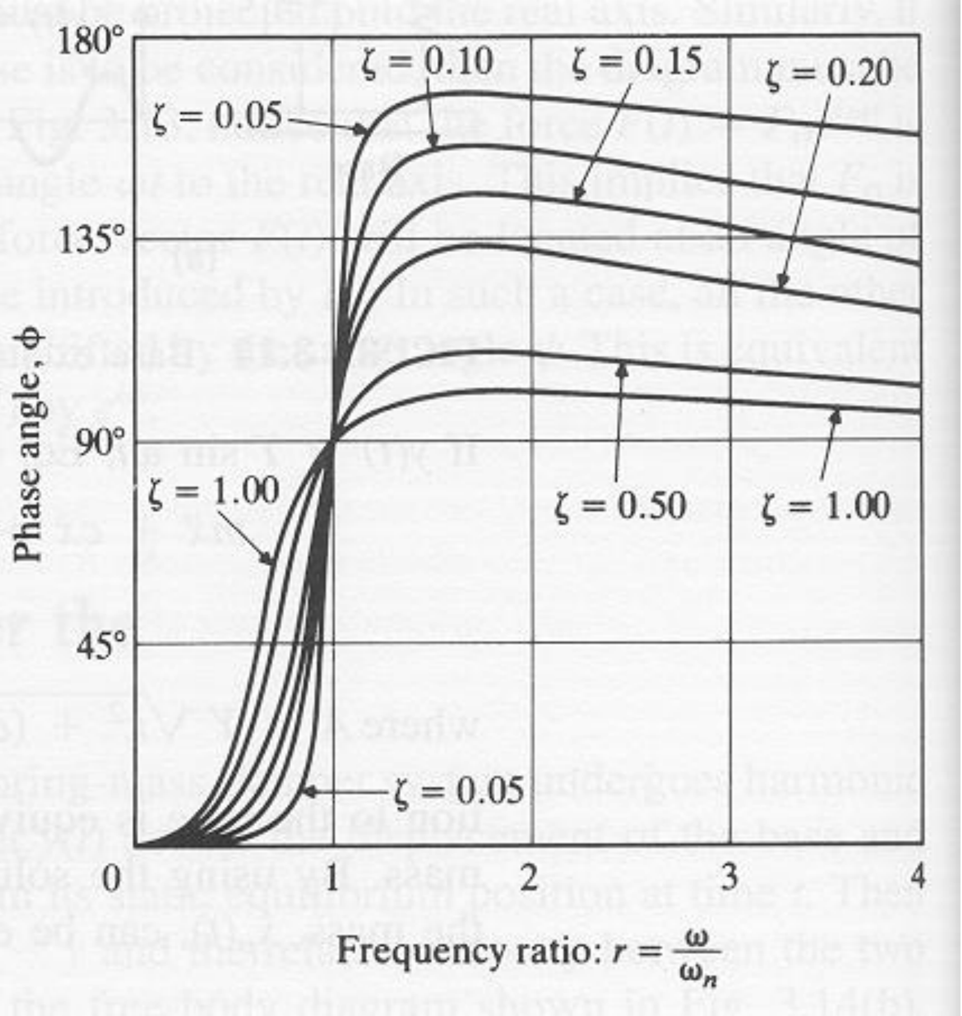
$$\frac{X}{Y} = \left[\frac{k^2 + (c\omega)^2}{\left(k - m\omega^2\right)^2 + (c\omega)^2}\right]^{1/2} = \left[\frac{1 + (2\zeta r)^2}{\left(1 - r^2\right)^2 + (2\zeta r)^2}\right]^{1/2}$$

$$\phi = \tan^{-1}\left(\frac{mc\omega^3}{k\left(k - m\omega^2\right) + (c\omega)^2}\right) = \tan^{-1}\left(\frac{2\zeta r^3}{1 + \left(4\zeta^2 - 1\right)r^2}\right)$$

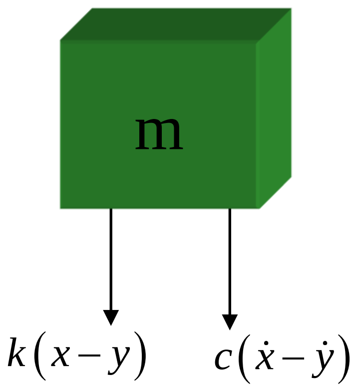
Displacement Transmissibility



Displacement Transmissibility (Phase)



Base Excitation – Force Transmitted to Mass



$$F = k(x-y) + c(\dot{x}-\dot{y})$$

For Dynamic Equilibrium:

$$k(x-y) + c(\dot{x}-\dot{y}) = -m\ddot{x}$$

Force Transmitted to Mass $= \; -m\ddot{x}$

For Steady State Base Excitation

$$\ddot{x}_{ss}(t) = -\omega^2 X \sin(\omega t - \phi)$$

$$F = m\omega^2 X \sin(\omega t - \phi)$$

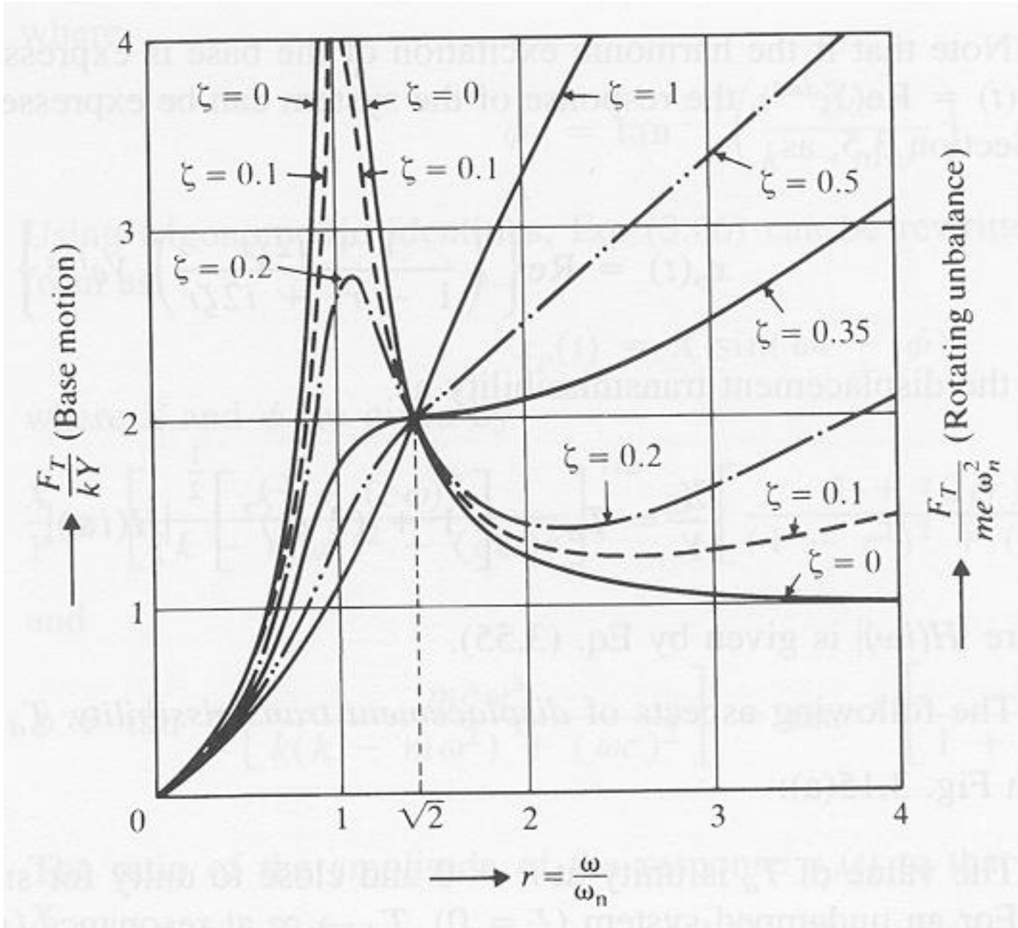
Base Excitation – Force Transmitted to Mass

$$F = m\omega^2 X \sin(\omega t - \phi) = F_T \sin(\omega t - \phi)$$

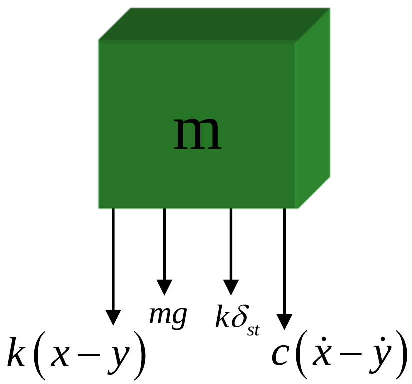
F_T = magnitude of transmitted force due to base excitation

$$\frac{F_T}{kY} = r^2 \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$

Base Excitation – Force Transmitted to Mass



Base Excitation – Relative Motion



$$z = x - y$$

$$\Sigma Fx = m\ddot{x}$$

$$-kz - c\dot{z} = m\ddot{x} = m(\ddot{z} + \ddot{y})$$

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y}$$

$$y = Y\sin(\omega t)$$

$$mz + cz + kz = m\omega^2 Y\sin(\omega t)$$

Base Excitation – Relative Motion

$$m\ddot{z} + c\dot{z} + kz = m\omega^2 Y \sin(\omega t)$$

$$z_{ss}(t) = \frac{m\omega^2 Y}{\left[\left(k - m\omega^2\right)^2 + (c\omega)^2\right]^{1/2}} \sin(\omega t - \phi_1)$$

$$z_{ss}(t) = Z \sin(\omega t - \phi_1) \qquad \phi_1 = \tan^{-1}\left(\frac{2\zeta r}{\left(1 - r^2\right)}\right)$$

$$\frac{Z}{Y} = \frac{m\omega^2}{\left[\left(k - m\omega^2\right)^2 + (c\omega)^2\right]^{1/2}} = \frac{r^2}{\left[\left(1 - r^2\right)^2 + (2\zeta r)^2\right]^{1/2}}$$

Base Excitation – Relative Motion